EUROfusion

TSVV-G meeting

Geometric Orbital Spectrum Analysis of Resonant Ion-Mode Interactions and Transport in Tokamaks

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Collaboration with

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OUTLINE

- **MOTIVATION & METHODOLOGY OVERVIEW**
- **GEOMETRICAL CALCULATION OF THE ORBITAL FREQUENCY SPECTRUM (OFS)**
  – PREDICTION OF RESONANCES LOCATIONS
  – OFS DEPENDENCE ON THE SHAPE OF THE BACKGROUND MAGNETIC FIELD
  – ORBITAL SPECTRAL ANALYSIS (OSA) OF PERTURBATIONS
- **FORMATION OF TRANSPORT BARRIERS**
- **ONGOING AND FUTURE WORK**
  – STELLARATORS
  – COLLECTIVE PARTICLE DYNAMICS
- **SUMMARY AND CONCLUSIONS**

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MOTIVATION & PROPOSED METHODOLOGY

The confinement of energetic particles and their collective interaction with modes remain central challenges for the performance of magnetic confinement fusion devices.

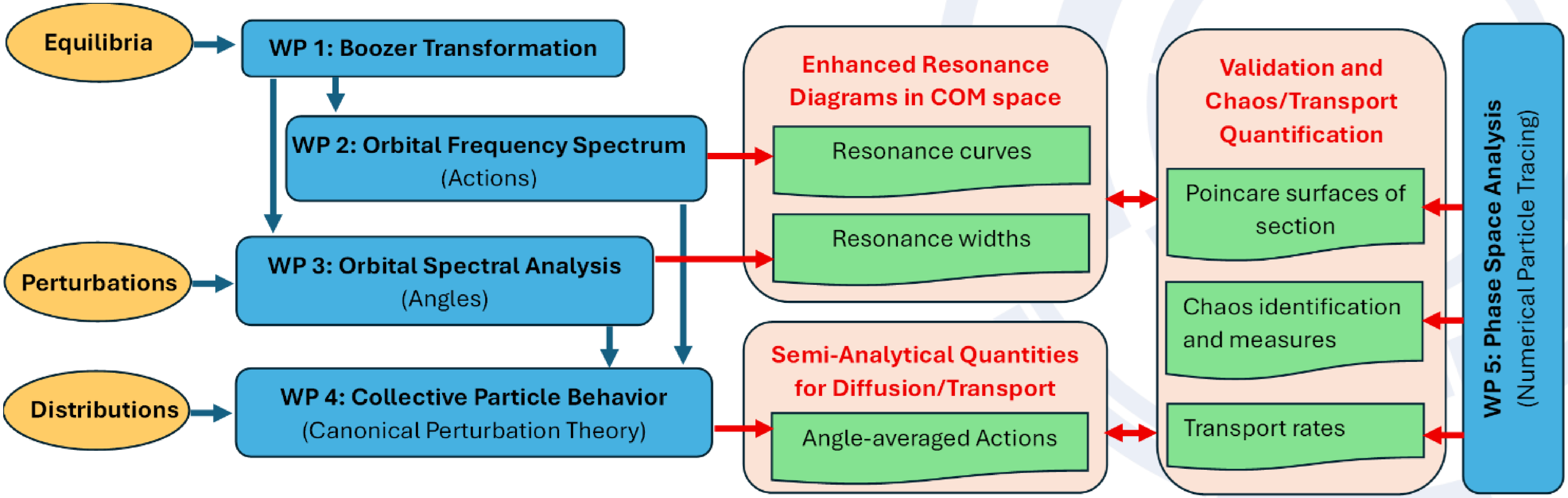
Proposed methodology:

- **Hamiltonian formulation (Action-Angle)** of Guiding-Center (GC) dynamics.
- Exploitation of the integrability of the GC motion in symmetric configurations.
- **Geometric calculation of Orbital Frequency Spectra (OFS)** determining conditions for **resonant mode-particle interactions** (taking into account **finite drifts** of thermal and energetic particles).
- **Computational efficiency** based on unperturbed GC motion (calculations performed **once per equilibrium** configuration, regardless of the perturbations).
- Applied across both **tokamak and stellarator** configurations.
- Extensive **parametric studies** for scenario simulations (equilibria & perturbations).

METHODOLOGY OVERVIEW

Enabling Research Project (2026-2027):
GOSARIT

NTUA P. Zestanakis, G. Anastassiou, Y. Antonenas, C. Tsironis, Y. Kominis
Consorzio RFX I. Predebon, G. Spizzo, B. Momo
ENEA Frascati M. Falessi, N. Carlevaro
MPG/IPP A. Zocco



OUTCOME

Results and Tools:

- Direct link between plasma **equilibrium shaping** - including elongation, triangularity, shear, and quasi-symmetry - and the **resonance structure** governing transport and stability.
- Construction of predictive **resonance diagrams**, conditions for the formation of **transport barriers**, and characterization of **island overlap**, providing overview and intuitive understanding of the underlying transport mechanisms under the presence of **multi-scale perturbations**.
- **Reduced model** based on first-principles.
- Systematic and **computationally efficient procedure** (workflow), supported by appropriately developed **tools**.
- Applications to **scenario exploration** over a wide range of equilibria and **geometry optimization to tailor EP transport**.

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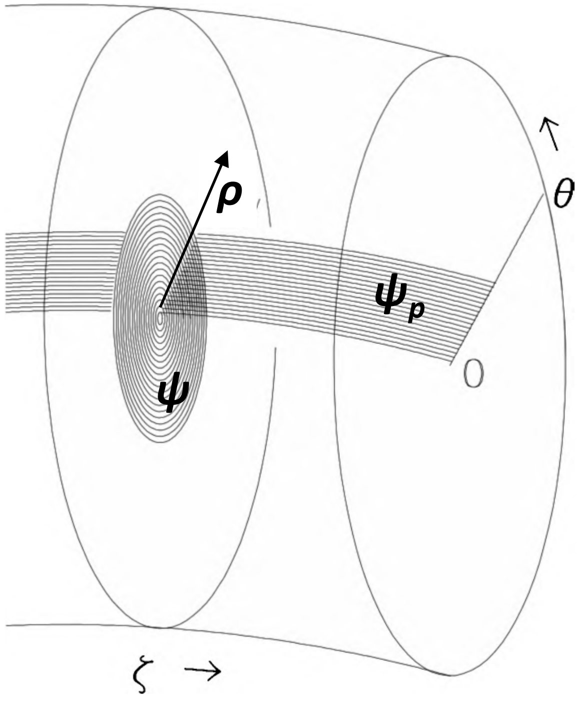
HAMILTONIAN (CANONICAL) DESCRIPTION OF THE GC MOTION

Guiding Center Hamiltonian:

$$\mathcal{H} = \frac{\rho_\parallel^2 B^2}{2} + \mu B + \Phi$$

$\rho_\parallel = v_\parallel / B$ normalized $v_\parallel$

$\mu = v_\perp^2 / 2B$ magnetic moment (magnetic flux through gyration orbit)



Boozer coordinates:

$$\mathbf{B} = g(\psi)\nabla\zeta + I(\psi)\nabla\theta$$

covariant representation

$g(\psi)$: poloidal current

$I(\psi)$: toroidal current

Canonical variables:

$P_\theta = \psi + \rho_\parallel I(\psi)$ θ poloidal angle

$P_\zeta = \rho_\parallel g(\psi) - \psi_p(\psi)$ ζ toroidal angle

$P_\xi = \mu$ ξ gyroangle

the canonical momenta:
- are not mechanical momenta
- take into account the magnetic field

$$H = \frac{(P_\zeta + \psi_p(P_\zeta, P_\theta))^2}{2g^2(P_\zeta, P_\theta)} B^2(P_\zeta, P_\theta, \zeta, \theta) + \mu B(P_\zeta, P_\theta, \zeta, \theta) + \Phi(P_\zeta, P_\theta, \zeta, \theta)$$

ACTION-ANGLE FORMULATION OF THE INTEGRABLE GC MOTION

Action-Angle variables:

$$(\mu,\xi)\rightarrow(J_\xi,\hat{\xi})$$
$$(P_\zeta,\zeta)\rightarrow(J_\zeta,\hat{\zeta})$$
$$(P_\theta,\theta)\rightarrow(J_\theta,\hat{\theta})$$

$$J_\xi=\mu$$
$$J_\zeta=P_\zeta$$
$$J_\theta=\frac{1}{2\pi}\oint P_\theta(E,J_\xi,J_\zeta,\theta)d\theta$$

$$\hat{\xi}=\xi+\partial f_2(J,\theta)/\partial J_\xi$$
$$\hat{\zeta}=\zeta+\partial f_2(J,\theta)/\partial J_\zeta$$
$$\hat{\theta}=\partial f_2(J,\theta)/\partial J_\theta \quad (*)$$

phase-like variables (linear time evolution)

generating function: $$F_2(J_\xi,J_\zeta,J_\theta,\xi,\zeta,\theta)=\xi J_\xi+\zeta J_\zeta+\overbrace{\int^\theta P_\theta\left(J_\xi,J_\zeta,J_\theta(J_\xi,J_\zeta),\theta'\right)d\theta'}^{f_2(\mathbf{J},\theta)}$$

Orbital Spectrum & Dynamical Reduction (bounce/transit averaged dynamics):

$$T_{\hat{\theta}}=T_\theta=2\pi/\hat{\omega}_\theta \quad (*)$$

$$\frac{\hat{\omega}_\zeta}{\hat{\omega}_\theta}=\frac{\partial H/\partial J_\zeta}{\partial H/\partial J_\theta}=-\frac{\partial J_\theta(J_\xi,J_\zeta)}{\partial J_\zeta}$$

$$\frac{\hat{\omega}_\zeta}{\hat{\omega}_\theta}=\frac{(\Delta\zeta)_{T_\theta}}{2\pi}$$

$$\langle\dot{\zeta}\rangle_{T_\theta}\equiv\frac{1}{T_\theta}\int_0^{T_\theta}\dot{\zeta}\,dt=\frac{\hat{\omega}_\theta}{2\pi}(\Delta\zeta)_{T_\theta}$$

The poloidal Action (- J_θ) is the Hamiltonian for the other degree(s) of freedom when time is normalized to T_θ [Perkins & Bellan, PRL 2010]

$\hat{\omega}_\zeta$ is the bounce/transit averaged **toroidal precession**

SYMMETRY BREAKING PERTURBATIONS – NONINTEGRABLE GC DYNAMICS

- **Symmetry-breaking perturbations** (general form):

$$H = \hat{H}_0(J_\zeta, J_\theta, J_\xi) + \sum_{m,n} \hat{H}_{mnl}(J_\zeta, J_\theta, J_\xi)\exp[i(m\hat{\theta} + n\hat{\zeta} + l\hat{\xi} - \omega t)]$$

- The perturbations affect particle and momentum transport in a resonant fashion, in the specific locations of the phase space, where the **resonance condition** is fulfilled locally in the phase space

$$m\omega_\theta + n\omega_\zeta + l\omega_\xi = \omega$$

action-dependent condition

  - Case $l = 0$: **low frequency (magnetic) perturbations**
    - for $\omega = 0$, the interactions take place in a **constant energy surface**

$$s = -n / m = \hat{\omega}_\theta / \hat{\omega}_\zeta$$

orbital frequencies calculated once (without perturbations)
→ a priori knowledge of the resonance locations for all types of (multi-scale) perturbations

Non-axisymmetric, time-dependent magnetic perturbations

- Flute-type perturbations: $\delta\mathbf{B} = \nabla\times\alpha\mathbf{B}$ → redefinition of canonical variables: $\rho_c = \rho_{\|} + \alpha$

$$H = \frac{(P_\zeta + \psi_p(P_\zeta, P_\theta))^2}{2g^2(P_\zeta, P_\theta)} B^2 + \mu B - \alpha\rho_c B^2 + \frac{1}{2}\alpha^2 B^2$$

$$\alpha = \alpha_{n,m}(\psi)\exp\left[i(n\zeta + m\theta - \omega t)\right]$$

$$B(P_\zeta, P_\theta, \zeta, \theta)$$

Orbital Spectrum Analysis

$$H = H_0 + H_1 + H_2$$

$$H_1 = \sum_{k=0}^{\infty} H_1^k(J_\theta, P_\zeta)\exp\left[i(n\hat{\zeta} + k\hat{\theta} - \omega t)\right]$$

$$H_2 = \sum_{k=0}^{\infty} H_2^k(J_\theta, P_\zeta)\exp\left[i(2n\hat{\zeta} + k\hat{\theta} - 2\omega t)\right] + \sum_{k=0}^{\infty} \tilde{H}_2^k(J_\theta, P_\zeta)\exp\left[ik\hat{\theta}\right]$$

$$\hat{\zeta} = \zeta + \partial f_2(J,\theta)/\partial J_\zeta$$

$$\hat{\theta} = \partial f_2(J,\theta)/\partial J_\theta$$

→ for each (n,m) mode
- (n,k) harmonics (1st order)
- $(2n,k)$ and $(0, k)$ harmonics (2nd order)

Resonance condition when phases are stationary in the particle frame: $n\omega_\zeta + k\omega_\theta - \omega = 0$

$$q_{kin} \equiv \frac{\omega_\zeta}{\omega_\theta} = -\frac{k}{n}$$

static perturbations ($\omega=0$)

ANALYTICAL EQUILIBRIA: LARGE ASPECT RATIO

Large Aspect Ratio (LAR) equilibrium:

$$g = 1, \quad I = 0, \quad B = 1 - r\cos\theta, \quad \psi = r^2/2$$

$$P_\theta = \psi, \quad P_\zeta = \rho_\parallel - \psi_p(\psi)$$

Analytical results for a Drift-Center Approximation (DCA):

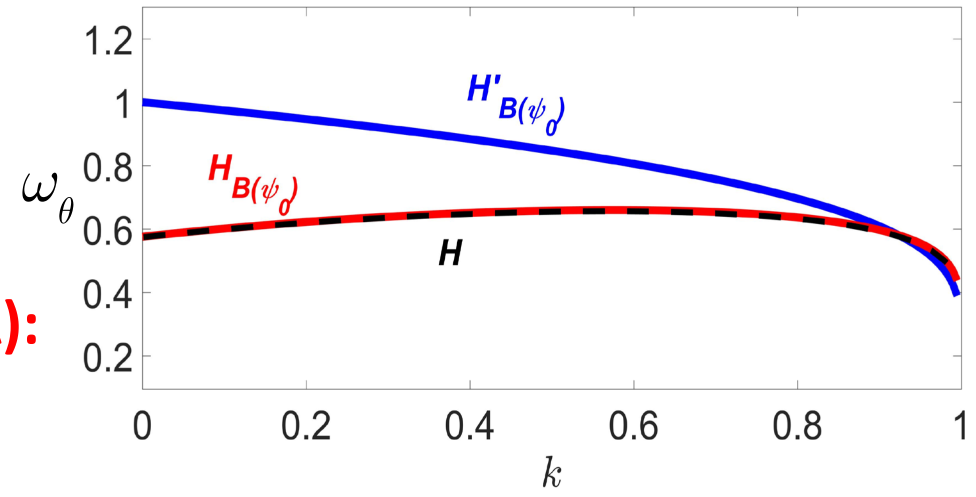
all quantities related to the magnetic configuration are calculated at a **flux surface of reference**

(also applicable to realistic equilibria)

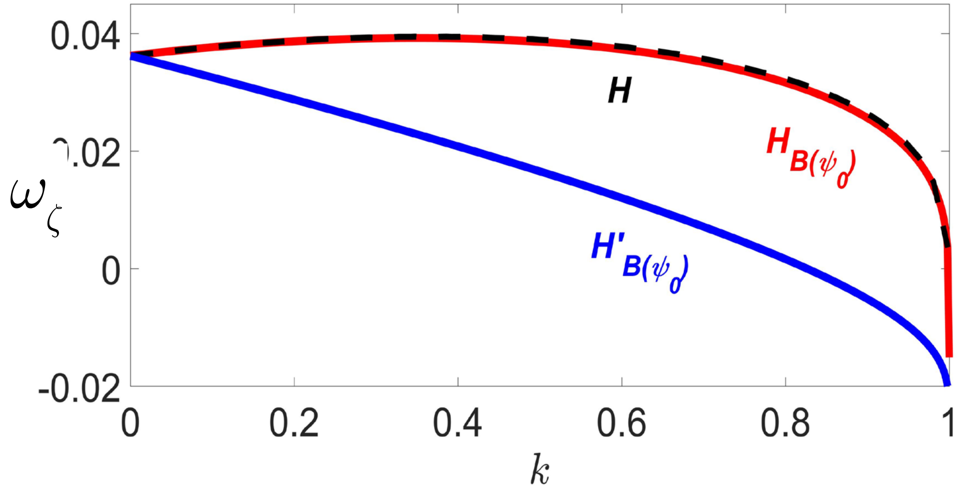
For large pitch angles (deeply trapped particles) the standard pendulum-like Hamiltonian is obtained:

$$H'_{B(\psi_0)}(\rho_\parallel, \theta; \psi_0, \mu) = \frac{\rho_\parallel^2}{2} + \mu\left[1 - \sqrt{2\psi_0}\cos(\theta)\right]$$

Bounce frequency (trapped particles)



Bounce-averaged toroidal precession frequency



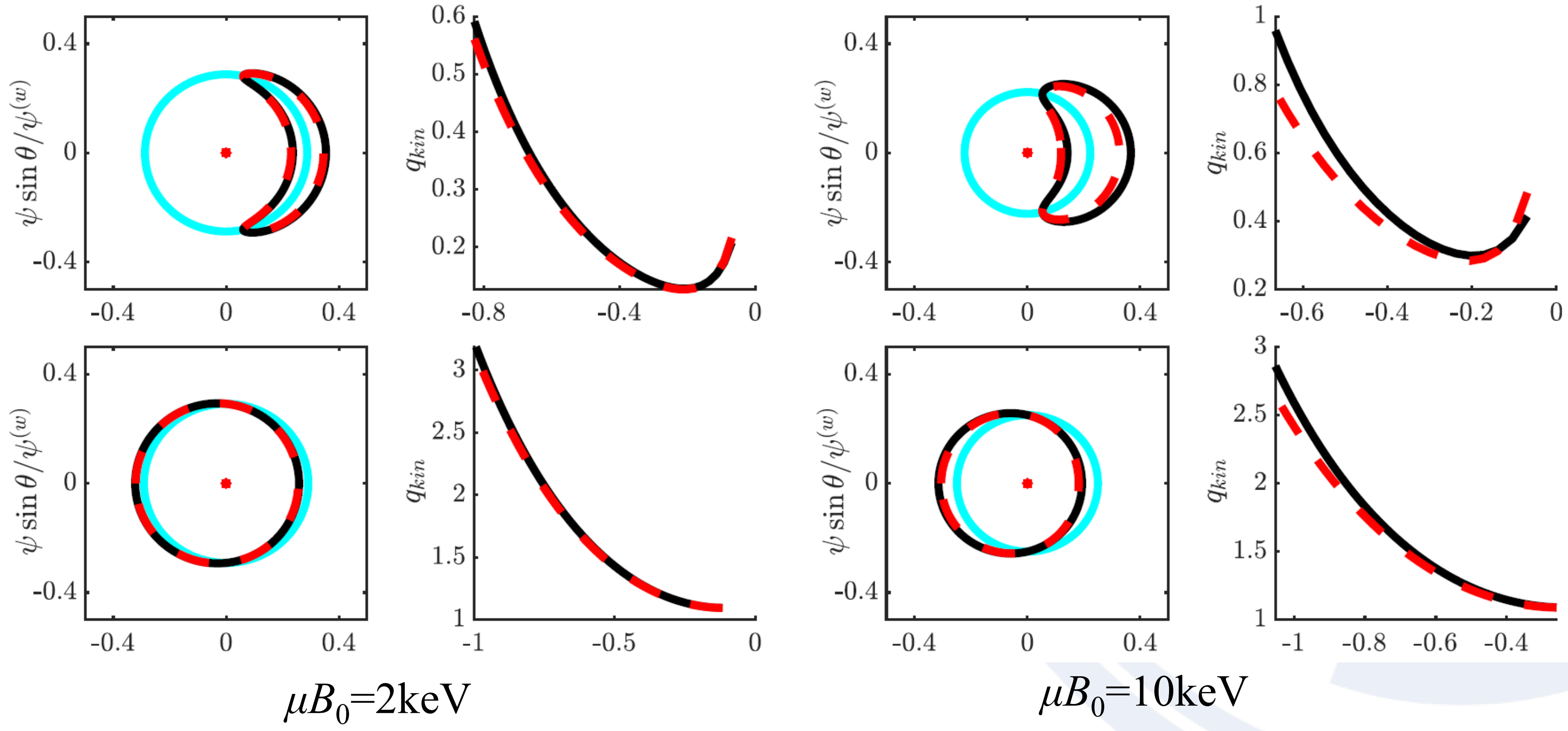
[Antonenas, Anastasiou, Kominis, JPP 2021]

ANALYTICAL EQUILIBRIA: LARGE ASPECT RATIO

The accuracy of the approximate (DCA) analytical results for the kinetic q factor depends on the GC drift width

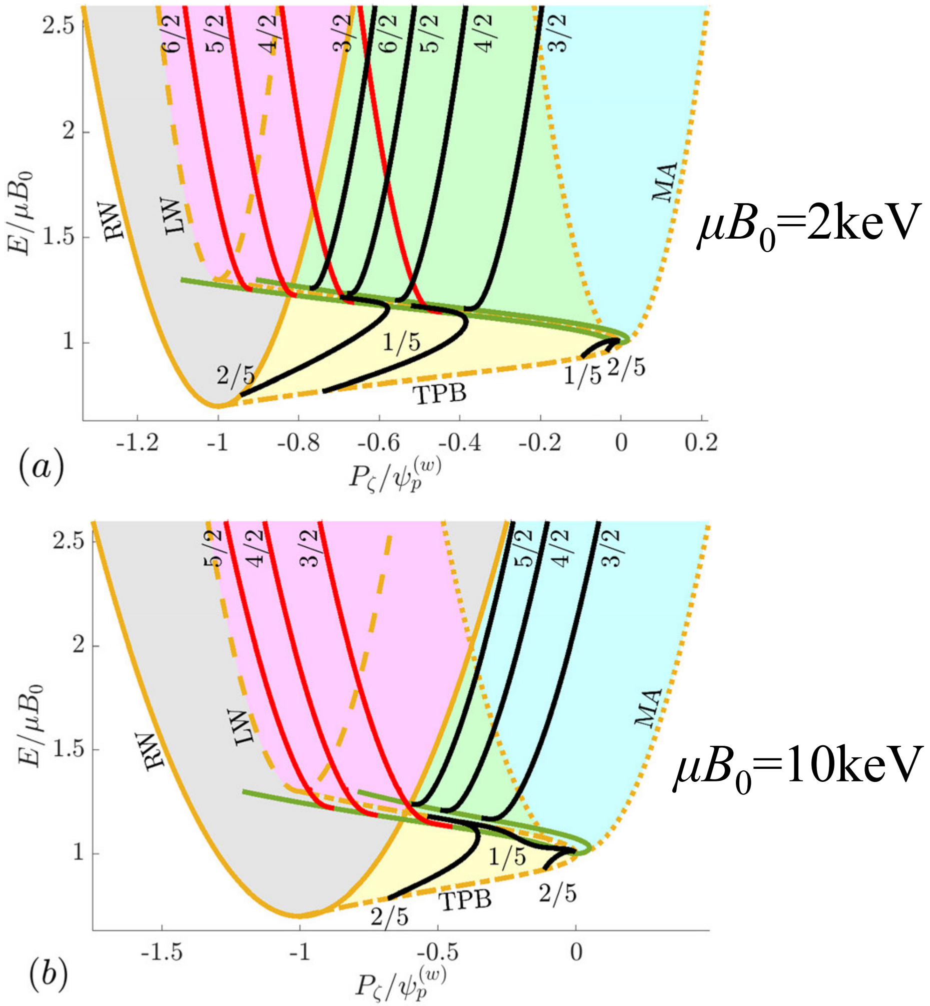
trapped

passing



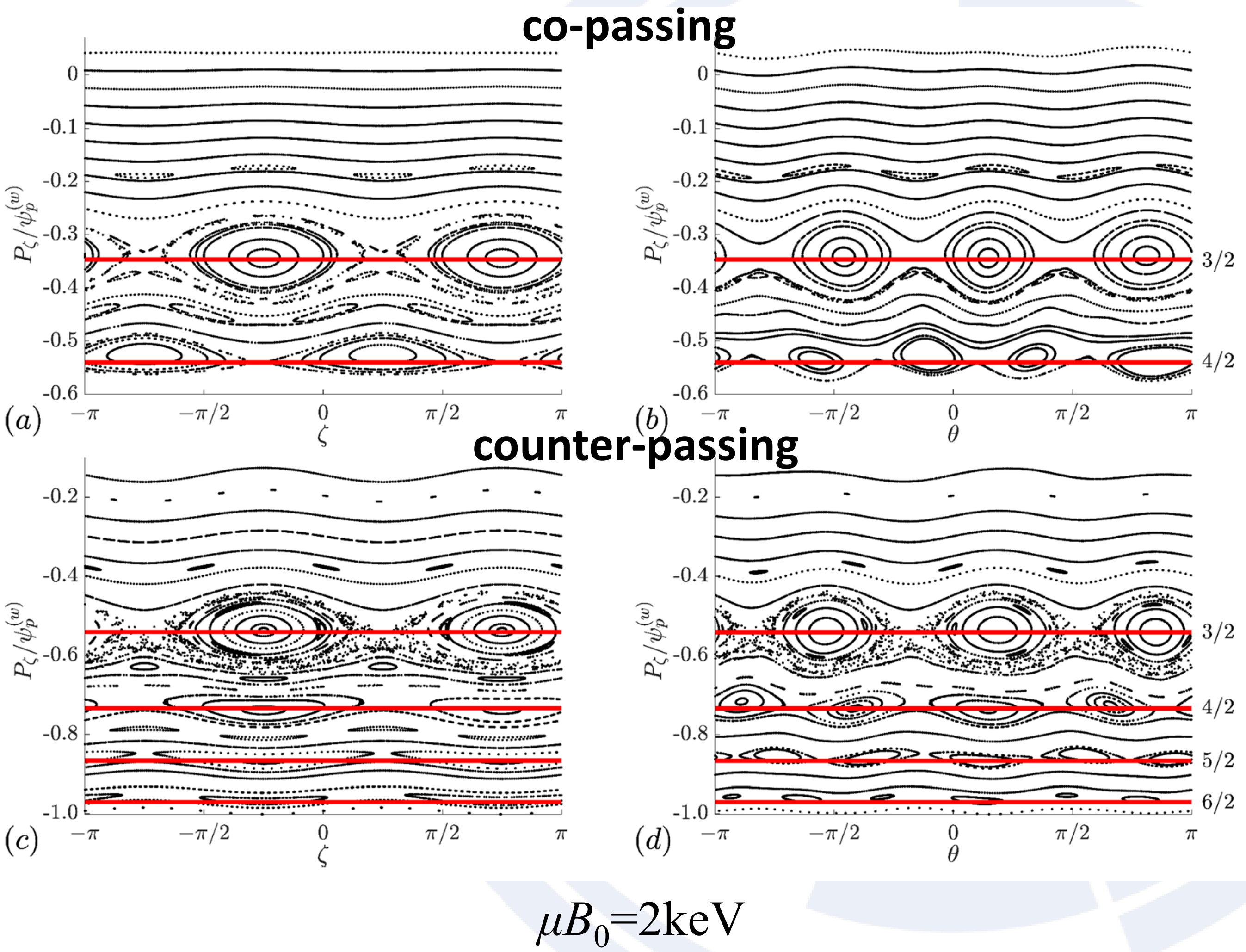
Note: the kinetic q factor has a local minimum for trapped particles, although q is monotonic

RESONANCE DIAGRAMS AND PREDICTION OF RESONANCE LOCATIONS



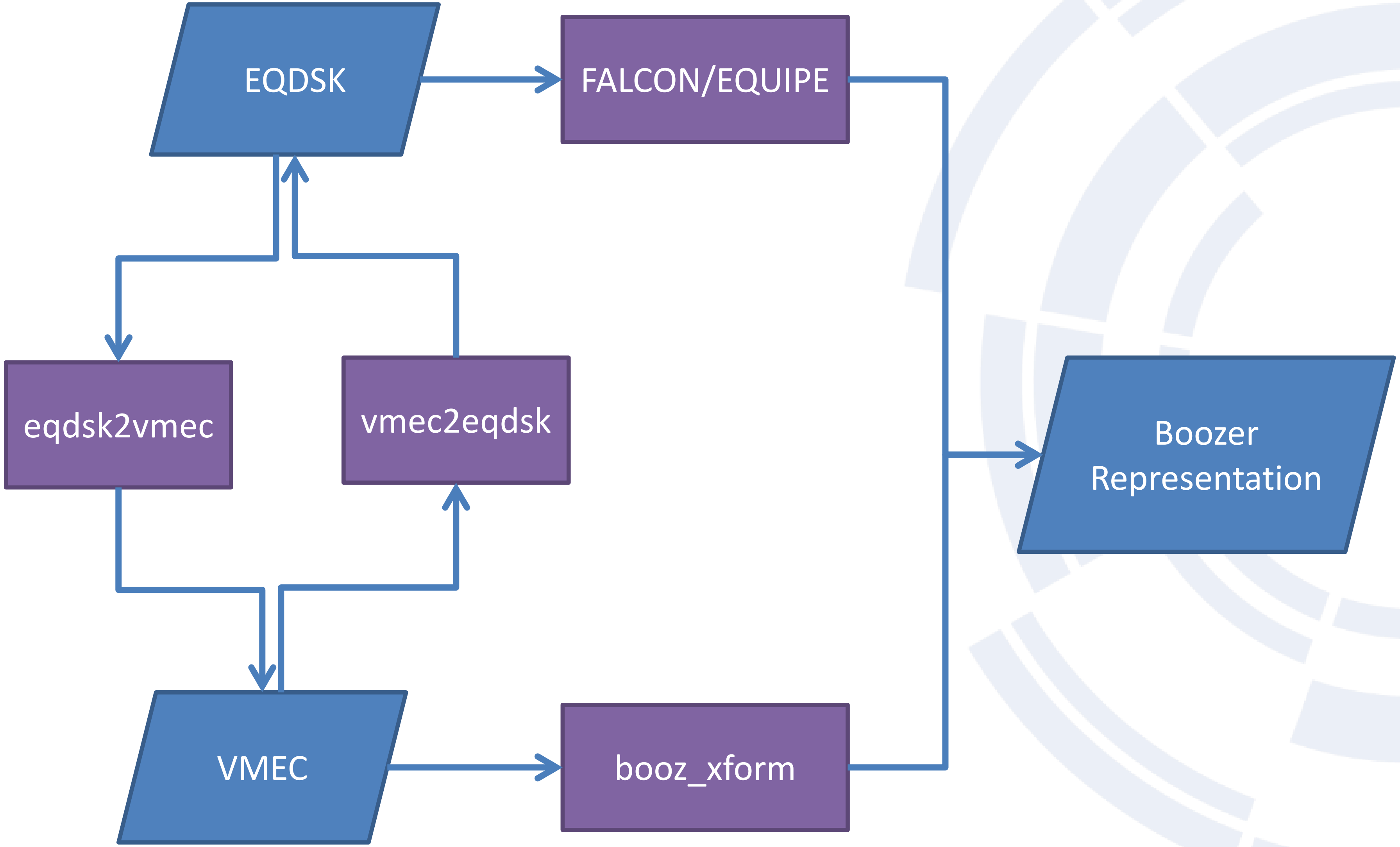
Resonance Diagrams in the COM space

[Antonenas, Anastasiou, Kominis, PoP 2024]



Poincare surface of section under the presence of a single perturbative mode with $(m,n) = (3,2)$ and $a_{(3,2)}=0.5\text{x}10^{-4}$

REALISTIC EQUILIBRIA: PROCESSING WORKFLOW



REALISTIC EQUILIBRIA: EXACT GEOMETRICAL CALCULATIONS

Axisymmetric (tokamak) equilibrium
[or quasi-symmetric (stellarator)]

→ **Integrable GC motion (P_ζ is conserved)**

$$H = \frac{(P_\zeta + \psi_p(P_\zeta, P_\theta))^2}{2g^2(P_\zeta, P_\theta)} B^2(P_\theta, \theta; P_\zeta) + \mu B(P_\theta, \theta; P_\zeta)$$

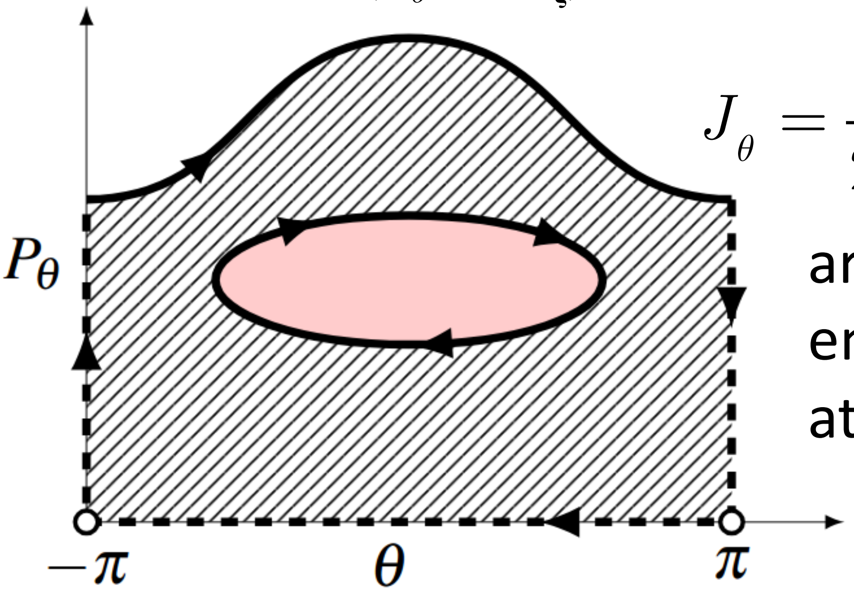
Exact calculation of the OFS:

- **Integration of the eqs of motion for a single period**

  or

- **Action-based geometrical approach**

Contours: $H(P_\theta, \theta\, ; P_\zeta) = E = \text{const.}$



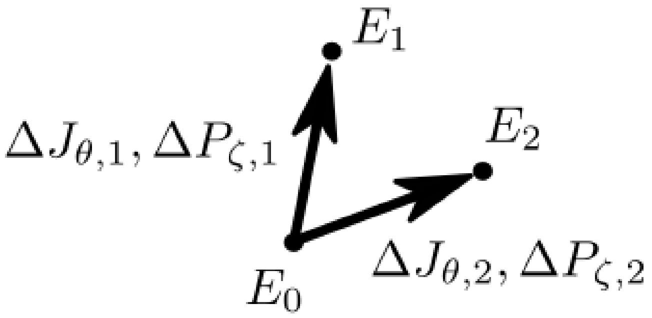
$$J_\theta = \frac{1}{2\pi} \oint P_\theta(\theta; P_\zeta, E) d\theta$$

area enclosed by energy contours at constant μ, P_ζ

$$\omega_\zeta = \partial E / \partial P_\zeta$$

$$\omega_\theta = \partial E / \partial J_\theta$$

$$q_{kin} = \omega_\zeta / \omega_\theta$$



calculate by solving

$$\Delta E_1 \approx \omega_\theta \Delta J_{\theta,1} + \omega_\zeta \Delta P_{\zeta,1}$$

$$\Delta E_2 \approx \omega_\theta \Delta J_{\theta,2} + \omega_\zeta \Delta P_{\zeta,2}$$

RESULTS: AUG & DTT (static non-axisymmetric perturbations)

Demonstration for two cases:

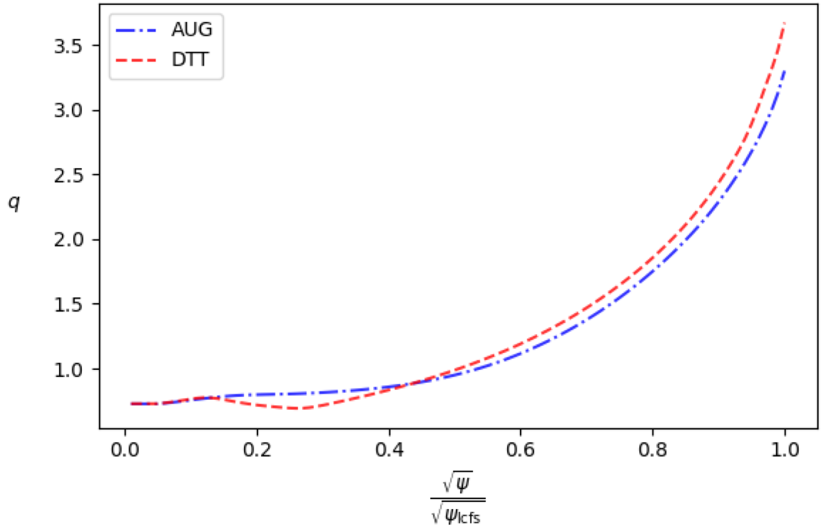
- ASDEX Upgade (AUG) - reduced field scenario with shaping
- Divertor Tokamak Test facility (DTT) - full power reference plasma scenario

	B_0(T)	R_0(m)
AUG	1.45	1.66
DTT	5.96	2.26

Normalizations and Comparison

- To cancel out the effects of tokamak scaling, comparison between the two equilibria is carried out between orbits with equal **normalized kinetic quantities**.
- If the configurations of the two equilibria were exactly the same, apart from a scale in size and field strength, the corresponding **GC orbits would be the same up to a length and a time scaling factor**.
- In absolute units, **the energies of the compared orbits differ by a factor** of

$$(B_0^{\mathrm{DTT}} R_0^{\mathrm{DTT}})^2 / (B_0^{\mathrm{AUG}} R_0^{\mathrm{AUG}})^2 \approx 30$$



Comparable safety factor profiles (with the exception that the DTT is in a reverse shear configuration)

RESULTS: AUG & DTT (static non-axisymmetric perturbations)

Resonances with a static $(n, m)=(11, -8)$ perturbative mode

$$\alpha = \epsilon \sqrt{\frac{\psi}{\psi_{lcfs}}} \exp[i(11\zeta - 8\theta)]$$

Radial dependence chosen to obtain an approximately flat $\delta B/B$ profile (to isolate the intrinsic localization of the resonances)

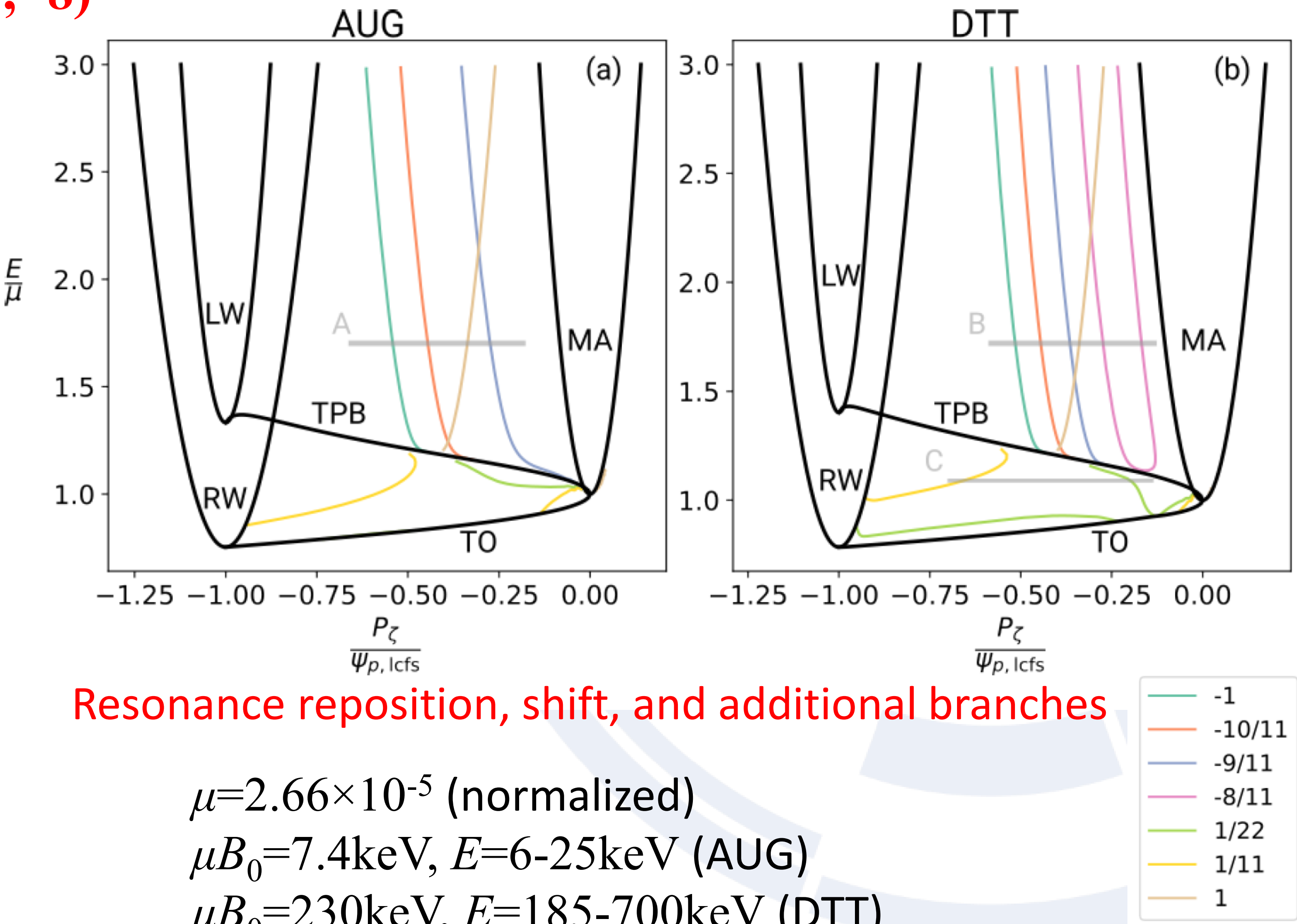
First-order resonances:

$q_{\mathrm{kin}}=r/11$

Second-order resonances:

$q_{\mathrm{kin}}=s/22$

[Zestanakis, Antonenas, Anastasiou, Falessi, Kominis, submitted to NF (2026)]



Resonance reposition, shift, and additional branches

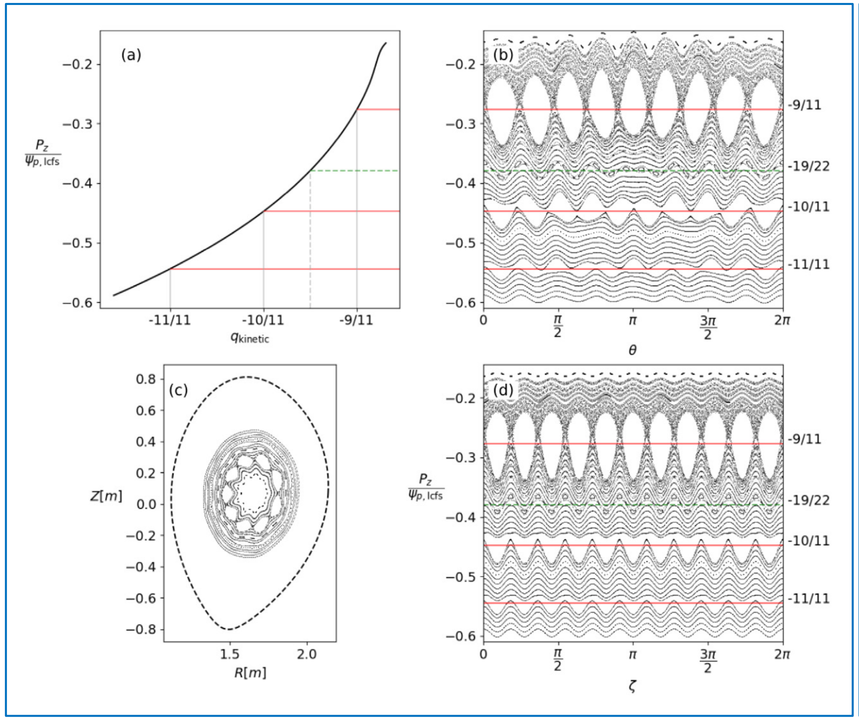
$\mu=2.66\times10^{-5}$ (normalized)

$\mu B_0=7.4\mathrm{keV}$, $E=6\text{-}25\mathrm{keV}$ (AUG)

$\mu B_0=230\mathrm{keV}$, $E=185\text{-}700\mathrm{keV}$ (DTT)

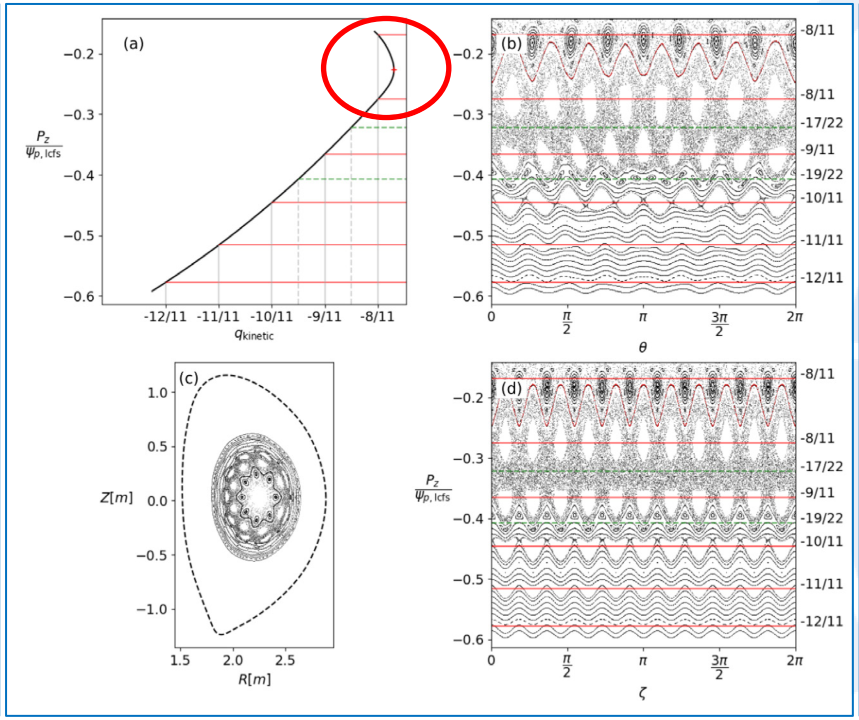
RESULTS: AUG & DTT (static non-axisymmetric perturbations)

Resonances with a static $(n, m)=(11, -8)$ perturbative mode (amplitude $\varepsilon=2.5\times10^{-4}$)



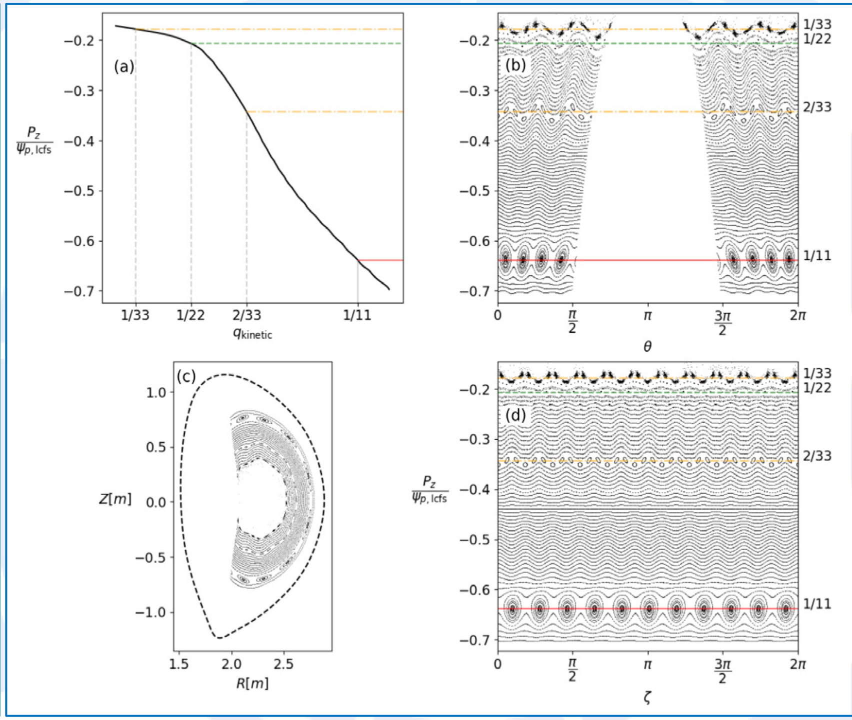
AUG

E=12.8keV (passing)



DTT

E=400keV (passing)



DTT

E=250keV (trapped)

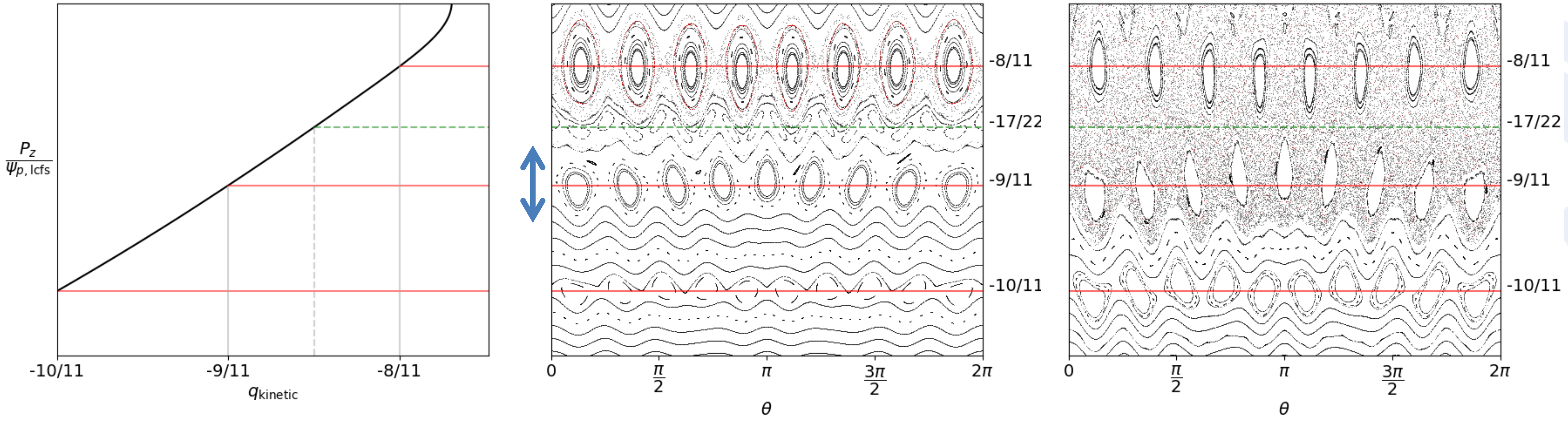
$\mu=2.66\times10^{-5}$ (normalized)

μB_0=7.4keV (AUG)

μB_0=230keV (DTT)

[Zestanakis, Antonenas, Anastasiou, Falessi, Kominis, submitted to NF (2026)]

ORBITAL SPECTRUM ANALYSIS OF PERTURBATIONS & RESONANCE WIDTH



Island width $\sim \sqrt{H_i^k}$ → conditions for **resonance overlap** and extended transport

Orbital Spectral Analysis (OSA) :

The AA transformation is used to express all perturbations in terms of Fourier series of the Angle variables providing the amplitude of each harmonic in Angles.

$$\alpha = \alpha_{n,m}(\psi)\exp\left[i(n\zeta + m\theta - \omega t)\right]$$

$$H = H_0 + H_1 + H_2$$

$$H_1 = \sum_{k=0}^{\infty} H_1^k(J_\theta, P_\zeta)\exp\left[i(n\hat{\zeta} + k\hat{\theta} - \omega t)\right]$$

$$H_2 = \sum_{k=0}^{\infty} H_2^k(J_\theta, P_\zeta)\exp\left[i(2n\hat{\zeta} + k\hat{\theta} - 2\omega t)\right] + \sum_{k=0}^{\infty} \tilde{H}_2^k(J_\theta, P_\zeta)\exp\left[ik\hat{\theta}\right]$$

RESULTS: AUG (time-dependent axisymmetric perturbations - GAM)

Resonances with a time-dependent axisymmetric (GAM-like) mode

$$\alpha = \epsilon \sqrt{\frac{\psi}{\psi_{lcfs}}} \exp\left[i(m\theta - \omega t)\right]$$

$m=\pm 2$,
$\omega = 2.4\times 10^{-3}$
($f \approx 53\text{kHz}$)

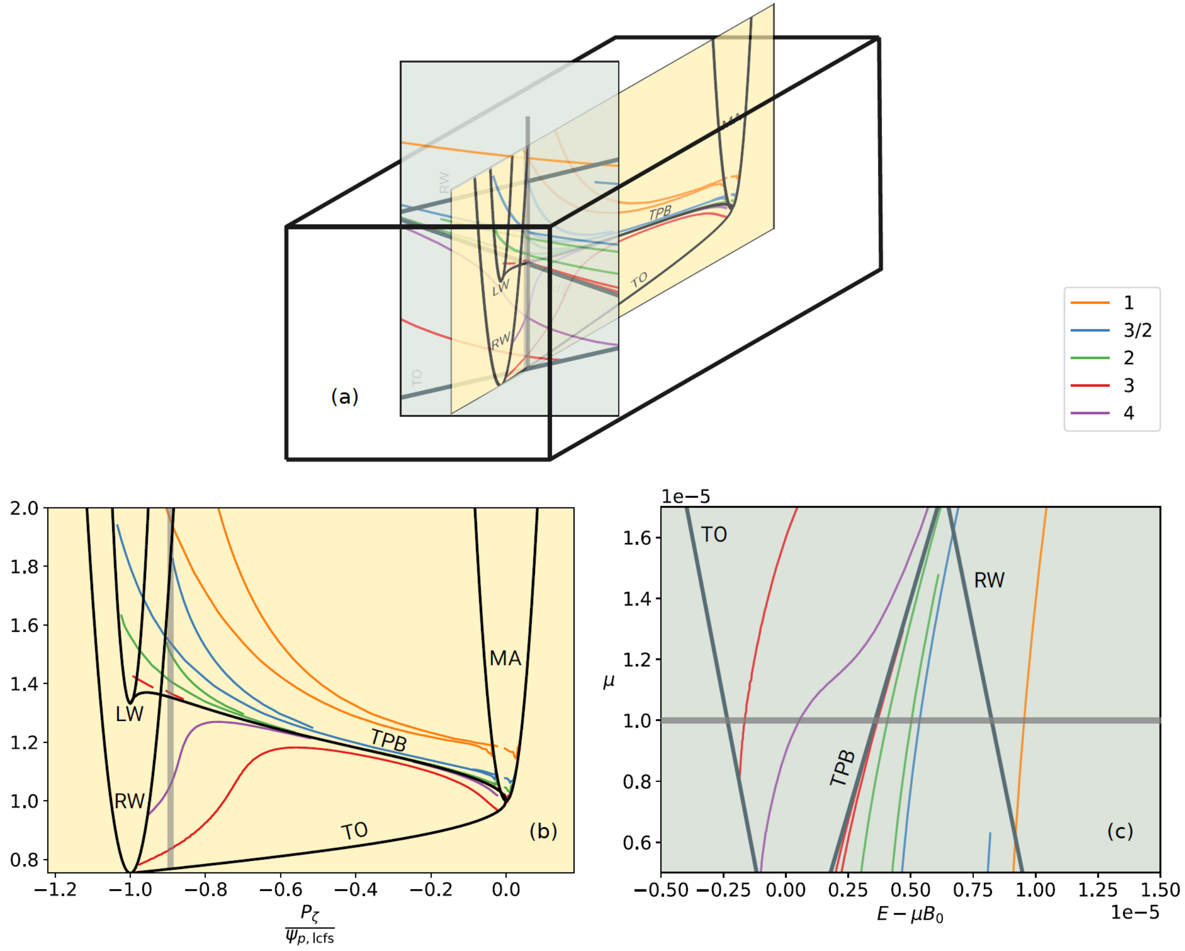
- The toroidal canonical momentum P_ζ (but not E) is conserved.
- The GC motion is constrained on drift surfaces defined by P_ζ=const.
- The resonance condition in this case does not involve q_{kin}, but is given by

$$\lambda = \omega/\omega_\theta$$

where λ takes integer values for primary resonances and rational values for higher order resonances.

[Zestanakis, Antonenas, Anastasiou, Falessi, Kominis, submitted to NF (2026)]

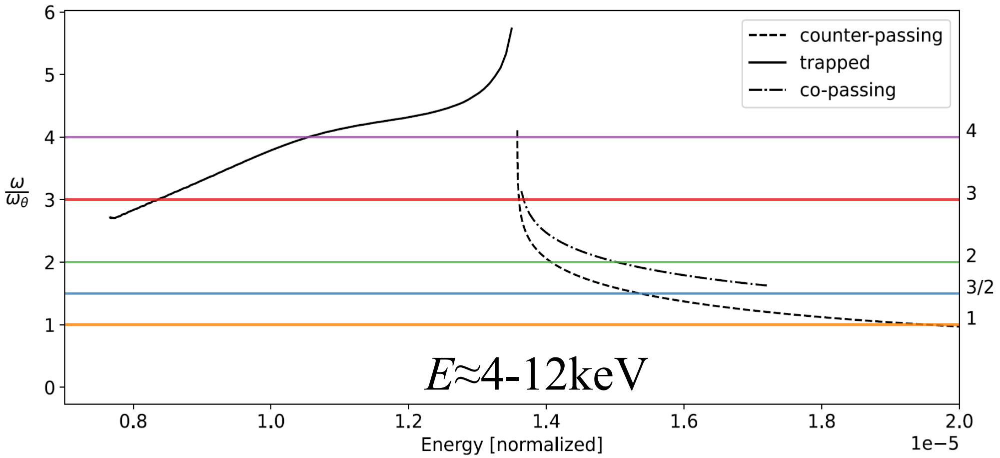
Tomography of the COM space resonance structure



$\mu = 1\times 10^{-5}$ (μB_0=5keV)

$P_\zeta = -4.8\times 10^{-2}$

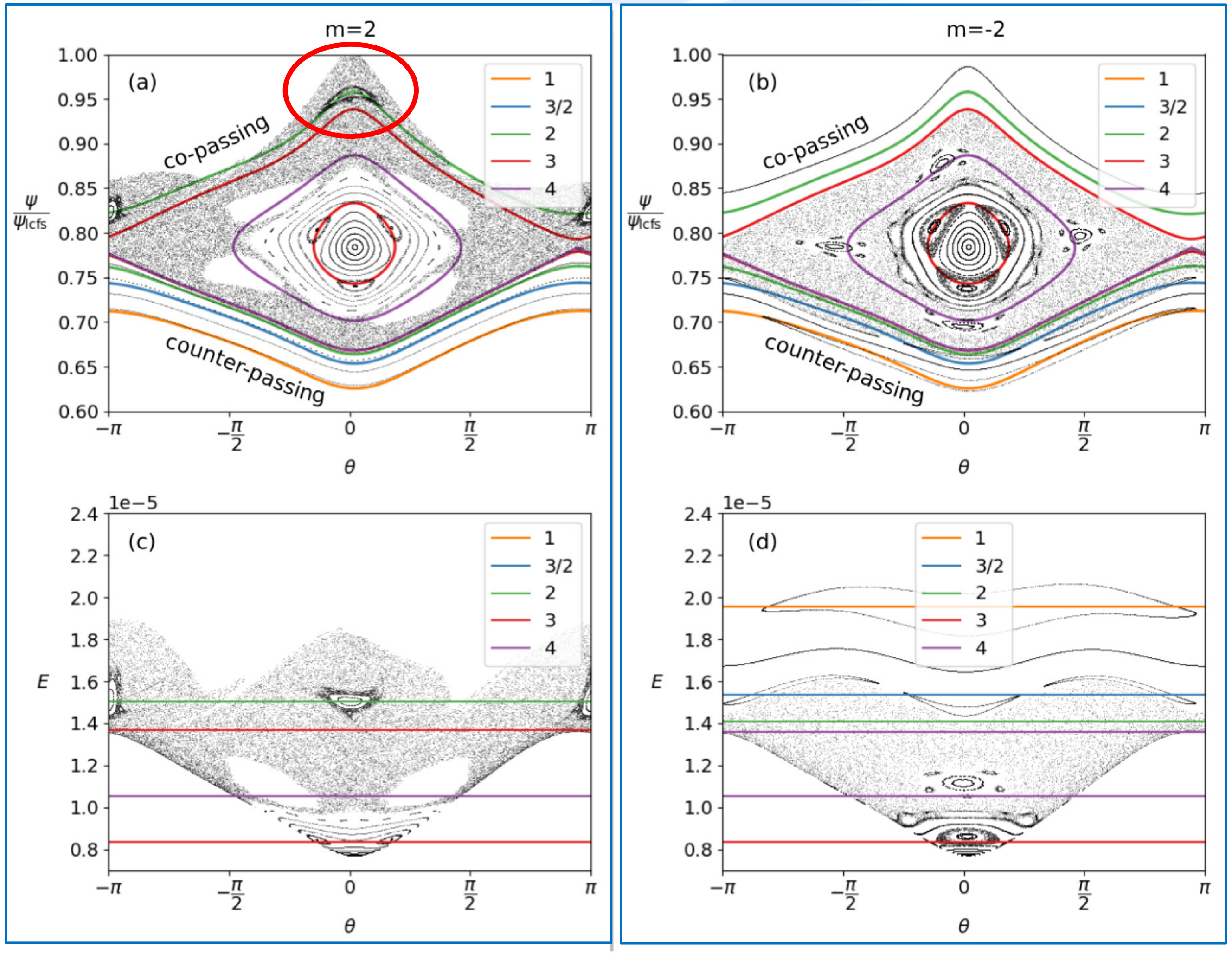
RESULTS: AUG (time-dependent axisymmetric perturbations - GAM)



Although the original poloidal mode number m is not present in the resonance condition and does not therefore affect the location of the resonances, it does affect the strength of the interaction through the modification of the harmonic amplitudes in perturbed Hamiltonian.

As a result, different resonant islands and chaotic domains appear for different directions of propagation.

→ Importance of the Orbital Spectral Analysis (OSA) of the perturbative modes



$\rho_\parallel > 0$ $\rho_\parallel < 0$

$\varepsilon = 1\times10^{-4}$

[Zestanakis, Antonenas, Anastasiou, Falessi, Kominis, submitted to NF (2026)]

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TRANSPORT BARRIERS: effect of an edge-localized radial electric field

Transport barriers:

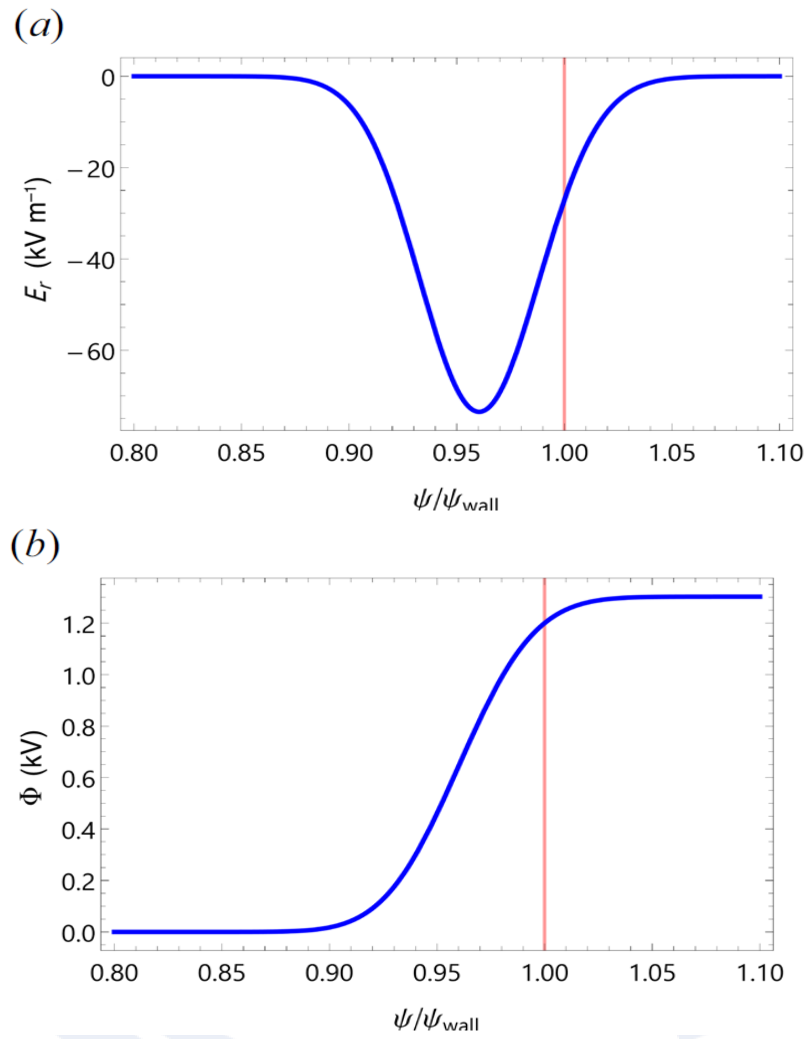
- correspond to shearless particle motion (non-twist condition)
- are formed at local extrema of the kinetic q factor
- low energy, small pitch-angle particles following magnetic field lines have transport barriers at minima of the magnetic q factor

The presence of an edge-localized radial electric field:

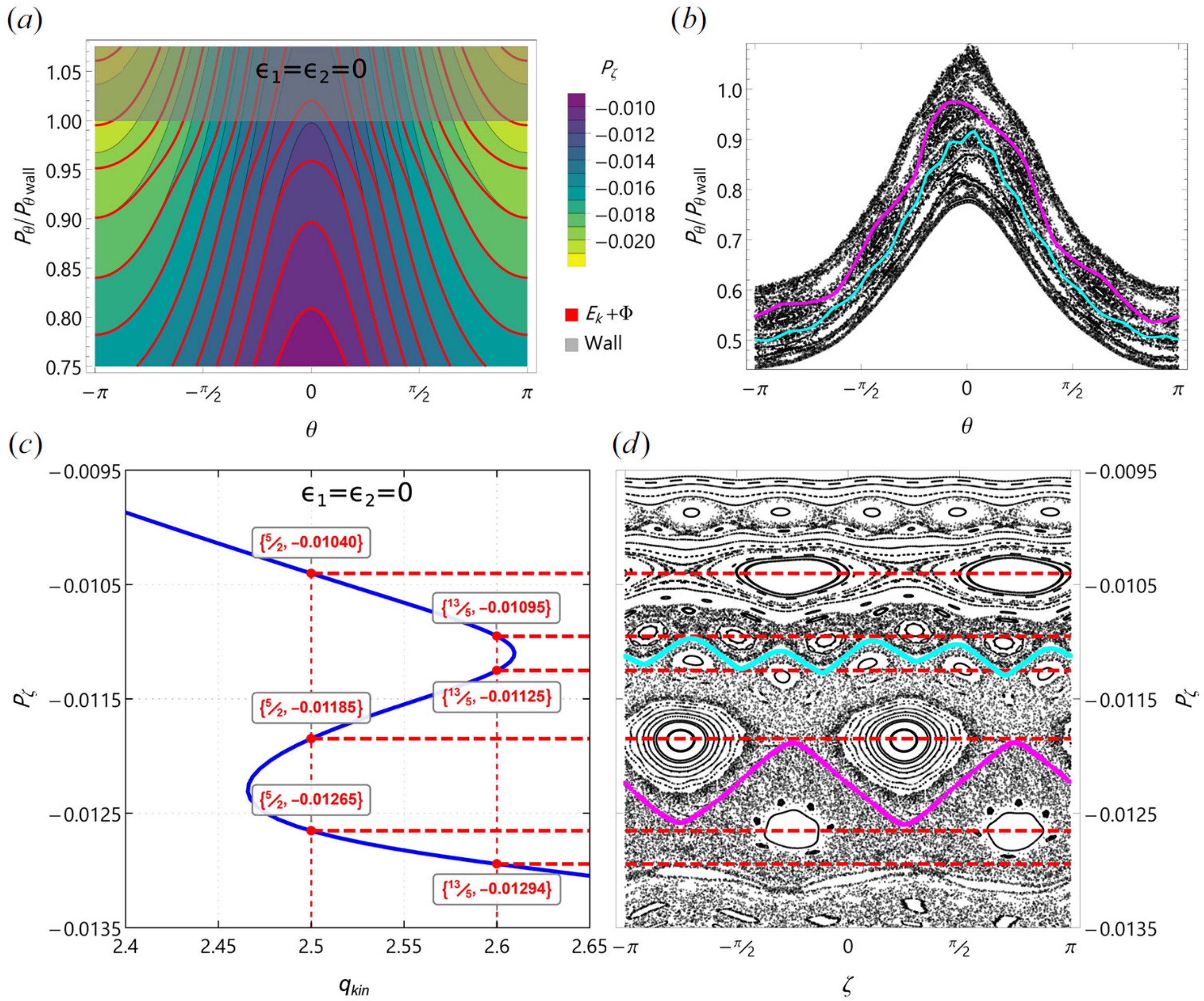
- does not break integrability of the GC motion in an axisymmetric equilibrium

$$H = \frac{(P_\zeta + \psi_p(P_\zeta, P_\theta))^2}{2g^2(P_\zeta, P_\theta)} B^2(P_\theta, \theta; P_\zeta) + \mu B(P_\theta, \theta; P_\zeta) + \Phi\big(\psi(P_\zeta, P_\theta)\big)$$

- **drastically modifies the kinetic q factor near the edge**



TRANSPORT BARRIERS: effect of an edge-localized radial electric field



Large Aspect Ratio (LAR)

E=18keV, μB_0=10keV

(m_1,n_1)=(5,2) & (m_2,n_2)=(13,5)

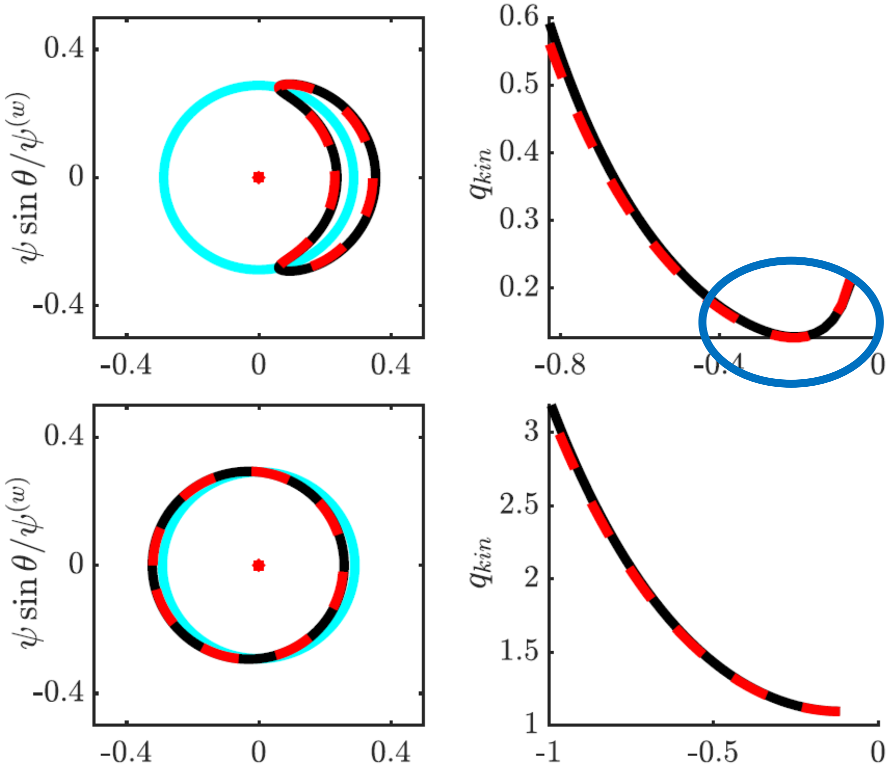
[G. Anastasiou, P. Zestanakis, Y. Antonenas, E. Viezzer, Y. Kominis, JPP 2024]

TRANSPORT BARRIERS: intrinsic conditions

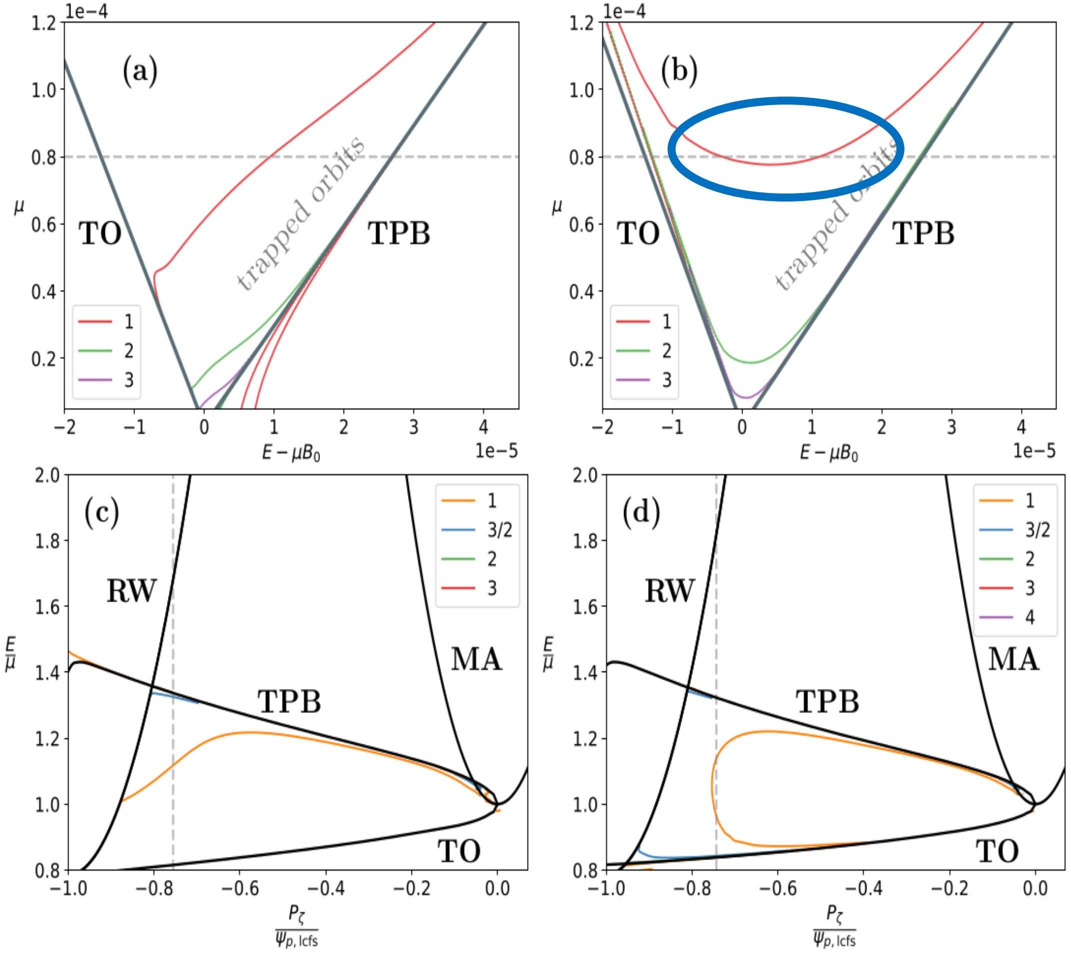
Transport barriers also appear without a radial electric field:

the same resonance condition is fulfilled on both sides of a local extremum of the kinetic q factor or a limit point of a resonance branch

LAR



DTT - PT **DTT - NT**



OUTLINE

- MOTIVATION & METHODOLOGY OVERVIEW
- GEOMETRICAL CALCULATION OF THE ORBITAL FREQUENCY SPECTRUM (OFS)
  – PREDICTION OF RESONANCES LOCATIONS
  – OFS DEPENDENCE ON THE SHAPE OF THE BACKGROUND MAGNETIC FIELD
  – ORBITAL SPECTRAL ANALYSIS (OSA) OF PERTURBATIONS
- FORMATION OF TRANSPORT BARRIERS
- **ONGOING AND FUTURE WORK**
  – STELLARATORS
  – COLLECTIVE PARTICLE DYNAMICS
- SUMMARY AND CONCLUSIONS

ONGOING AND FUTURE WORK

Stellarators:

In quasi-helical and quasi-symmetric equilibria where unperturbed GC orbits are near-integrable. Such equilibria in general can be decomposed into a dominant helically symmetric integrable part and a secondary non-integrable part which will be treated here as perturbation.

Collective particle dynamics:

- Utilization of the Canonical Perturbation Method
- Distribution functions in the Action space and semi-analytical calculation of ensemble-averaged quantities, such as radial positions and momentum variations.
- Applications:
  - calculation of transport/diffusion coefficients (quasi-linear, higher-order?)
  - calculation of the particle-dependent source terms driving the perturbative mode evolution (closing the self-consistent loop the mode-particle interactions)
  - radial transport optimization of stellarator equilibria

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SUMMARY AND CONCLUSIONS

Outcomes of the proposed methodology:

- **Orbital Frequency Spectrum** and mode-particle resonance conditions calculations for across **tokamak and stellarator configurations**, enabling **precise prediction of resonances and transport barriers** in COM space for all ion species.
- **Orbital Spectral Analysis** of perturbative modes, allowing the accurate **calculation of resonance widths** and **prediction of enhanced transport** through **resonance overlap**.
- Construction of **comprehensive resonance diagrams** incorporating essential mode-particle interaction information, including both resonance locations and island widths.
- Systematic investigation of the **effects of equilibrium shaping parameters** (Shafranov shift, elongation, positive and negative triangularity) on **resonance diagram topology**, and their implications for **transport**.
- **Collective particle behavior** (ensemble-averaged quantities) **radial transport optimization in stellarators** and determination of **particle-dependent driving terms** of the mode evolution equations for self-consistent consideration of the mode-particle interactions.

QUESTIONS & DISCUSSION

Thank you for your attention!

Questions?