GENE-X

Turbulent Impurity Transport in the Edge and Scrape-Off-Layer

Focus: Development of Moment Approach for Coulomb Collisions

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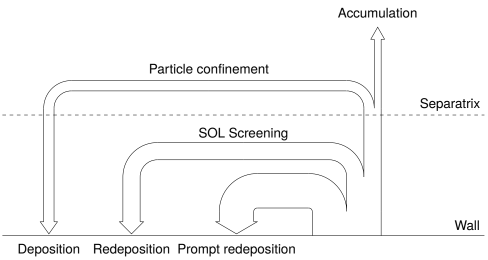
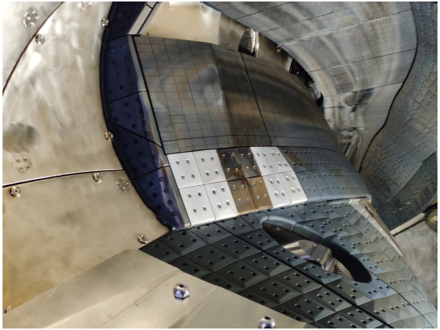
Impurities





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Coulomb collisions

$$C_{\alpha\beta}(f_\alpha) = \underbrace{\frac{\Gamma_{\alpha\beta}}{2} \int \mathrm{d}^3\mathbf{v}' \frac{\partial}{\partial \mathbf{v}} \cdot \frac{u^2\mathbb{1} - \mathbf{u}\mathbf{u}}{u^3} \cdot \left(-\frac{m_\alpha}{m_\beta} f_\alpha(\mathbf{v}) \frac{\partial f_\beta(\mathbf{v}')}{\partial \mathbf{v}'} + \frac{\partial f_\alpha(\mathbf{v})}{\partial \mathbf{v}} f_\beta(\mathbf{v}') \right)}_{\text{Landau form}}$$

$$= \underbrace{\Gamma_{\alpha\beta} \frac{\partial}{\partial \mathbf{v}} \cdot \left(-\frac{m_\alpha}{m_\alpha + m_\beta} \frac{\partial H_{\alpha\beta}}{\partial \mathbf{v}} f_\alpha + \frac{1}{2} \frac{\partial^2 G_{\alpha\beta}}{\partial \mathbf{v} \partial \mathbf{v}} \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} \right)}_{\text{Rosenbluth form}}$$

with

$$G_{\alpha\beta} = \int \mathrm{d}^3\mathbf{v}'\, f_\beta(\mathbf{v}')|\mathbf{v} - \mathbf{v}'|, \qquad H_{\alpha\beta} = \frac{m_\alpha + m_\beta}{m_\beta} \int \mathrm{d}^3\mathbf{v}'\, \frac{f_\beta(\mathbf{v}')}{|\mathbf{v} - \mathbf{v}'|},$$

satisfying

$$\nabla_v^2 H_{\alpha\beta} = -4\pi \frac{m_\alpha + m_\beta}{m_\beta} f_\beta, \qquad \nabla_v^2 G_{\alpha\beta} = \frac{2m_\beta}{m_\alpha + m_\beta} H_{\alpha\beta}.$$

Model operators

Examples of model operators:

- Lorentz operator: $f_\beta = \delta(\mathbf{v})$
- Lenard-Bernstein/Dougherty operator: $f_\beta = \mathcal{M}_\beta,\ \partial f_\alpha / \partial\theta \approx 0$
- Linearized operator $f_\beta = \mathcal{M}_\beta$

Linearization:

$$C_{\alpha\beta}(f_\alpha, f_\beta) = \underbrace{C_{\alpha\beta}(\mathcal{M}_\alpha, \mathcal{M}_\beta) + C_{\alpha\beta}(\delta f_\alpha, \mathcal{M}_\beta)}_{C_{\alpha\beta}(f_\alpha, \mathcal{M}_\beta):\ \text{test particle operator easy to solve}} + \underbrace{C_{\alpha\beta}(\mathcal{M}_\alpha, \delta f_\beta)}_{\text{field particle operator hard to solve!}} + \underbrace{C_{\alpha\beta}(\delta f_\alpha, \delta f_\beta)}_{\text{field particle operator usually ignored}}$$

Multipole expansion

Linearized operators assumes $f \approx \mathcal{M}$. Better: $f = \mathcal{M}\left(1 + 2\frac{\mathbf{v}\cdot\mathbf{u}}{v_T^2}\right)$.

Even better with an expansion in Sonine polynomials and irreducible tensors:

$$
\begin{aligned}
f_\beta = \mathcal{M}_\beta \Bigg(&\left(N_0 L_0^{(1/2)} + N_1 L_1^{(1/2)} + N_2 L_2^{(1/2)} + \dots\right) \\
&+ 2\frac{\mathbf{v}}{v_T^2} \cdot \left(\mathbf{u}_0 L_0^{(3/2)} + \mathbf{u}_1 L_1^{(3/2)} + \mathbf{u}_2 L_2^{(3/2)} + \dots\right) \\
&+ 2\frac{\mathbf{v}\mathbf{v} - (v^2/3)\mathbb{1}}{m_\beta n_\beta v_T^4} : \left(\Pi_0 L_0^{(5/2)} + \Pi_1 L_1^{(5/2)} + \Pi_2 L_2^{(5/2)} + \dots\right) + \dots \Bigg). \\
= \mathcal{M} \sum_{lk} & L_k^{(l+1/2)}(v^2/v_T^2) \mathsf{P}_l(\hat{\mathbf{v}}) \odot \mathsf{M}_{kl}
\end{aligned}
$$

Connection to transport/fluid theory

Assuming that $f_\alpha \approx \mathcal{M}_\alpha$, we approximate Vlasov equation as

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial \mathcal{M}_\alpha}{\partial \mathbf{x}} + \frac{q_\alpha}{m_\alpha}\mathbf{E} \cdot \frac{\partial \mathcal{M}_\alpha}{\partial \mathbf{v}} = -\frac{q_\alpha}{m_\alpha c}\mathbf{v} \times \mathbf{B} \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} + \sum_\beta C_{\alpha\beta}(f_\alpha).$$

Using

$$\frac{\partial \mathcal{M}}{\partial n} = \frac{\mathcal{M}}{n}, \qquad \frac{\partial \mathcal{M}}{\partial T} = \left(\frac{mv^2}{2T} - \frac{3}{2}\right)\frac{\mathcal{M}}{T}, \qquad \frac{\partial \mathcal{M}}{\partial v} = -\frac{mv}{T}\mathcal{M},$$

the LHS becomes

$$\frac{\partial f_\alpha}{\partial t} + \underbrace{\mathbf{v}}_{\mathsf{P}_1} \cdot \left[\underbrace{\left(\frac{\partial \ln(n_\alpha T_\alpha)}{\partial \mathbf{x}} + \frac{q_\alpha}{T_\alpha}\mathbf{E}\right)}_{L_0^{(3/2)}(x_\alpha)\mathbf{A}_{\alpha,0}} + \underbrace{\left(\frac{mv^2}{2T} - \frac{5}{2}\right)\left(\frac{\partial \ln T_\alpha}{\partial \mathbf{x}}\right)}_{L_1^{(3/2)}(x_\alpha)\mathbf{A}_{\alpha,1}} \right] \mathcal{M}_\alpha$$

Generalized Spitzer problem

The RHS are moments of the collision operator acting on moments of the distribution function.
The off-diagonal terms couple different transport channels

$$\frac{\partial}{\partial t}\begin{pmatrix} u_{\alpha,0} \\ u_{\alpha,1} \\ u_{\alpha,2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2}A_{\alpha,0} \\ \frac{5}{4}A_{\alpha,1} \\ 0 \\ \vdots \end{pmatrix} n_\alpha v_{T,\alpha}^2 = \underbrace{\begin{pmatrix} C_{00} & C_{01} & C_{02} & \cdots \\ C_{10} & C_{11} & C_{12} & \cdots \\ C_{20} & C_{21} & C_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \cdot \begin{pmatrix} u_{\alpha,0} \\ u_{\alpha,1} \\ u_{\alpha,2} \end{pmatrix}}_{\sum_\beta \int \mathrm{d}^3\mathbf{v}\ v_{L_i^{(3/2)}} C_{\alpha\beta}\left(L_j^{(3/2)} u_{\alpha,j}\right)}$$

Burnett polynomials expansion

The Burnett expansion uses $Y_{lm}(\theta, \phi)$ instead of P$(\hat{\mathbf{v}})$:

$$f_\alpha = \sum_{klm} x_\alpha^{l/2} \exp(-x_\alpha) L_k^{(l+1/2)}(x_\alpha) Y_{lm}(\theta, \phi) f_{\alpha,klm}$$

Information of velocity-space is compressed into few degrees of freedom.

Spherical harmonics are "eigenfunctions" of the Poisson equation

$$\nabla_v^2 Y_{lm}(\theta, \phi) = -v^{-2} l(l+1) Y_{lm}(\theta, \phi).$$

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[0]See also: JY Ji and ED Held Phys. Plasmas 13 102103 (2006); R Jorge et al Jour. Plasma Phys. 85.6 (2019); P Donnel et al, Plasma Phys. Control. Fusion 63 025006 (2021); von Boetticher et al, Plasma Phys. Control. Fusion 66 105016 (2024)

Moment expansion of the Rosenbluth potentials

Moment-expansion of the Rosenbluth $H_{\alpha\beta}$ potential

$$\frac{m_\beta}{m_\alpha + m_\beta} H_{\alpha\beta} = \sum_{klmn} \underbrace{\frac{2\pi}{2l+1} v_{T,\beta}^2}_{H_{c,l}} c_{lkn} f_{\beta,lmk} Y_{lm} \underbrace{\left(x_\beta^{-(l+1)/2} l + n + \frac{3}{2}, x_\beta + x_\beta^{l/2} n + 1, x_\beta \right)}_{H_{v,ln}(x_\beta)}$$

and the Rosenbluth $G_{\alpha\beta}$ potential

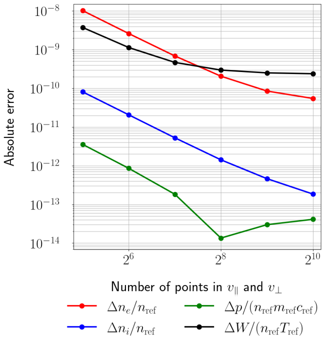
$$G_{\alpha\beta} = \sum_{klmn} \underbrace{\frac{2\pi}{(2l+1)(2l+3)} v_{T,\beta}^4}_{G_{c1,l}} c_{lkn} f_{\beta,lmk} Y_{lm} \underbrace{\left(x_\beta^{-(l+1)/2} l + n + \frac{5}{2}, x_\beta + x_\beta^{(l+2)/2} n + 1, x_\beta \right)}_{G_{v1,ln}(x_\beta)}$$

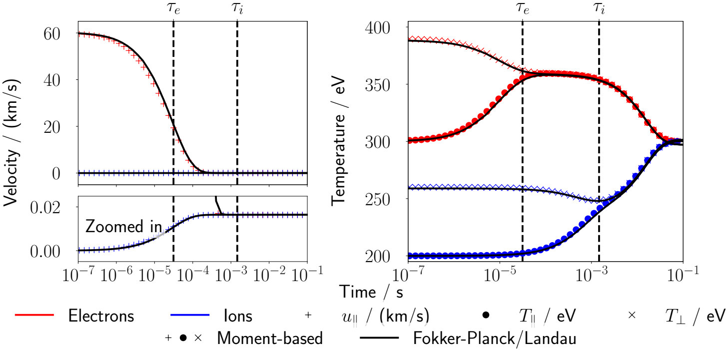
$$- \sum_{klmn} \underbrace{\frac{2\pi}{(2l+1)(2l-1)} v_{T,\beta}^4}_{G_{c2,l}} c_{lkn} f_{\beta,lmk} Y_{lm} \underbrace{\left(x_\beta^{-(l-1)/2} l + n + \frac{3}{2}, x_\beta + x_\beta^{l/2} n + 2, x_\beta \right)}_{G_{v2,ln}(x_\beta)},$$

Implementation

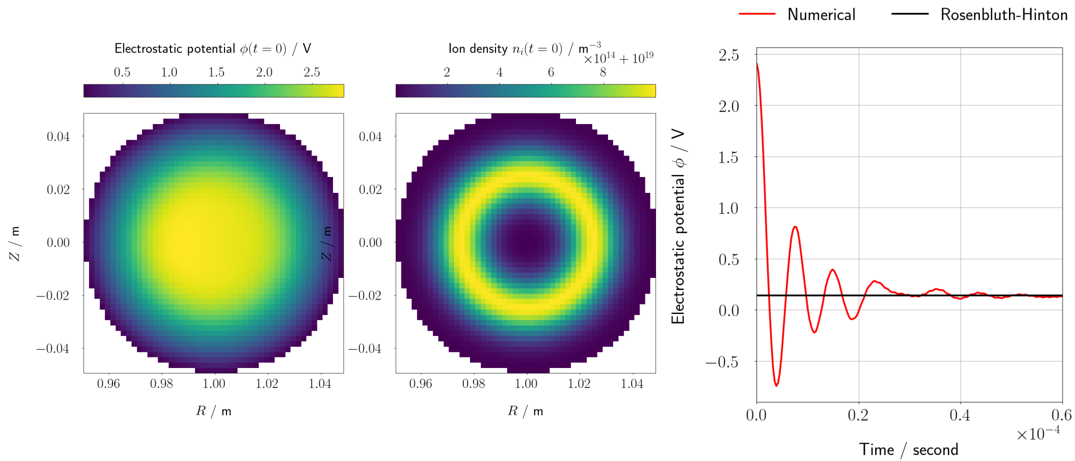
- **Gyrokinetic:** suitable for microturbulence in strongly magnetized plasmas
- **Global:** simulates whole system
- **Full-f:** solves the full distribution function, not only the fluctuations
- **Flux-coordinate-independent:** Suitable for edge region
- **Grid-based and spectral velocity discretization**

Classical physics verifications: Relaxation and conservation tests





Neoclassical physics verifications: Zonal flow damping (future work)



Conclusion and remarks

What was done:

- Derivation of nonlinear Fokker-Planck collision operator using a moment approach
- Implementation to grid and spectral velocity discretizations in `GENE-X`

Future works:

- Running impurity simulations with `GENE-X`
- Investigation of non-linear collisional effects on turbulence