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FÜR PLASMAPHYSIK

Present status of the EUTERPE code

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Introduction

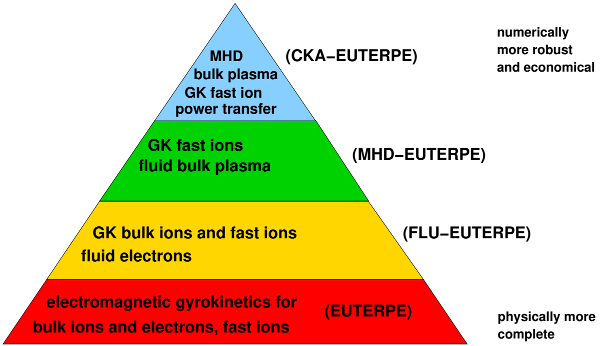
- Interest in global (full-volume) gyrokinetic particle-in-cell simulations for stellarators.
- Three main areas of activity:
  - (Micro) instabilities and turbulence
  - MHD modes and their interaction with fast particles
  - Global neoclassics
- MHD modes provide an excellent testing ground for codes since a very high quality of numerics is required.
- Fully gyrokinetic simulations are very time-consuming and difficult.
  Simplified models sacrifice physics details for gain in speed.

General features of EUTERPE

- δf particle-in-cell code
- global simulation domain: full-volume for 3D stellarator equilibria
- multiple kinetic species (ions, electrons, fast ions/impurities)
- linear (with phase factor) / nonlinear
- electrostatic/electromagnetic (includes $\delta B_{||}$)
- pullback mitigation scheme
- arbitrary wavelength (Padé approximation)
- linearised collision operators with moment conservation (e.g. pitch angle, slowing down)
- multiple distribution functions (Maxwellian, slowing-down, ...)

Hierarchy of models

EUTERPE, FLU-EUTERPE, MHD-EUTERPE, CKA-EUTERPE



CKA: Code for Kinetic Alfvén waves

Full electromagnetic gyrokinetic model

All equations are derived from a Lagrangian via a variational principle
using the mixed formulation $A_\| = A_\|^s + A_\|^h$

$$\mathcal{L}_{\mathrm{mixed}} = \sum_s \left\{ \int f_s \left[q_s \vec{A}^* \cdot \dot{\vec{R}} + \frac{m_s^2}{q_s}\mu\dot{\alpha} - \frac{m_s}{2}u_\|^2 - \mu B - q_s \left(\langle \Phi \rangle - u_\| \langle A_\|^h \rangle \right) + \right.\right.$$
$$\left.\left. - \frac{q_s^2}{2m_s}\langle A_\|^h \rangle^2 \right] \mathrm{d}^6 \tilde{Z}_0 + \int \frac{m_s n_{0,s}}{2B^2} (\nabla_\perp \Phi)^2 \mathrm{d}\tilde{V} \right\} - \frac{1}{2\mu_0} \int (\nabla_\perp A_\|)^2 \, \mathrm{d}\tilde{V} +$$
$$+ \int \lambda \left(\frac{\partial A_\|^s}{\partial t} + \vec{b} \cdot \nabla \Phi \right) \mathrm{d}\tilde{V}$$

Simple $\delta B_\|$ treatment added on top:

$$-\frac{B}{\beta}\delta B_\| = \sum_s m_s \int \mu B \delta f_s dW$$

Simple non-ideal electron fluid model

Electron density

$$\frac{\partial n_\mathrm{e}}{\partial t} + \nabla \cdot \left[n_{0,\mathrm{e}} u_{\|,\mathrm{e}} \vec{b} + n_{0,\mathrm{e}} \frac{\vec{b} \times \nabla \phi}{B} - \frac{j_{\|,0}}{q_\mathrm{e}} \frac{\vec{b} \times \nabla A_\|}{B} + \right.$$
$$\left. + \frac{j_{\|,0}}{q_\mathrm{e} B} \vec{b} \times \vec{\kappa} A_\| + \frac{2}{q_\mathrm{e} B} \left(\vec{b} \times \vec{\kappa} - \alpha \frac{(\nabla \times B)_\perp}{B} \right) p_\mathrm{e} \right] = 0$$

Parallel electron velocity

$$u_{\|,\mathrm{e}} = -\frac{1}{q_\mathrm{e} n_{0,\mathrm{e}}} \left(\sum_{s=\mathrm{i,f}} \langle j_{\|,s} \rangle + \frac{1}{\mu_0} \nabla \cdot \nabla_\perp A_\| \right)$$

Non-ideal Ohm law

$$\frac{\partial}{\partial t} \left(A_\| - d_\mathrm{e}^2 \nabla \cdot \nabla_\perp A_\| \right) = -\vec{b} \cdot \nabla \phi + \frac{\eta}{\mu_0} \nabla \cdot \nabla_\perp A_\| .$$

Simple pressure closure

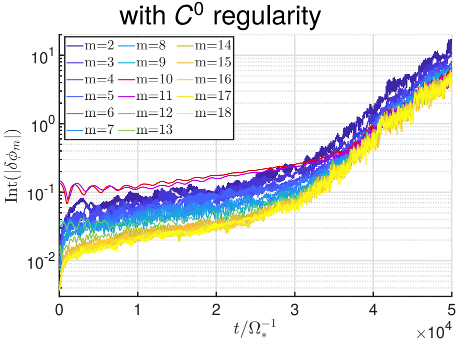
$$\frac{\partial p_\mathrm{e}}{\partial t} = -\vec{v}_{E\times B} \cdot \nabla p_{0,\mathrm{e}} + D_\mathrm{p} \nabla \cdot \nabla_\perp p_\mathrm{e}$$

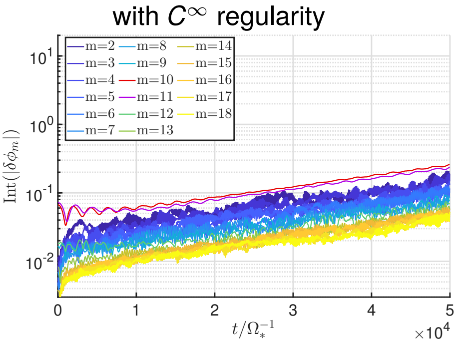
Smooth polar splines

- regular functions must behave $\sim r^{|m|}$ near the axis ($r = 0$)
- standard tensor product splines guarantee only continuity (C^0)
- constructed new spline basis near the origin (Jiang 2026)
- new basis has correct regularity for a finite grid for $0 \leq m \leq p$ (with p the spline order)
- leads to much better numerical behaviour, for low m modes
- easy to implement in existing codes (in contrast to Toshniwal splines), just matrix projection to regularity subspace

Smooth polar splines

Example: tokamak TAE eigenmode (ITPA) without/with smooth polar splines
low *m* modes are numerical unstable without regularity





(Jiang 2026)

Global neoclassics

- classical neoclassic is local, i.e. particles can not leave flux surface
- E_r determined by ambipolarity: $\Gamma_i = \Gamma_e$ (only works for flux surface averaged E_r)
- electron-ion root transition region can not be calculated with local theory, gives discontinuity (Maxwell construction)
- transition width important for possible turbulence stabilisation
- neoclassical extension of EUTERPE (M.D.Kuczyński 2024)

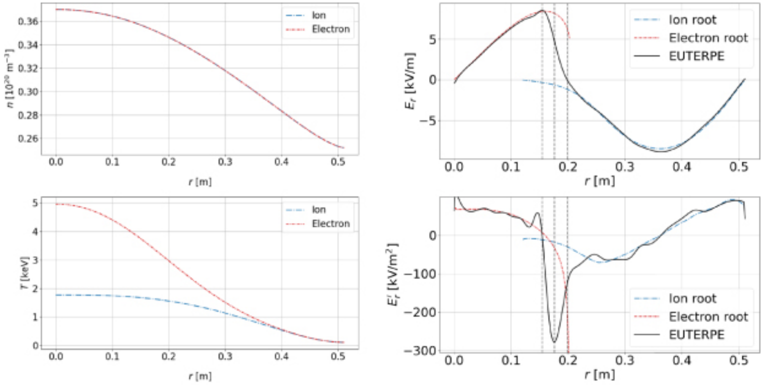
$$\frac{\mathrm{d}\delta f}{\mathrm{d}t} = -\frac{\mathrm{d}^{(1)}}{\mathrm{d}t} f^{(0)} - \frac{\mathrm{d}^{(0)}}{\mathrm{d}t} f^{(0)} + C(\delta f) \tag{1}$$

- replace ambipolarity by quasineutrality, works for general E_r

$$\alpha \nabla_\perp^2 \phi = n_i - n_e \tag{2}$$

- runs are very time consuming $O(10^5 - 10^6)$ CPUh
- develop cheaper alternatives: e.g. non-local ions, local electrons (newly developed CHIMAERA code, A.Runov) to investigate e.g. hysteresis effects

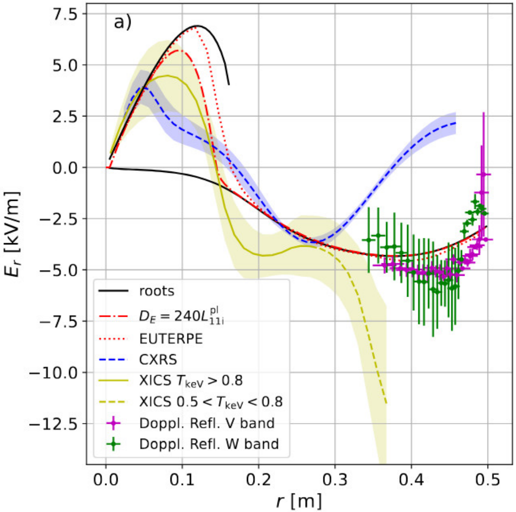
Global neoclassics



(a) (b)

(M.D.Kuczyński 2024)

Global neoclassics, W7-X discharge 20250402.03



(H.M.Smith submitted)

Island model

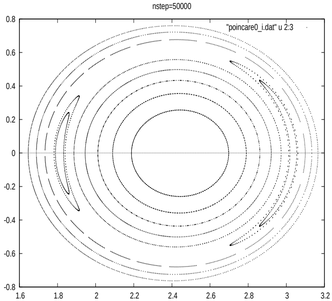
Including islands from e.g. HINT solution requires irregular grid.

- add ad-hoc island model on top of existing regular grid

$$B_{isl} = \nabla \times (\lambda B_0), \quad \lambda = A \cos\left[m(\theta - \theta_0) - n\phi\right] \tag{3}$$

- neglect many island related terms in the equations of motion

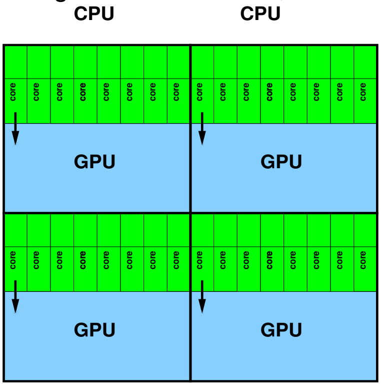
Poincaré plot for tokamak



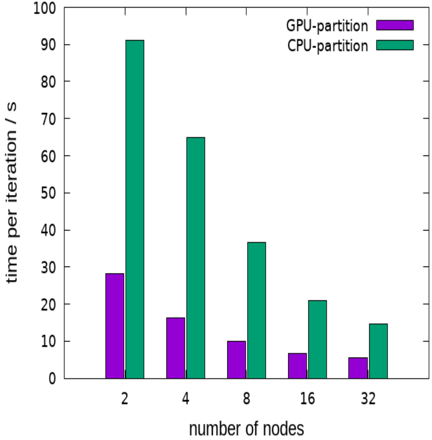
- neoclassics
- Alfvén modes
- turbulence

GPU version for Nvidia

particles on GPU, field solver on CPU
Pitagora node: 64 core, 4 GPU



speed-up of around three



Other extensions

- multi mode version of CKA-EUTERPE
- non up-down symmetric tokamak equilibrium
- mapping for CHEASE input files
- exponential integrators if phase factor is used
- nonlinear terms in the equations of motion
- full 3D field output
- Julia GUI tools for standard diagnostics

Planned/running projects

- field line aligned coordinates
  should reduce number of necessary grid points considerably
- include regularity splines for Fourier solver
- extend simulation domain to $r > 1$ (but neglect ergodic region and divertor)
  important for modes localised near the boundary, islands near boundary
- antenna version
- arbitrary number of species
- extend fluid model to include GAM terms
- solve current numerical problems for neoclassics + turbulence
- version for AMD GPU currently in implementation
- big refactoring to simplify and improve code structure