`stella` Update: Full-Flux Annulus

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TSVV-J meeting

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`stella`: Introduction

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- Each term in the gyrokinetic (GK) equation is solved independently

$$\frac{\partial g_\nu}{\partial t} + \underbrace{\underbrace{\mathcal{S}_\nu[g_\nu,\varphi_\nu]}_{\text{streaming}} + \underbrace{\mathcal{M}_\nu[g_\nu]}_{\text{mirror}}}_{\text{Implicit}} + \underbrace{\underbrace{\mathcal{D}_\nu[g_\nu,\varphi_\nu] + \mathcal{G}_\nu[\varphi_\nu]}_{\text{drifts}} + \underbrace{\mathcal{N}_\nu[g_\nu,\varphi_\nu]}_{\text{non-linear}} = \underbrace{\mathcal{C}_\nu[\{g_{\nu'}\},\{\varphi_{\nu'}\}]}_{\text{collisions}}}_{\text{Explicit}},$$

$$\partial_t g_\nu = \textstyle\sum_{i=1}^{3}(\partial_t g_\nu)_i \quad \left| \quad \begin{aligned} (\partial_t g_\nu)_1 + \mathcal{D}_\nu[g_\nu,\varphi_\nu] + \mathcal{G}_\nu[\varphi_\nu] + \mathcal{N}_\nu[g_\nu,\varphi_\nu] &= \mathcal{C}_\nu \\ (\partial_t g_\nu)_2 + \mathcal{M}_\nu[g_\nu] &= 0 \\ (\partial_t g_\nu)_3 + \mathcal{S}_\nu[g_\nu,\varphi_\nu] &= 0 \end{aligned} \right.$$

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$$\mathcal{S}_\nu \doteq \quad v_{\rm th} \quad v_\parallel \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} z \left(\frac{\partial g_\nu}{\partial z} + \frac{Z_\nu}{T_\nu} \frac{\partial \varphi_\nu}{\partial z} F_{0,\nu} \right)$$

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$$\frac{g_\nu^{n+1} - g_\nu^{n}}{\Delta t} = -v_{\mathrm{th}} v_{\|} \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} z \left(\frac{\partial g_\nu^{n}}{\partial z} + \frac{Z_\nu e}{T_\nu} \frac{\partial \varphi_\nu^{n}}{\partial z} F_{0,\nu} \right) \tag{1}$$

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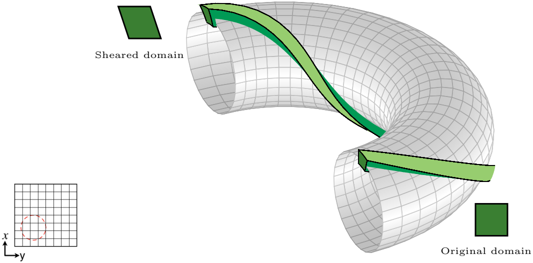
- Simulates turbulence on gyroradius scales. Solves for $g = \langle \delta f \rangle_{\mathbf{R}_\nu}$
- Uses $\{v_\parallel, \mu\}$ for the velocity space coordinates → good for stellarators

`stella`: Introduction



- Simulation coordinates: $(x, y, z) \rightarrow (\psi, \alpha, \zeta)$

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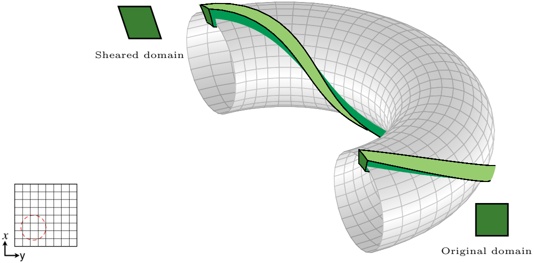

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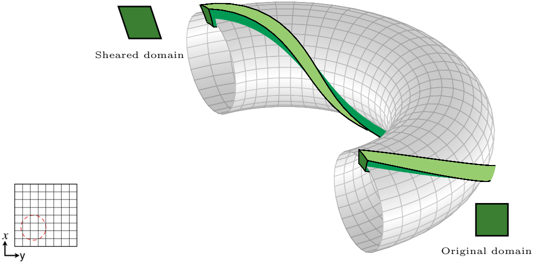

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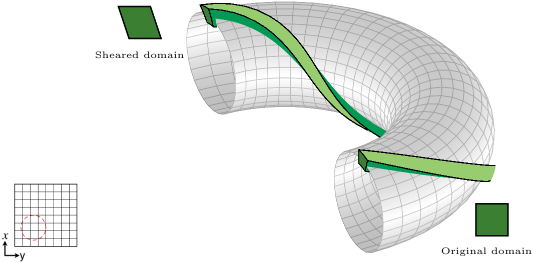

- Simulation coordinates: $(x, y, z) \to (\psi, \alpha, \zeta)$
- Initialise some δf and ϕ at $t = 0$ on simulation domain
- Evolve gyrokinetic equation pseudo-spectrally
  - Decay in $v_\parallel$; $g(t, \boldsymbol{x}, v_\parallel \to \pm\infty, \mu) \to 0$
  - Turbulence is taken as periodic in perpendicular directions, $k_x, k_y \gg 1/L$
  - Use twist-and-shift boundary conditions in z to capture extended modes

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Flux tube formalism

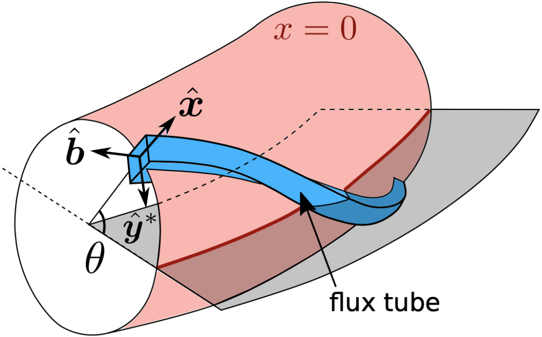

Image: Nicolas Christen, Bistable turbulent transport in fusion plasmas with rotational shear (2021)

- Flux-tube simulations are sufficient for tokamaks because we can stitch our flux tubes together

Flux tube formalism

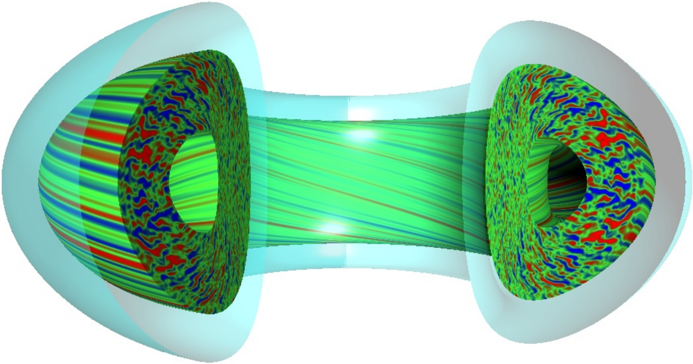

Image: J.Candy, Waltz, GYRO simulation of DIII-D

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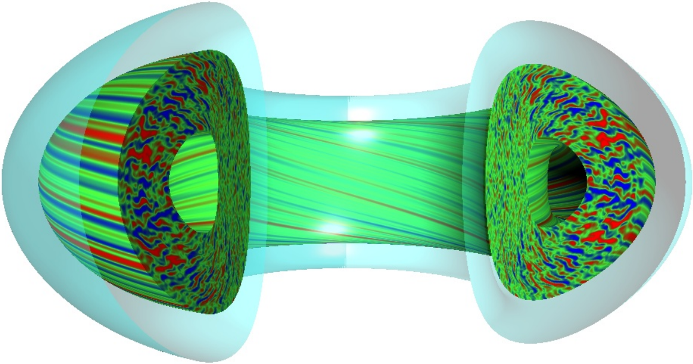
Flux tube formalism: pros and cons



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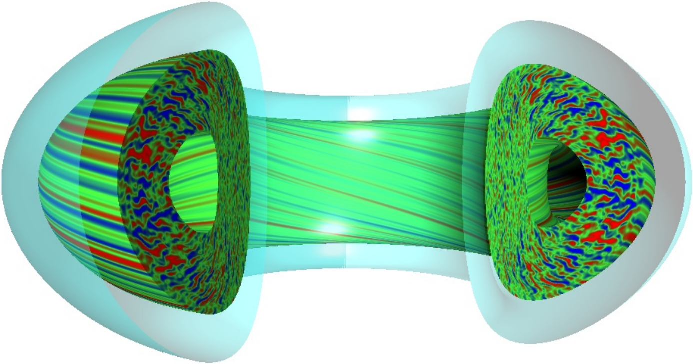

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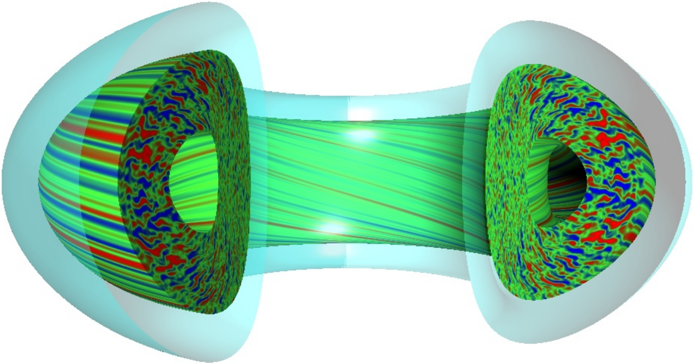

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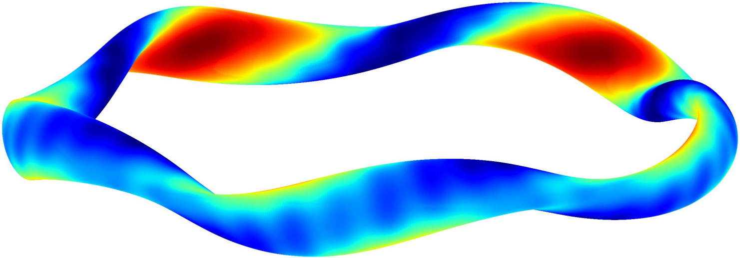
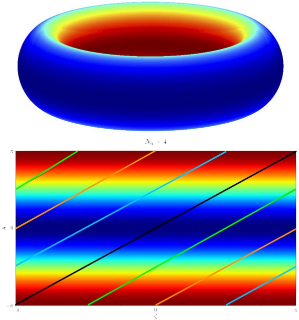


- Fast codes: perform many simulations in quick succession
- Easy to interpret: normal modes are well defined
- Retains spectral accuracy in spacial derivatives
- Does not capture global effects like coupling between different field lines

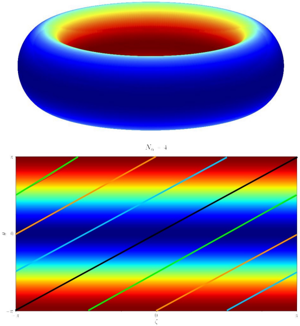
Motivation

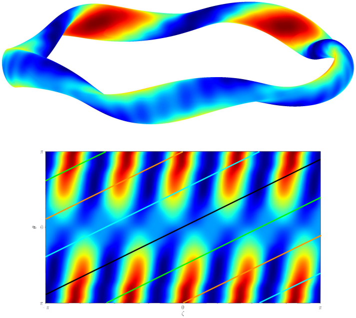
- Axisymmetry means all field lines are equivalent



Motivation

- Axisymmetry means all field lines are equivalent
- Stellarator magnetic geometry varies with field line: single field line is **not** sufficient!





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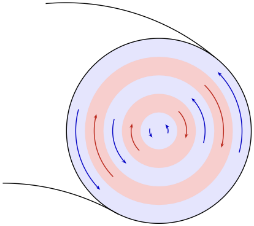
- Modes on different field lines interact → complicates algorithms due to α-inhomogeneity
- Particularly relevant for zonal flows



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Flux annulus formalism: motivation

- Stellarator geometry varies with field line
- Method of stitching together flux tubes no longer holds

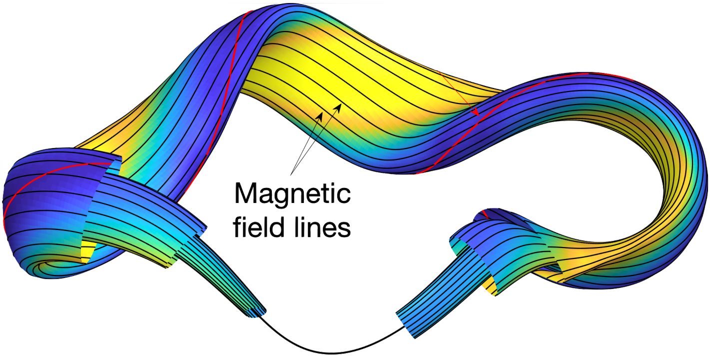
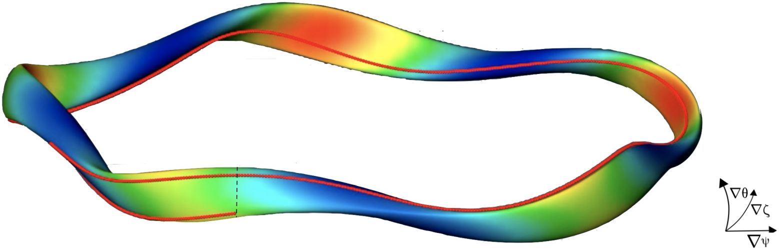


Image: UMD stellarator group

Flux annulus framework

- Domain is now 2π in ζ, not 2π in θ



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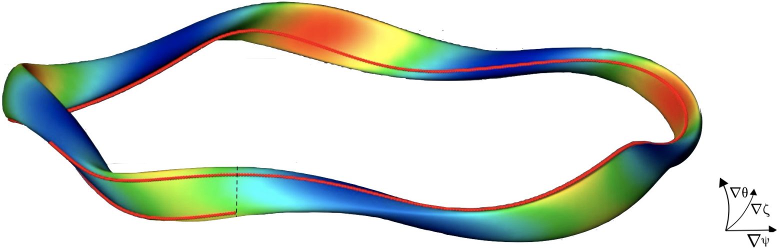

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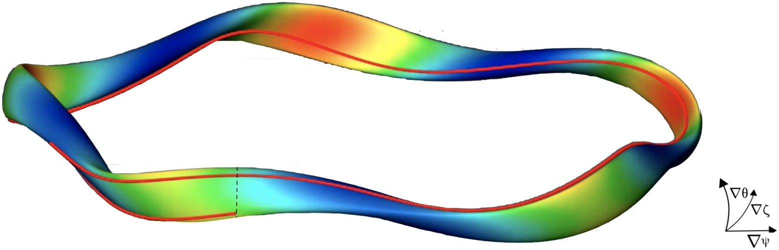

- Evolve pseudo-spectrally to retain spectral accuracy in derivatives
- N_y field lines that cover different geometry and are now coupled together

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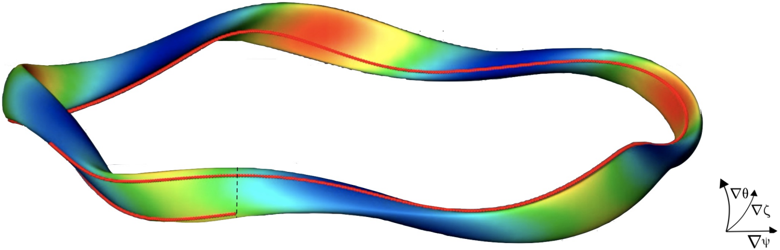

- Evolve pseudo-spectrally to retain spectral accuracy in derivatives
- N_y field lines that cover different geometry and are now coupled together
- Apply twist-and-shift to entire poloidal domain

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- Geometric factors are α-dependent
- Bessel functions $J_0(a_{\boldsymbol{k}})$ with $a_{\boldsymbol{k}} = \frac{k_\perp v_\perp}{\Omega_s}$

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- α-inhomogeneity leads to convolutions

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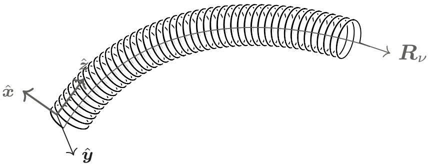
- Gyro-averaging introduces coupling between different k_y-modes → no longer a local operation



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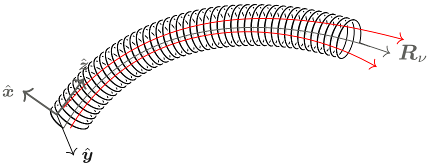

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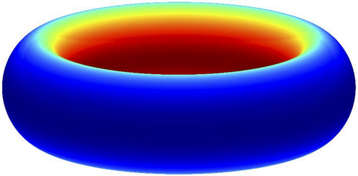
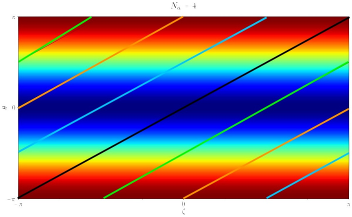
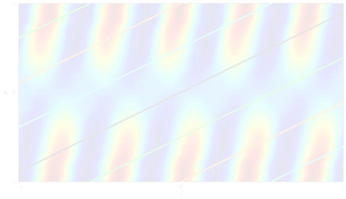
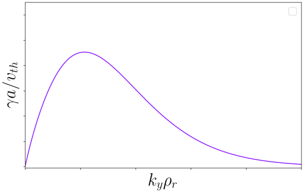
- Develop iterative-implicit method → accurate treatment without stringent CFL restriction

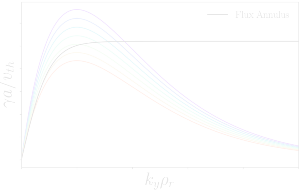
How 3D geometry modifies expectations











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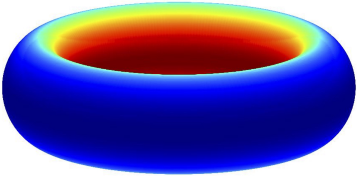

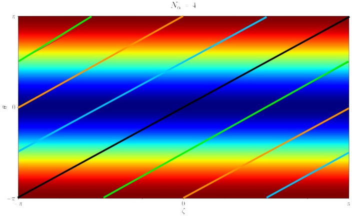

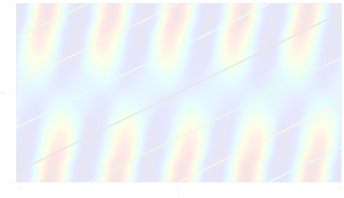

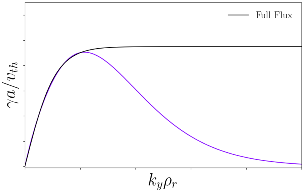

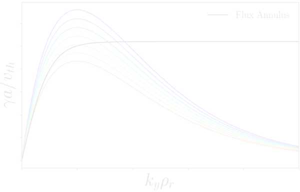

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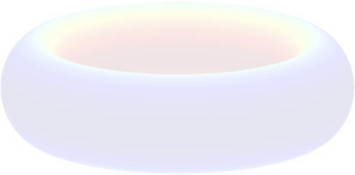

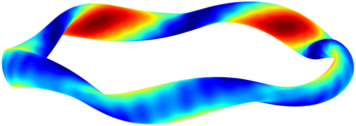

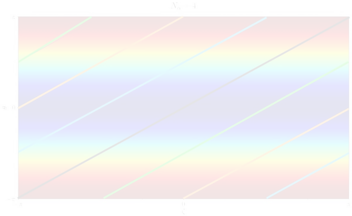

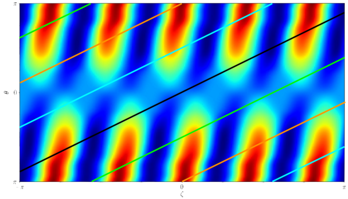

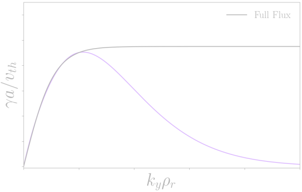

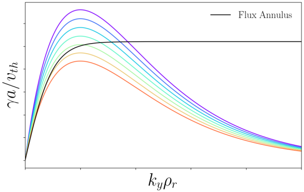

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Benchmarked

- Simulates gyrokinetic turbulence on a full-flux annulus in general 3D geometry
- Capable of simulating adiabatic and kinetic electrons

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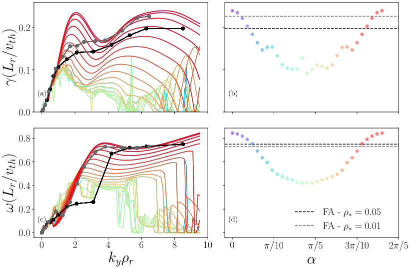
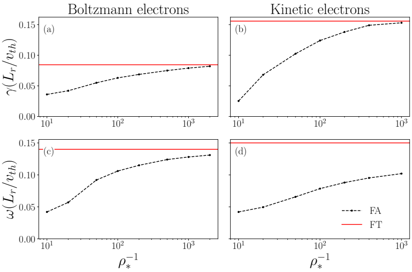
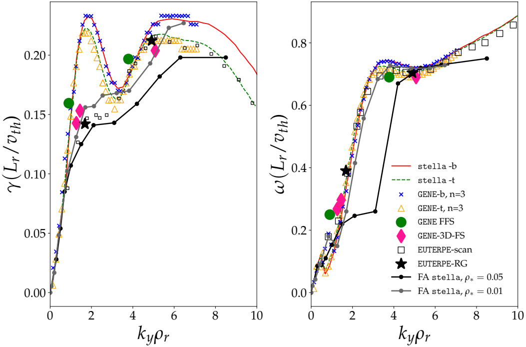
- Capable of simulating adiabatic and kinetic electrons
- Has been benchmarked against stella flux tube code in appropriate limits





Benchmarked

- Simulates gyrokinetic turbulence on a full-flux annulus in general 3D geometry
- Capable of simulating adiabatic and kinetic electrons
- Has been benchmarked against stella flux tube code in appropriate limits
- Shows good agreement with other full-surface codes



E. Sánchez, J. M. García-Regaña, A. B. Navarro, J. H. E. Proll, C. M. Moreno, A. González-Jerez, I. Calvo, R. Kleiber, J. Riemann, J. Smoniewski, et al.
Gyrokinetic simulations in stellarators using different computational domains.
Nuclear Fusion, 61(11):116074, 2021.

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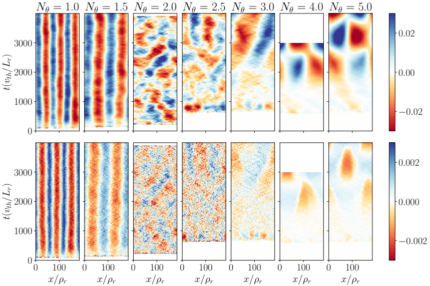

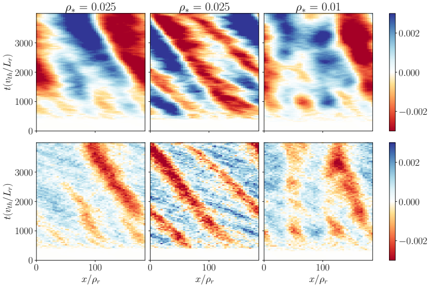

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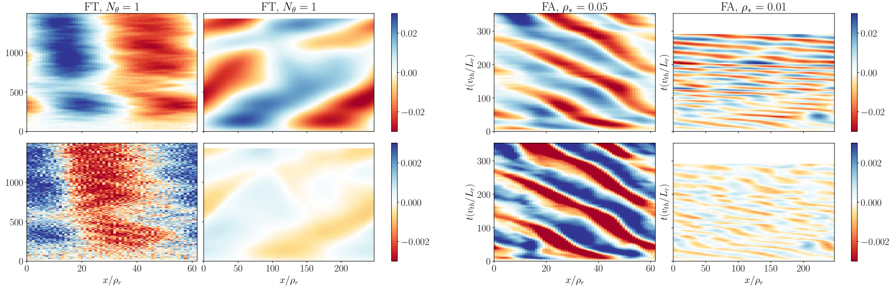
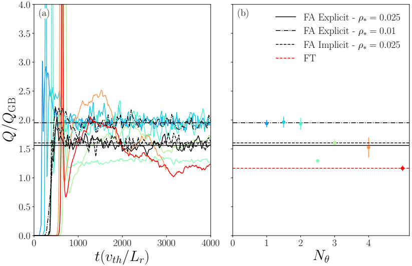
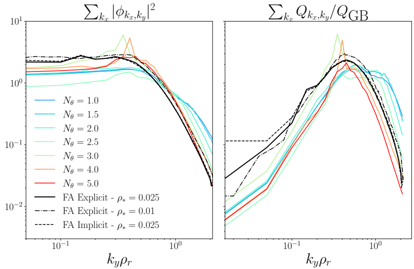


Simulation results: code efficiency

- Boltzmann electrons:

Simulation Case	Time Step (dt)	CPU Hours
FT, $N_\theta = 1$	0.049666	26,880
FT, $N_\theta = 5$	0.030470	80,640
FA (explicit), $\rho_* = 0.025$	0.1	43,550
FA (implicit), $\rho_* = 0.025$	0.1	51,690
FA (explicit)[1], $\rho_* = 0.01$	0.1	35,420





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[1] Reduced resolution

Simulation results: code efficiency

- Kinetic electrons:

Parameter	FT_1	FT_2	FA_1	FA_2
Δt	1.6577×10^{-2}	2.1195×10^{-2}	7.7464×10^{-3}	5.6368×10^{-3}
t_{final}	1500	1470	355	270
CPU hours	46,080	76,800	107,520	128,640

- $\Delta t =$ `1E-006` for explicit kinetic electron simulations → numerically unstable
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Improvements

- NB: this was before `stella` improvements → need to re-run with updated version
- Already identified inefficiencies
- Anticipate this can be reduced to $\sim \times 5$ speed of flux-tube
- Removing strong coupling (e.g. for CBC) improves code by ~ 10 times for KE

Summary and future work

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- Developed an iterative-implicit algorithm to deal with full-flux annulus effects, with kinetic electrons, in arbitrary geometry
- FFA `stella` has been benchmarked in appropriate limits and against existing codes

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Future Work

- Investigate how zonal flows are supported in stellarators
- Combine full-flux with electromagnetic `stella`
- Optimise for computational speed-up

Backup slides: Derivation of twist-and-shift boundary conditions

*

$$A(t,x,y,z) = \sum_k \hat{A}_{k_x,k_y}(t,z)e^{ik_y(y-y_0)+ik_x(x-x_0)} \tag{1}$$

Set $y_0 = 0$, $x_0 = 0$ $A(t,x,y(x,\theta,z),z) = A(t,x,y'(\theta,z+2p\pi),z+2p\pi)$
But $y = y(\theta,z)$

$$\sum_k \hat{A}_{k_x,k_y}(t,z)e^{ik_y y+ik_x x} = \sum_k \hat{A}_{k_x,k_y}(t,z')e^{ik_y(y'(\theta,z'))+ik_x x} \tag{2}$$

So $y'(\theta,z') = y + \frac{\partial y}{\partial z}2\pi p = y + 2\pi p\frac{\partial y}{\partial \alpha}\frac{\partial \alpha}{\partial z} = y - 2\pi p\iota(\psi)\frac{\partial y}{\partial \alpha}$ Remembering $\iota(\psi) = \iota(\psi_0) + \iota'\frac{\partial \psi}{\partial x}$ so $y' = y - 2\pi p\iota(\psi_0)\frac{\partial y}{\partial \alpha} - 2(x-x_0)\pi p\iota'\frac{\partial y}{\partial \alpha}\frac{\partial \psi}{\partial x}$

$$\sum_k \hat{A}_{k_x,k_y}(t,z)e^{ik_y y+ik_x x} = \sum_k \hat{A}_{k_x,k_y}(t,z{+}2\pi p)e^{ik_y y+i(k_x-2\pi p\iota'\frac{\partial y}{\partial \alpha}\frac{\partial \psi}{\partial x}k_y)x'-i2\pi pk_y\iota p\frac{\partial y}{\partial \alpha}} \tag{3}$$

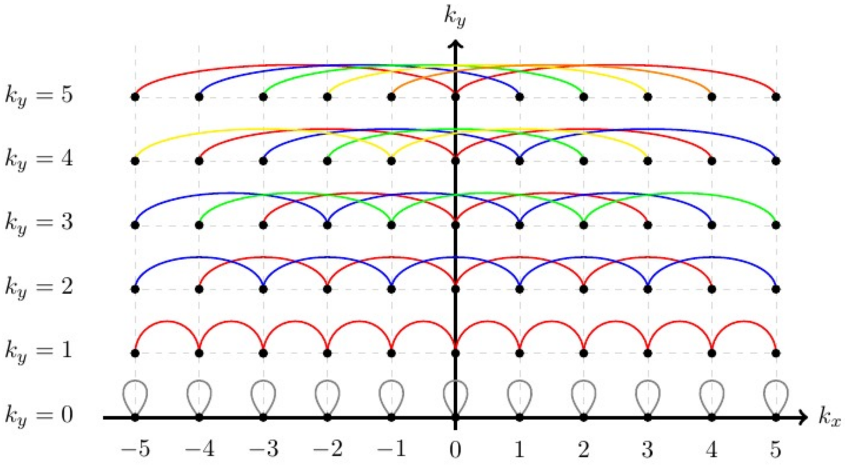
Let $\delta k_x = 2\pi p\iota'\frac{\partial y}{\partial \alpha}\frac{\partial \psi}{\partial x}k_y$ and $\Delta = -2\pi pk_y\iota\frac{\partial y}{\partial \alpha}$

$$\sum_k \hat{A}_{k_x,k_y}(t,z)e^{ik_y y+ik_x x} = \sum_k \hat{A}_{k_x,k_y}(t,z+2\pi p)e^{ik_y y+i(k_x-\delta k_x)x'}e^{i\Delta} \tag{4}$$

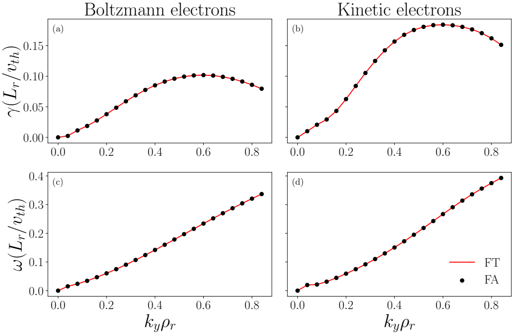
So relate $k_x = k_x' - 2\pi p\iota'\frac{\partial y}{\partial \alpha}\frac{\partial \psi}{\partial x}k_y$

Backup slides: Eigenmode chains

*



Backup slides: Miller geometry linear comparison



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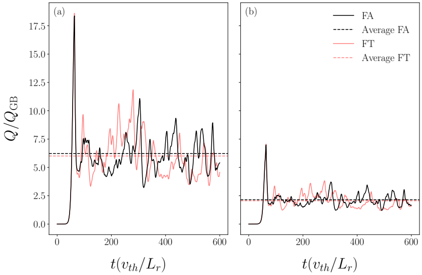

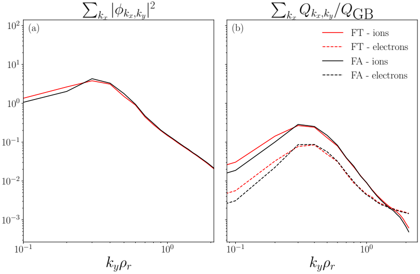

Backup slides: Adiabatic, W7X comparison between GX and stella