MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

ORB5: distribution functions

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EUROfusion

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Energetic Particles

Energetic Particles (EPs) are **supra**-thermal plasma species
Low density, low collisionality, moderate pressure
Sources:

- Alpha particles: born isotropically with E=3.5 MeV
- NBI particles: born **an**isotropically with E_{birth} (e.g. 93 keV, 1 MeV)
  - "PINI" (most present day machines): components E_{birth}, $E_{\mathrm{birth}}/2$, $E_{\mathrm{birth}}/3$
  - "NINI" (e.g. ITER): only E_{birth}
- Alpha + NBI: slow down from birth to thermal (collisions with electrons)
- ICRH, energy pumped into cyclotron resonance $\rightarrow$ energy into $v_{\perp}$ (μ)

Energetic Particles & instabilities

- AEs (e.g. Toroidal AE (TAE)) are a global problem, existence comes from profile variation and $k_{||}$ matching of Alfvén dispersion relation.
- TAE drive comes from radial pressure gradient of EPs
- TAE damping is mixture of non-local and local (radiative, continuum, and electron & ion Landau damping)

- EGAMs, (also GAE), can exist with n=0.
- Drive is from the **anistropy** of the EP distribution
- Sources of anisotropy/velocity space gradients:
  - Turn on a beam, initially “bump-on-tail”. This effect ends after $t_{\mathrm{slowingdown}}$
  - after this, dynamic equilibrium reached
- NBI beams have preferred pitch (geometric), injection anisotropic
- ICRH “pulls out” tails of distribution
- isotropic $\rightarrow$ anisotropic possible due to losses

ORB5

"ORB5: a global electromagnetic gyrokinetic code using the PIC approach in toroidal geometry" [for details, see Lanti 2020]

- delta-f modified distribution function discretized with PIC
- Fields solved using finite elements
  - Filter applied in toroidal and poloidal mode numbers
- Global electromagnetic (EM) simulations a difficult problem: small $k_\perp$ (e.g. MHD) particularly challenging for high beta.
  - Effectively mitigates with the so-called cancellation problem using the pullback scheme [Mishchenko 2019] (leads to an order of magn. increase of time step)
- ES: adiabatic, hybrid, or kinetic electrons, EM drift-kinetic electrons (or fluid)
- Previously used for turbulence studies as well as EP physics (separately, and interaction)

ORB5

ORB5's Vlasov equation (shown electrostatically, absence of collisions/sources)

$$\frac{d\delta f}{dt} = -\frac{df_0}{dt}$$

Total (Lagrangian) derivative (for plasma species s)

$$\frac{df_s}{dt} = \frac{\partial f_s}{\partial t} + \frac{\partial f_s}{\partial \mathbf{R}} \cdot \dot{\mathbf{R}} + \frac{\partial f_s}{\partial v_\parallel} \dot{v}_\parallel = 0$$

full-f Vlasov equation, requires evaluation of background f_0 and its gradients at particle positions

Note: No assumption (here) made about smallness of δf to f_0

Distribution functions

- Maxwellian: local, shifted, canonical
- Bump-on-tail
- Isotropic slowing down
- Anisotropic slowing down
- Bimaxwellian
- Constant of Motion distribution functions[1]
- Numerical distribution functions

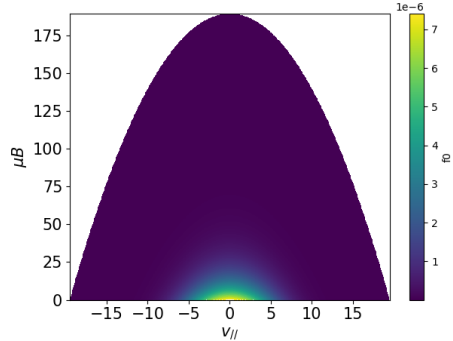
Analytical expressions, semi-analytical expressions, fully-numerical

[1] C. di Troia+, difficult to use for realistic experiments in practice

Maxwellian

$$F_{0,\mathrm{f,Max.}} = \frac{n_\mathrm{f}(r)}{(2\pi v_\mathrm{th}^2(r))^{3/2}} \exp\left(-E/v_\mathrm{th}^2\right) \exp\left(-\frac{u_\parallel}{2}\left(u_\parallel - 2v_\parallel\right)/\, v_\mathrm{th}^2\right)$$

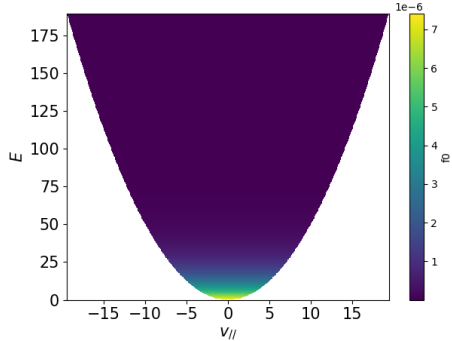
in absence of shift ($u_\parallel \to 0$), reduces to function of Energy, radius



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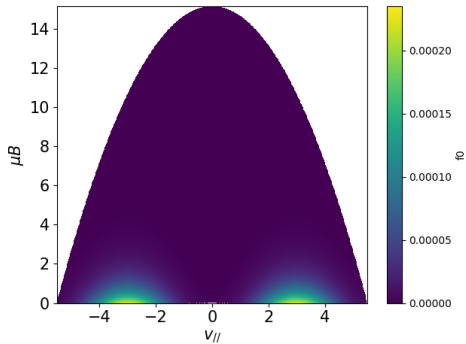


Bump-on-tail

$$F_{0,\mathrm{f},\mathrm{BoT}} = C \cdot n_\mathrm{f}(r) \exp(-E \cdot m_\mathrm{f}/T_\mathrm{f}) \exp(-v_{\parallel,f}^2/(2T_\mathrm{f})) \cosh(v_\parallel v_{\parallel,f}/T_\mathrm{f})$$

function of energy, radius, $v_\parallel$

- "Toy" distribution function with strong anisotropy (ideal to study EGAMs)
- Original version implemented for [Zarzoso+, NF, 2014]
  - Based on previous GYSELA work
- Originally zero radial dependence, since extended to include $n(r)$

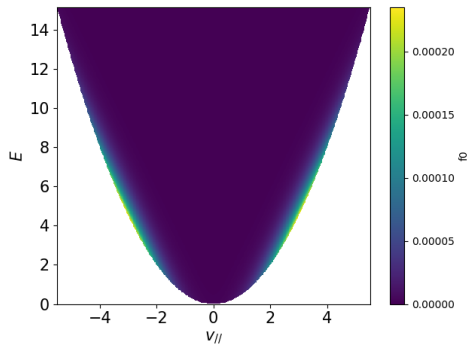


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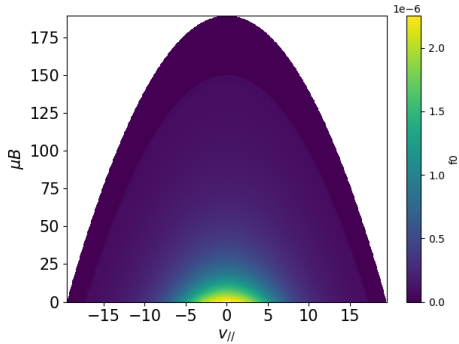


Isotropic Slowing down[2]

$$F_{0,\mathrm{f,SD}} = \frac{3n_\mathrm{f}(r)}{4\pi} \frac{\Theta(v_0 - |v|)}{(v_\mathrm{c}(r)^3 + |v|^3)\ln(1 + v_0/v_\mathrm{c}(r))}$$

also function of Energy ($|v|$), radius

- Decent approximation for alpha particles



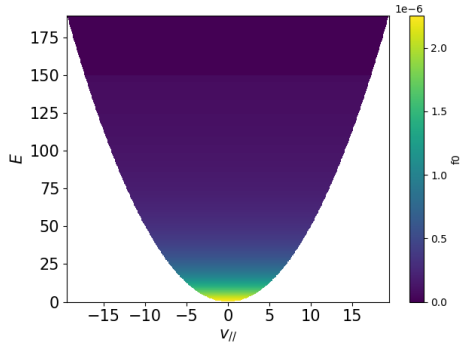
[2]Vannini+, Thesis+NF 2022

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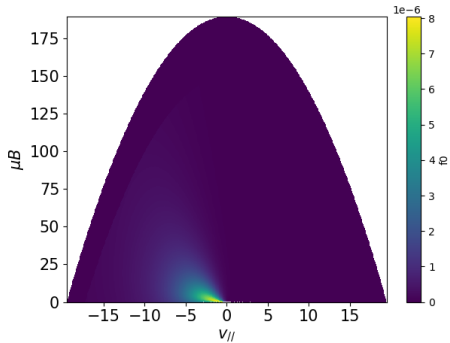
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Anisotropic Slowing down [3]

$$F_{0,\mathrm{f,ASD}} = F_{0,\mathrm{f,SD}}(r, E) \cdot \; C \exp\left(-(\xi - \xi_0)^2/(2\Delta\xi^2)\right)$$

where $\xi = v_{\parallel}/|v|$, $\rightarrow$ function of energy, radius, and parallel velocity

'C' is a messy term of error functions, …
derivatives 'tricky' $\rightarrow$ semi-analytical
F_0 analytic, but compute $\partial F_0/\partial X$ numerically



- Reasonable approximation for NBI

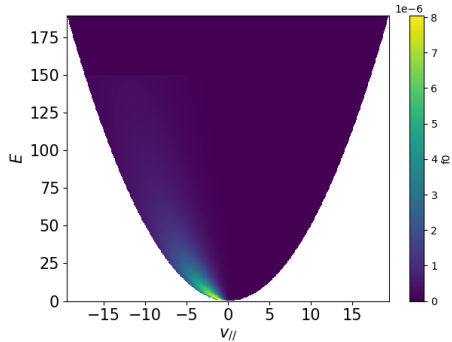
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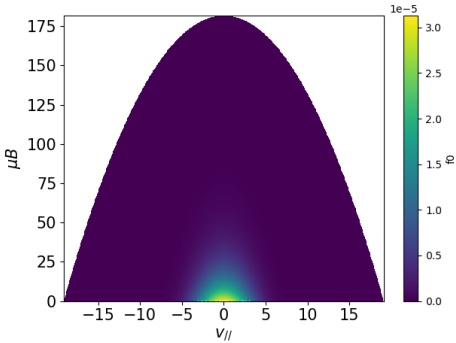
[3] Rettino+, NF 2022

Bi-Maxwellian[4]

$$F_{0,\mathrm{f},\mathrm{BiMax}} = n_{\mathrm{f}}(r) \left(\frac{m}{2\pi T_{\parallel}}\right)^{\frac{1}{2}} \left(\frac{m}{2\pi T_{\perp}}\right) \exp\left(-\frac{m}{2}\left(\frac{v_{\parallel}{}^2}{T_{\parallel}} + \frac{v_{\perp}{}^2}{T_{\perp}}\right)\right)$$

with effective temperature $T = \frac{T_{\parallel}}{3} + \frac{2T_{\perp}}{3}$

F_0 analytic, compute $\partial F_0 / \partial X$ numerically for simplicity



- Decent approximation for ICRH

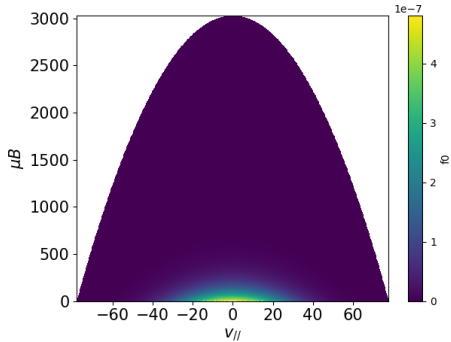
[4] J. N. Sama+, (arXiv:2209.14874)

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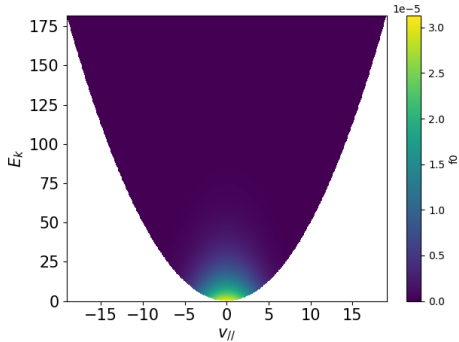
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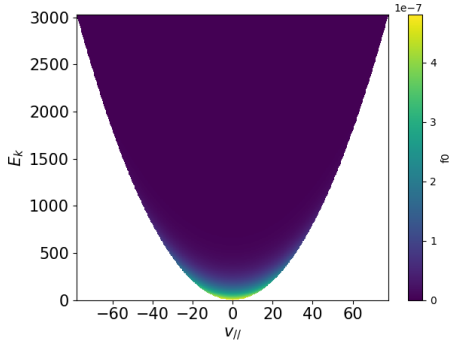
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Numerical validation for analytic distributions

When implementing purely analytical distribution functions:

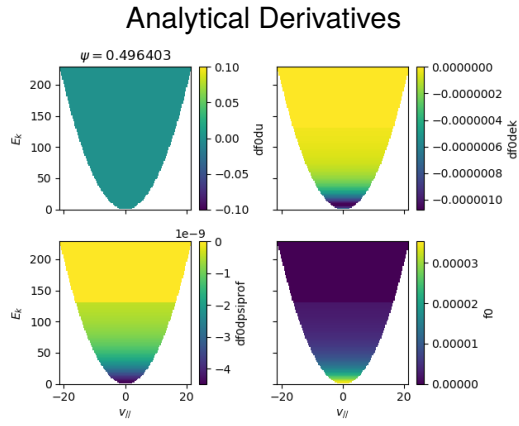
- Need analytical derivatives of F_0 w.r.t. E, $v_\parallel$, $\psi = r^2$
  - ... consistent with quirks of ORB5
- Validate analytical expressions by comparing to numerical derivatives

Validating F_0 numerically

- Output F_0 on mesh $(E, v_{\parallel}, \psi)$
- Implement finite differences evaluation of $\frac{\partial F_0}{\partial X}$
- Output $\frac{\partial F_0}{\partial X}$ on mesh
- Output $\frac{\partial F_0}{\partial X}_{\mathrm{FD}}$ on mesh

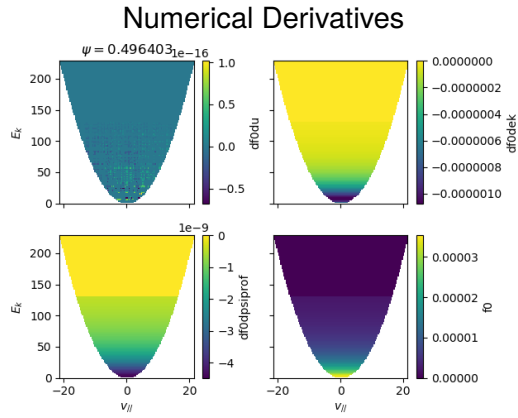
Validating F_0 numerically

- Output F_0 on mesh $(E, v_{\parallel}, \psi)$
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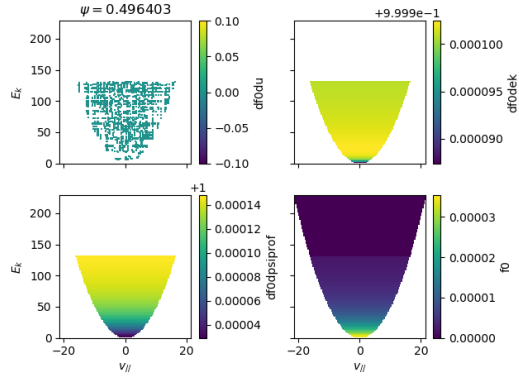
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Ratio analytical/numerical

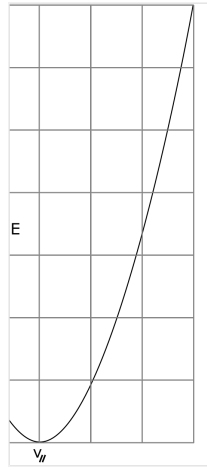


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- Output F_0 on mesh $(E, v_\parallel, \psi)$
- Implement finite differences evaluation of $\frac{\partial F_0}{\partial X}$
- Output $\frac{\partial F_0}{\partial X}$ on mesh
- Output $\frac{\partial F_0}{\partial X}_{\mathrm{FD}}$ on mesh
- We can probably reverse this?

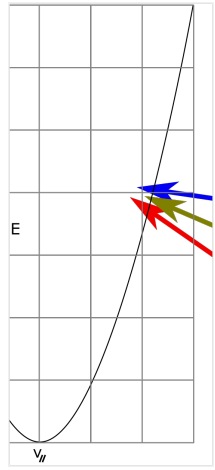
Numerical F_0

- Assume we have F_0 on mesh (vertices) $(E, v_{\parallel}, \psi)$
  - Can test with the output on previous slide
- F_0 for markers can be interpolated on the mesh



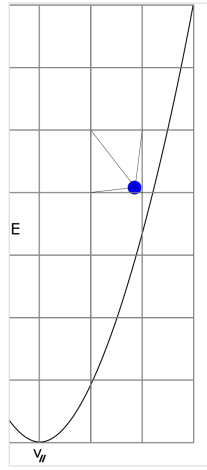
Numerical F_0

- Assume we have F_0 on mesh (vertices) $(E, v_{\parallel}, \psi)$
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- F_0 for markers can be interpolated on the mesh
- Consider 3 example markers



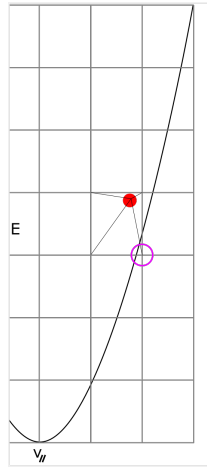
Numerical F_0

- Assume we have F_0 on mesh (vertices) $(E, v_{\parallel}, \psi)$
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- F_0 for markers can be interpolated on the mesh
- Consider 3 example markers
  - Blue is well behaved



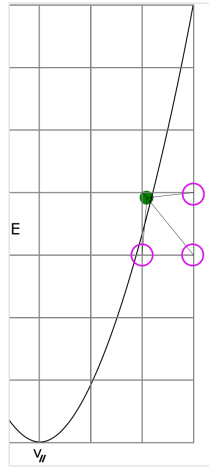
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  - Blue is well behaved
  - Red requires point with $\mu < 0$ for interpolation



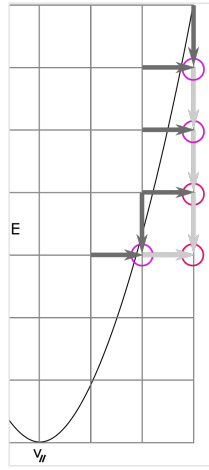
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Numerical F_0

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- Consider 3 example markers
  - Blue is well behaved
  - Red requires point with $\mu < 0$ for interpolation
  - Green even worse
  - Approach: smoothly continue F_0 across $\mu = 0$



RABBIT

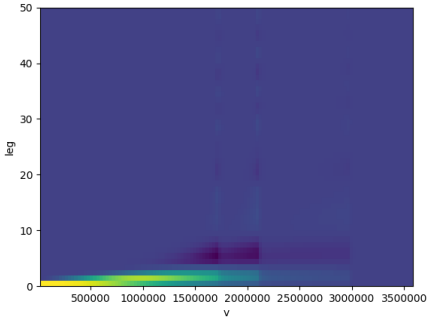
- RABBIT [Weiland+, NF, 2018+19]
  - real-time capable NBI code
- Describes NBI distribution function in experiment
- Non-Monte-Carlo method gives smooth function, good for derivatives
- We use RABBIT for ASDEX Upgrade (AUG) NBI F_0 (e.g. shot #31213 (NLED-AUG)) in the time-independent mode

$$f_-(v,\xi) = \frac{1}{2\pi}\frac{\tau_s}{v^3+v_c^3}\cdot \sum_{l=0}^{\infty}\left(l+\frac{1}{2}\right)P_l(\xi)S_l\cdot \left(\frac{v_0^3+v_c^3}{v^3+v_c^3}\frac{v^3}{v_0^3}\right)^{\frac{\beta}{3}l(l+1)}$$

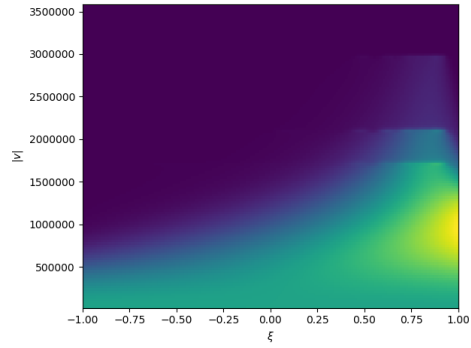
$$\xi = v_{\parallel}/v$$

RABBIT AUG example

AUG shot #31213 at t=0.84 s ("NLED-AUG" case[5])



$F_0(|v|, P_l(\xi))_{r=r_{\mathrm{ref}}}$

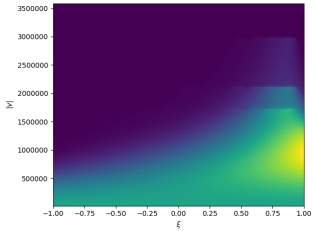


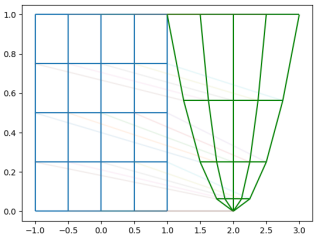
$F_0(\xi, |v|)_{r=r_{\mathrm{ref}}}$

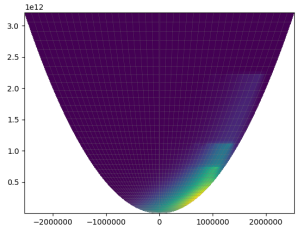
[5] Lauber+, IAEA FEC 2018

Mapping from RABBIT to ORB5

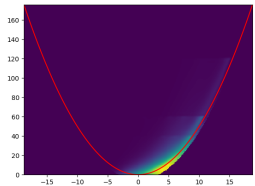
- Coordinate mapping from $(|v|, \xi, \rho_t) \rightarrow (E, v_\parallel, \psi)$
- Some additional details (interpolation objects, etc.)
- Fill in $F_0(\mu \lesssim 0)$

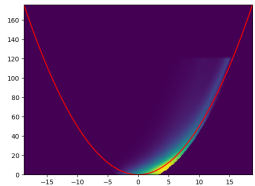


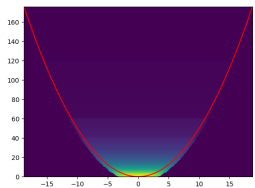


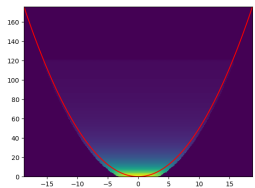


RABBIT AUG example









Clockwise (from TL): triple [nominal], single, single-no-pitch, triple-no-pitch.

Distribution functions

- Maxwellian: local, shifted, canonical
- Bump-on-tail
- Isotropic slowing down
- Anisotropic slowing down
- Bimaxwellian
- Constant of Motion distribution functions[6]
- Numerical distribution functions

CoM distribution functions, non-CoM anisotropic distribution functions, non-CoM isotropic* distribution functions.

[6] C. di Troia+, difficult to use for realistic experiments in practice

Constants of Motion

Constants of Motions (CoMs) in a tokamak:

1. $\mu = mv_{\perp}^2/2B$
2. $E = mv^2/2 + Ze\phi$
3. $P_\varphi = mv_\varphi - Ze\Psi_p$
4. ($sgn(v_{\parallel})$ for passing particles)

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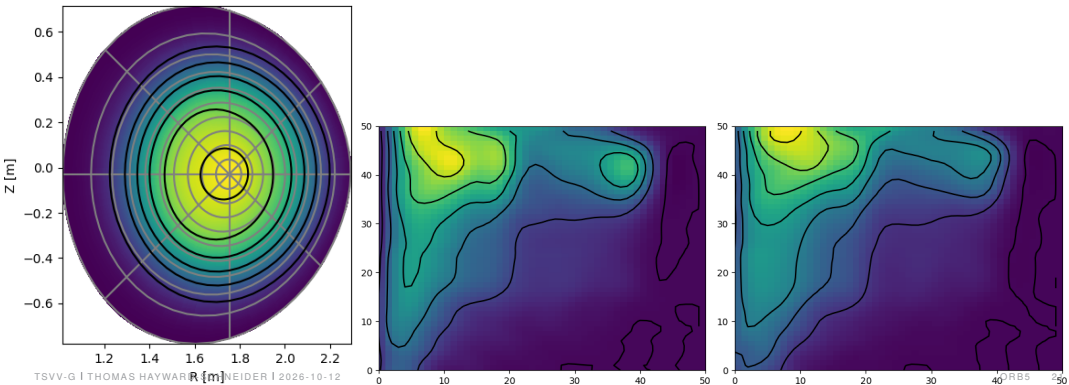
Guillaume Brochard wrote EPCoM tool for CoM distribution functions.
ITPEA-EP activity [JEX EP-28] to implement and compare in multiple codes (Done: GTC, XTOR-K, M3D-C1-K. In progress: JOREK, ORB5. Also promised: LIGKA, GENE, M3D-K, MEGA, X-TAPAS).

EPCoM

EPCoM provides F0 as a function of (P_φ, E, $\Lambda = \mu B_0/E$, sign($v_\parallel$)), along with derivatives. An interpolation routine was provided, along with notes on using chain-rule for GK-derivatives.

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Current status:

- ORB5 can interpolate the F0 function
  - Still to be checked
- Derivatives to be calculated numerically from (f0(X) - f0(X+dX))/dX
- QN must be disabled, role of “density” to be considered
  - Density is now a 2D function, how to best handle in the code, how to handle QN ($n_e(\psi, \theta)$)?

Summary

- ORB5 has a number of distribution function options. It's time to move away from local Maxwellians for EPs
- CoM numerical distribution functions based on EPCoM are being implemented (rather analogous to PSZS)
- Benchmark of beam-driven BAE in DIIID and ITPA-EP to follow