EPFL

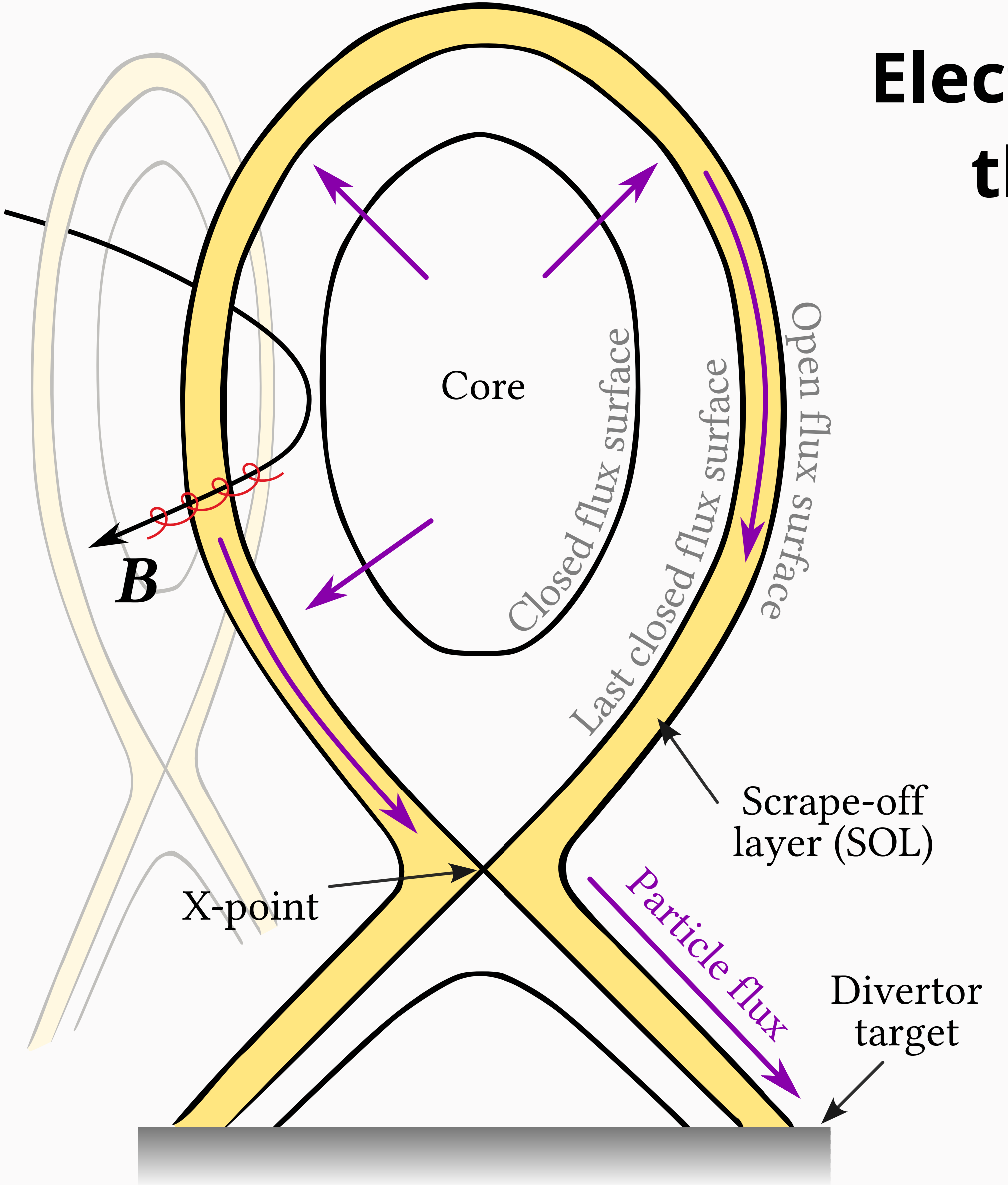
Kinetic simulations of the steady-state magnetized plasma sheath

TSVV-C meeting – 2026.06.02

Nicole Vadot, Thomas Stucker, Sam Zeegers,
Alessandro Geraldini and Stephan Brunner

Swiss
Plasma
Center

Electrons are faster to reach the wall, forming a small, non-neutral region



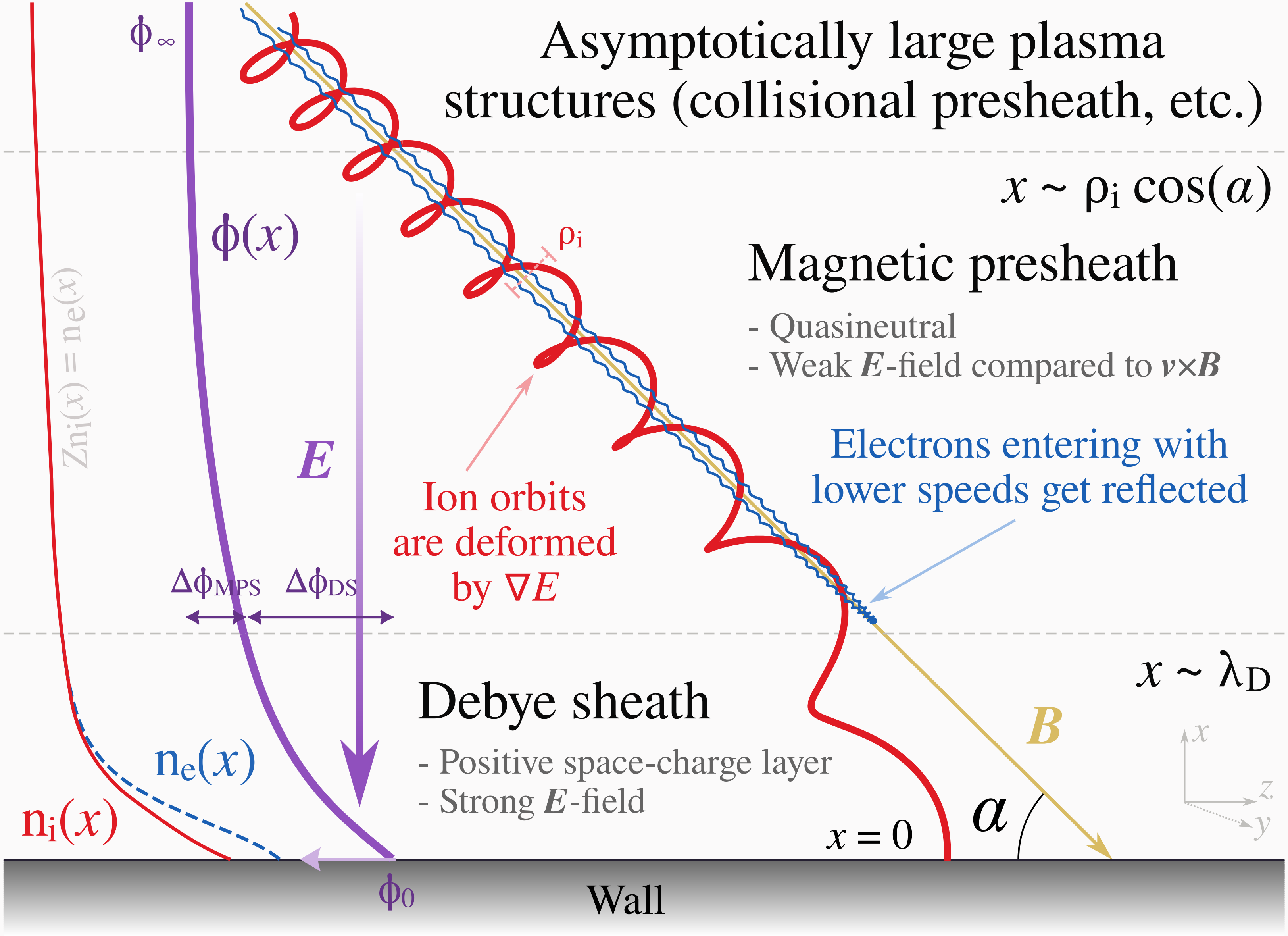
Imperfect confinement

⇒ Plasma from core flows into scrape-off layer, where the **open field lines** (and therefore trajectories) terminate at the wall

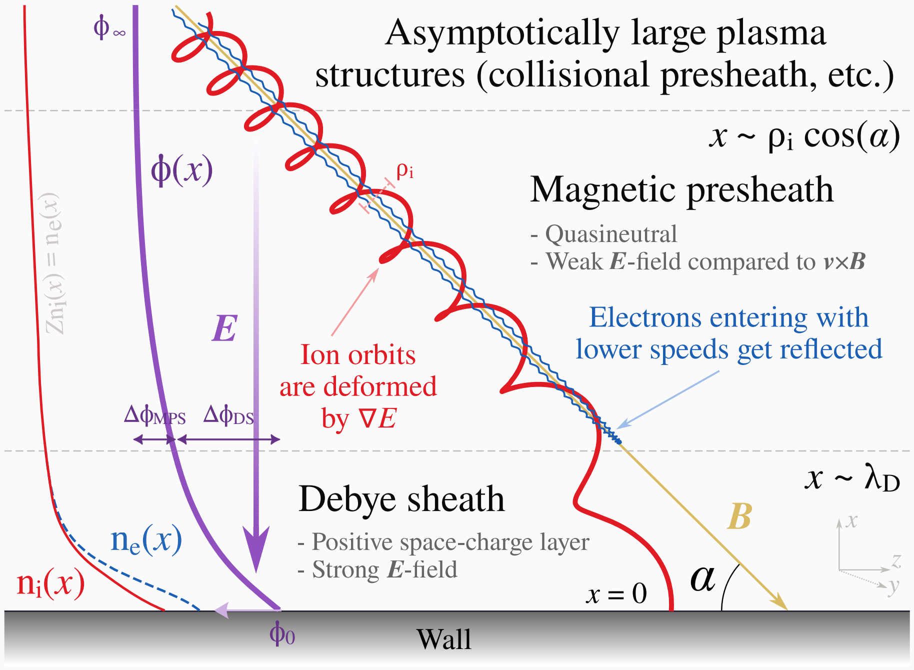
⇒ Electrons reach the wall first, charging it negatively

⇒ Debye sheath forms of scale ~ 10 µm

EPFL



Asymptotically large plasma structures (collisional presheath, etc.)
ϕ∞
ϕ(x)
x ~ ρi cos(α)
ρi
Magnetic presheath
- Quasineutral
- Weak E-field compared to v×B
Zni(x) = ne(x)
E
Ion orbits are deformed by ∇E
Electrons entering with lower speeds get reflected
ΔϕMPS
ΔϕDS
x ~ λD
Debye sheath
- Positive space-charge layer
- Strong E-field
ne(x)
ni(x)
B
x = 0
α
ϕ0
Wall
x
z
y



[1]: D. Tskhakaya, CPP 52, 490 (2012).
[2]: A. Geraldini, F. I. Parra, and F. Militello, PPCF 59, 25015 (2017).
[3]: Th. Daube and K.-U. Riemann, PoP 6, 2409 (1999).
[4]: A. Geraldini, F. I. Parra, and F. Militello, PPCF 60, 125002 (2018).
[5]: M. Abazorius, *Kinetic Analysis of the Collisional Layer*, PhD thesis, University of Oxford, (2025).

The magnetized sheath (DS+MPS) causes difficulties for standard codes:

- Cannot be resolved by gyrokinetics
- Particle-In-Cell codes expensive[1] ($\gtrsim 10^6$ time steps, $\gtrsim 10^5$ core-hours !)

Key insight: small scale of the magnetized sheath

⇒ Evolves much faster than turbulent timescales[2]
⇒ **Solve directly for the steady-state**
⇒ Iterative scheme

Prior art:

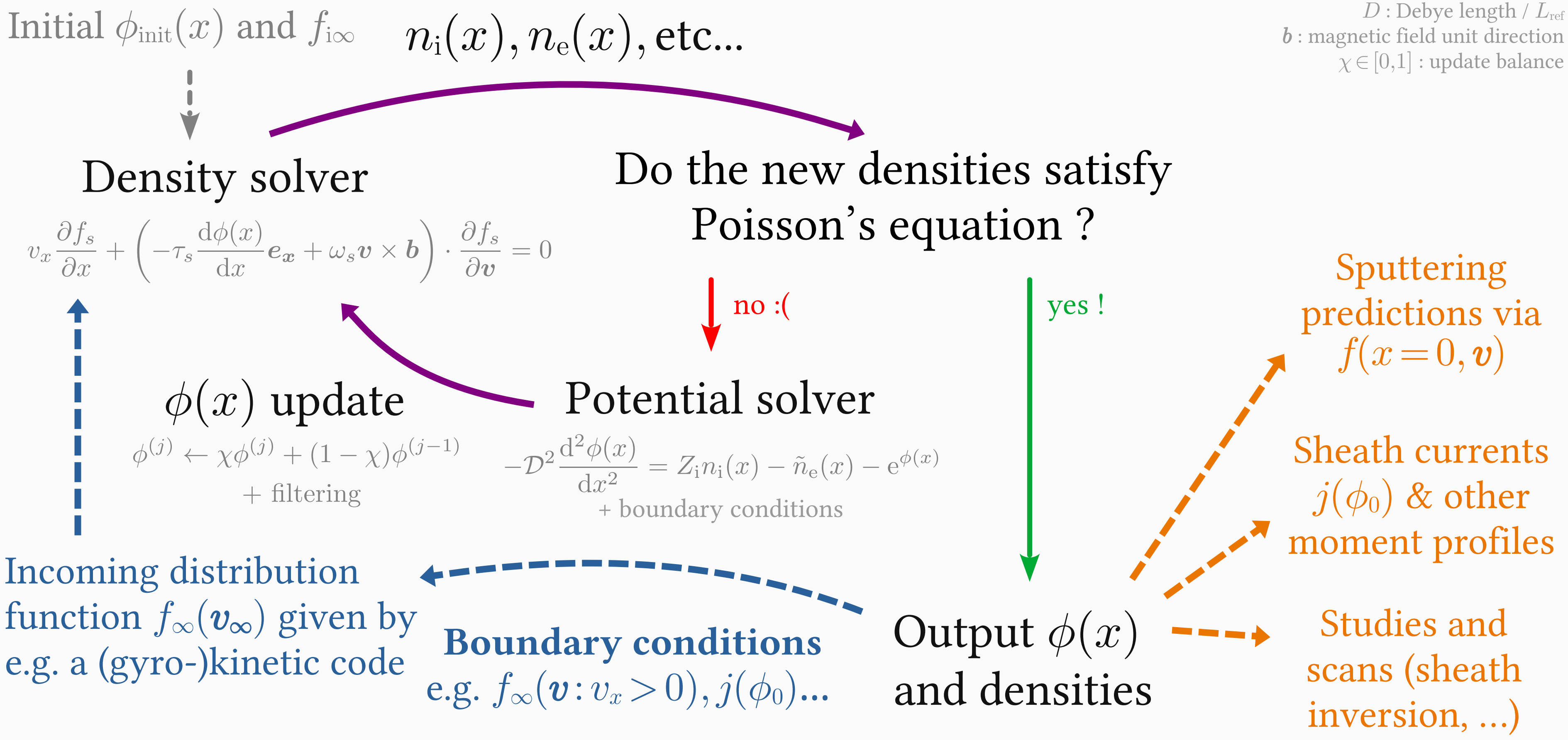
- Daube & Riemann[3]: magnetic presheath + cold neutral CX
- Geraldini, Parra & Militello[4]: scale-separated small-α gyrokinetics
- Abazorius[5]: drift-kinetics of the collisional presheath

Methods

Swiss
Plasma
Center

Goal: solve the steady-state Poisson-Vlasov system iteratively

Normalisations
$\boldsymbol{r} = (x, y, z)$: particle position / L_{ref}
$\boldsymbol{v}$: particle velocity / $v_{\mathrm{th},s}$
n_s : species' density / $n_{\mathrm{e}\infty}$
Z_s' : charge / $(Z_s T_{\mathrm{e}\infty} / T_{s\infty})$
Ω_s' : gyropulsation / $(\Omega_s L_{\mathrm{ref}} / v_{\mathrm{th},s})$
f_s : distribution function $n_{\mathrm{e}\infty} / v_{\mathrm{th},s}{}^3$
ϕ : electrostatic potential / $(T_{\mathrm{e}\infty} / e)$
D : Debye length / L_{ref}
$\boldsymbol{b}$: magnetic field unit direction
$\chi \in [0,1]$: update balance



Swiss Plasma Center

Trajectories "deposit" density onto basis fns

Project the density onto a basis function $\Lambda(x)$:

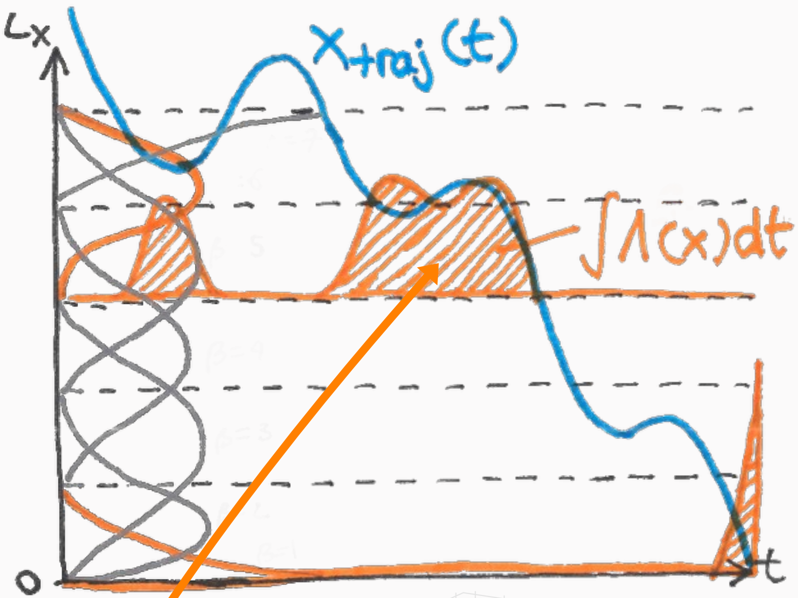
$$r = \int_0^{L_x} \mathrm{d}x\, n(x)\Lambda(x) = \int_0^{L_x} \mathrm{d}x \int \mathrm{d}^3\boldsymbol{v}\, f(x,\boldsymbol{v})\Lambda(x)$$

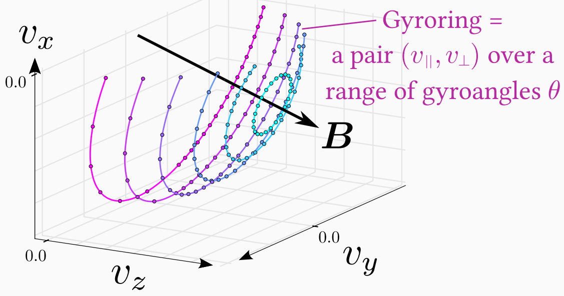
The Vlasov equation is solved via method of characteristics:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v_x,$$
$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\tau_s \frac{\mathrm{d}\phi}{\mathrm{d}x}\boldsymbol{e_x} + \omega_s \boldsymbol{v}\times\boldsymbol{b},$$
$$\frac{\mathrm{d}f_s}{\mathrm{d}t} = 0,$$

and integrating over a flux tube:

$$f(x,\boldsymbol{v})\,\mathrm{d}x\,\mathrm{d}^3\boldsymbol{v} = v_{\infty x} f(x_\infty, \boldsymbol{v_\infty})\,\mathrm{d}t\,\mathrm{d}^3\boldsymbol{v_\infty}$$





$$r = \int \mathrm{d}^3\boldsymbol{v_\infty}\, |v_{\infty,x}| f_\infty(\boldsymbol{v_\infty}) \int_0^{t_f} \mathrm{d}t\, \Lambda(x(t; L_x, \boldsymbol{v_\infty}))$$

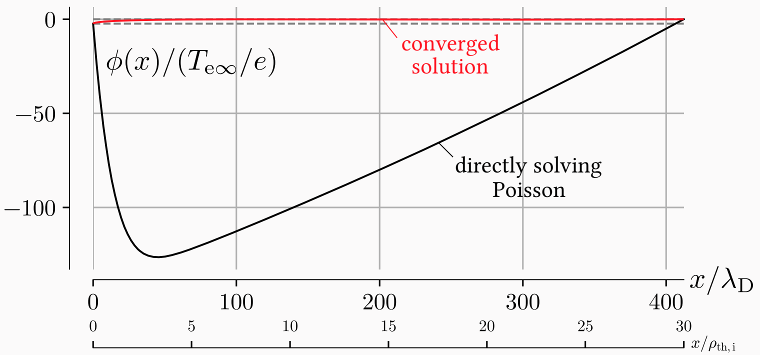
3D integral + **trajectories** = **main computational cost!**

$\phi(x)$ is obtained by solving a modified Poisson equation iteratively

Poisson's equation:

$$\mathcal{D}^2 \frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} = \underbrace{Z n_\mathrm{i} - n_\mathrm{e}}_{\text{implicit dependence on } \phi}$$

1. Consider RHS fixed ($\phi \mapsto \phi_\mathrm{prev}$)
   $\longrightarrow$ unstable !!



Potential profile after a single iteration by solving Poisson's equation

2. Isolate the adiabatic electron contribution

$$\mathcal{D}^2 \frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} + \mathrm{e}^{\phi} = \underbrace{Z n_\mathrm{i} - \left(n_\mathrm{e} + \mathrm{e}^{\phi_\mathrm{P}}\right)}_{\text{depend on } \phi_\mathrm{prev}}$$

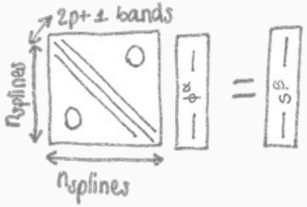
"Modified" Poisson's equation

$\longrightarrow$ when we converge, $\phi \to \phi_\mathrm{p}$ and recover Poisson's equation

$\longrightarrow$ implicit equation, solved iteratively via linearizing e^{ϕ}

$\longrightarrow$ Discarding *D* enforces quasineutrality, i.e. magnetic presheath simulations ($\lambda_\mathrm{D}/\rho_\mathrm{th,i} = 0$ limit)

Finite element projection: fast to compute and solve (banded matrices)



Results

Swiss
Plasma
Center

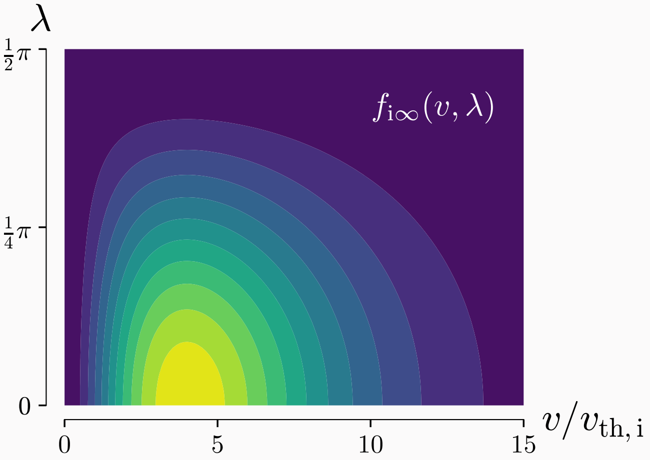
<aside>

Simulation parameters

- Inflow boundary condition marginally satifies the Bohm-Chodura condition
- Hydrogen plasma: $m_\mathrm{i}/m_\mathrm{e} = 1836$
- $T_{\mathrm{e}\infty} = 1$ eV, and $T_{\mathrm{i}\infty}/T_{\mathrm{e}\infty} = 1$
- $\phi(\infty) = 0$, and $\phi(0) = \phi_0$ fixed
  Note that asymptotic approach implies $\partial\phi(\infty) = 0$
- Varying α, $\zeta_\mathrm{i} = \rho_{\mathrm{th,i}}/\lambda_\mathrm{D}$, electron model (up next !)

$$f_{\mathrm{i}\infty} \propto \mathbb{1}_{\lambda>0}\left(\overbrace{\frac{v\cos\lambda}{v_{\mathrm{th,i}}}}^{v_\parallel}\right)^2 \exp\left(-\frac{v^2}{2v_{\mathrm{th,i}}}\right)$$

$$\frac{1}{n_\mathrm{i}}\int \mathrm{d}^3\boldsymbol{v}\,\frac{f_{\mathrm{i}\infty}}{v_\parallel^2} \leq \frac{m_\mathrm{i}}{T_{\mathrm{e}\infty}}$$



</aside>

<aside>

Electron models

Adiabatic (0V): electrons are Boltzmann distributed, no net current to the wall

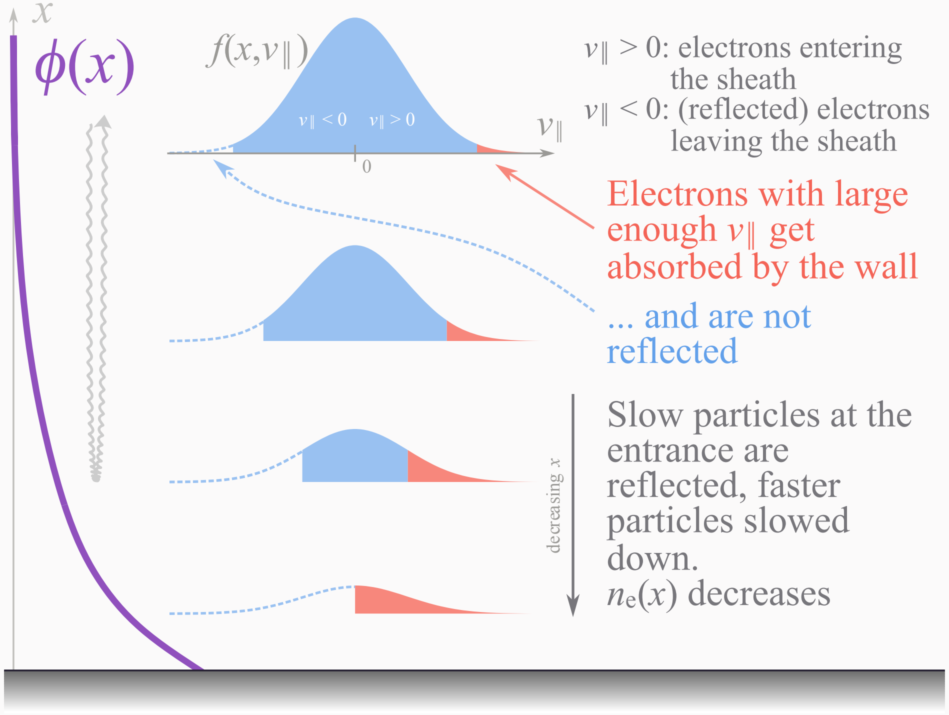
$$n_\mathrm{e} = \mathrm{e}^{\phi(x)}$$

Parallel streaming (1V): electrons are Boltzmann distributed, but taking into account parallel dynamics to account for reflection

$$n_\mathrm{e} \propto \frac{1}{2}\left(1 - \mathrm{erf}\left(-\sqrt{\phi(x) - \phi(0)}\right)\right)\mathrm{e}^{\phi(x)}$$

FLR (3V): electrons are fully kinetic, finite Larmor radius

</aside>



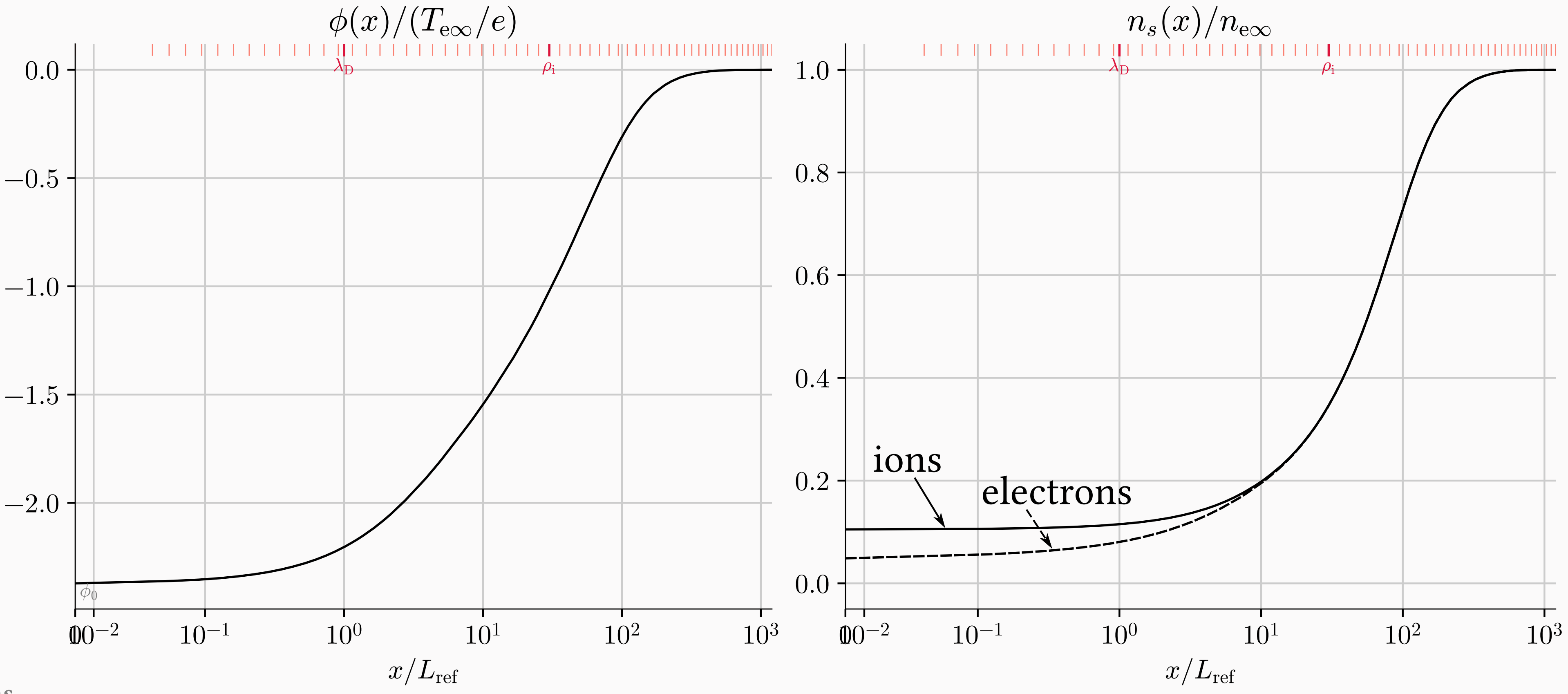
1V kinetic electron model

Here's what a "typical" run looks like

- $\alpha = 5°$
- $\zeta_\mathrm{i} = \rho_\mathrm{th,i} / \lambda_\mathrm{D} = 30$
- $\phi_0 = -2.37\, T_{\mathrm{e}\infty}/e$ (ambipolar)
- parallel streaming electrons

- Walltime ~ 20 mins over 4 CPUs

$\longrightarrow$ to animation



$\alpha = 5°$, $\zeta_i = \rho_{th,i}/\lambda_D = 30$,
parallel streaming electrons, ambipolar

EPFL

Intro
-
-
-

Methods
-
-
-

Results
Params
-
-
Verification
1
-
-
-
-
Ongoing
-
-
-

Conclusions

Swiss Plasma Center

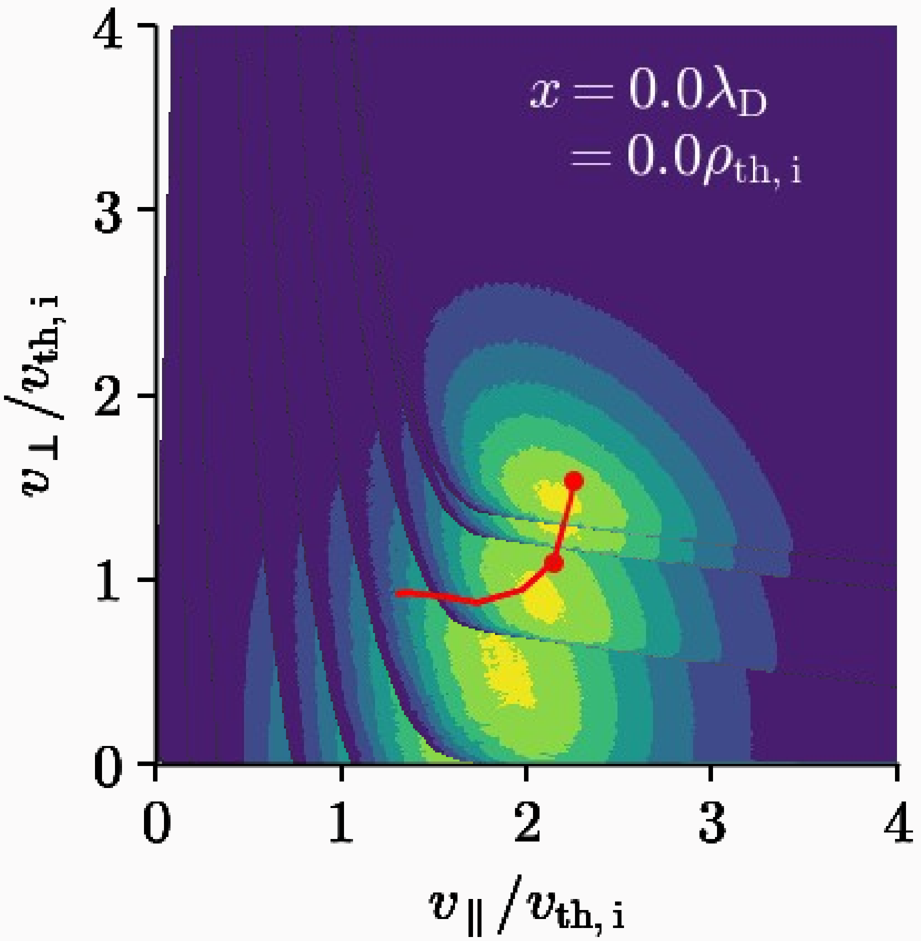
Results: $f(x, v)$ diagnostics

"Quick and dirty": log velocities at given positions as you deposit density

$$\left.\frac{\mathrm{d}f}{\mathrm{d}t}\right|_{\mathrm{traj}} = 0 \implies f(x, \boldsymbol{v}) = f_\infty(\boldsymbol{v}_\infty)$$

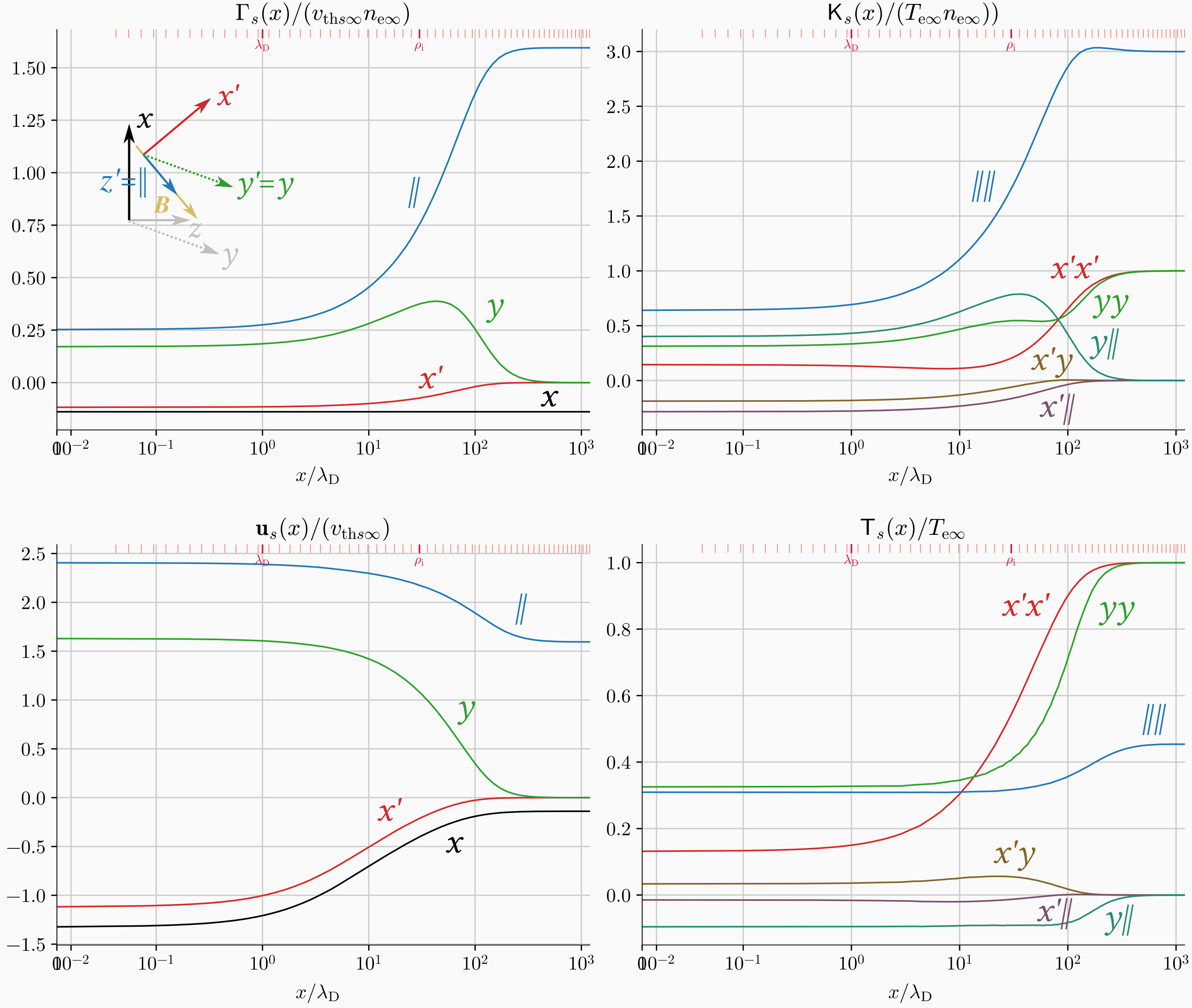
→ Acceleration in $v_{||}$ in the MPS

→ then in $v_\perp$ in the DS
(strong $\boldsymbol{E}$ field and $\boldsymbol{E}\times\boldsymbol{B}$ drift)



$\alpha = 5°$, $\zeta_i = \rho_{th,i}/\lambda_D = 30$,
parallel streaming electrons, ambipolar

Swiss Plasma Center



Results: moments

Density deposition easily generalises to moments:

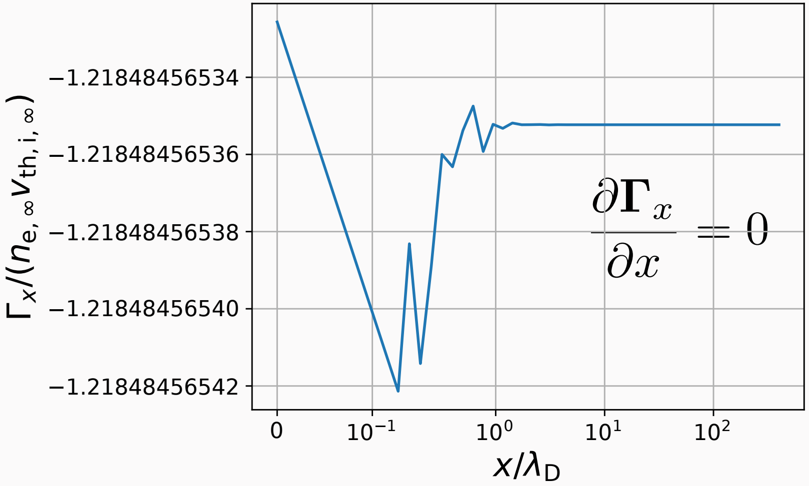
$$\int \mathrm{d}t\,\Lambda(x(t))$$

$$\rightarrow \int \mathrm{d}t\,h(\boldsymbol{v})\Lambda(x(t))$$

⟶ Ion acceleration
+ $\boldsymbol{E}\times\boldsymbol{B}$ drift in y
+ along field (∥)
+ across field (x')

⟶ Temperature
+ "Gyro-cooling"[1]
+ strong anisotropy!
⟶ ~~fluid approach~~

[1]: H. A. Claaßen and H. Gerhauser, *Contributions to Plasma Physics* 36, 381 (1996).



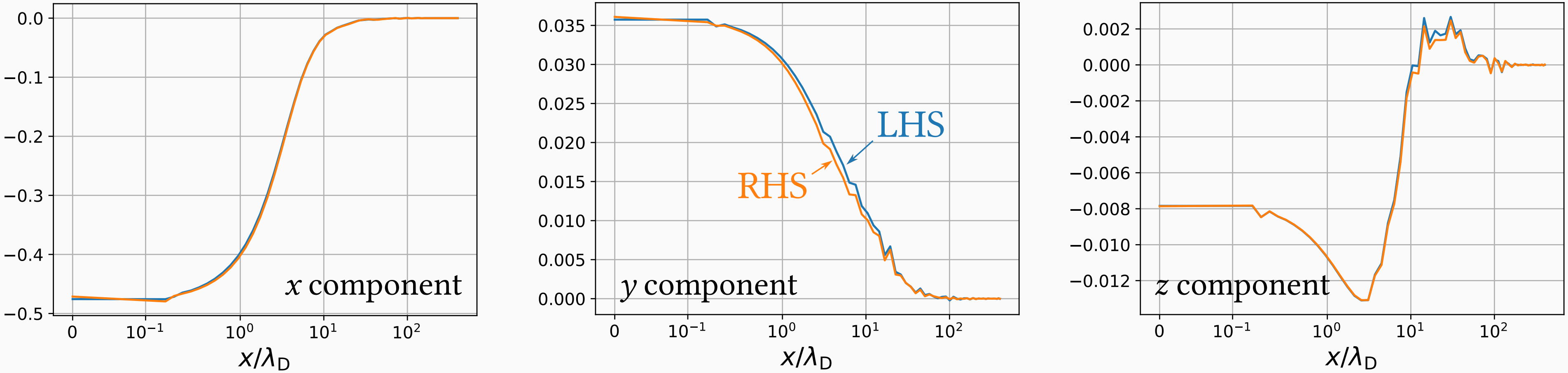
Verification: moments satisfy fluid equations

→ Should always satisfy Vlasov's equation and its moments!

[✓] Flux is conserved
[✓] Momentum equation satisfied

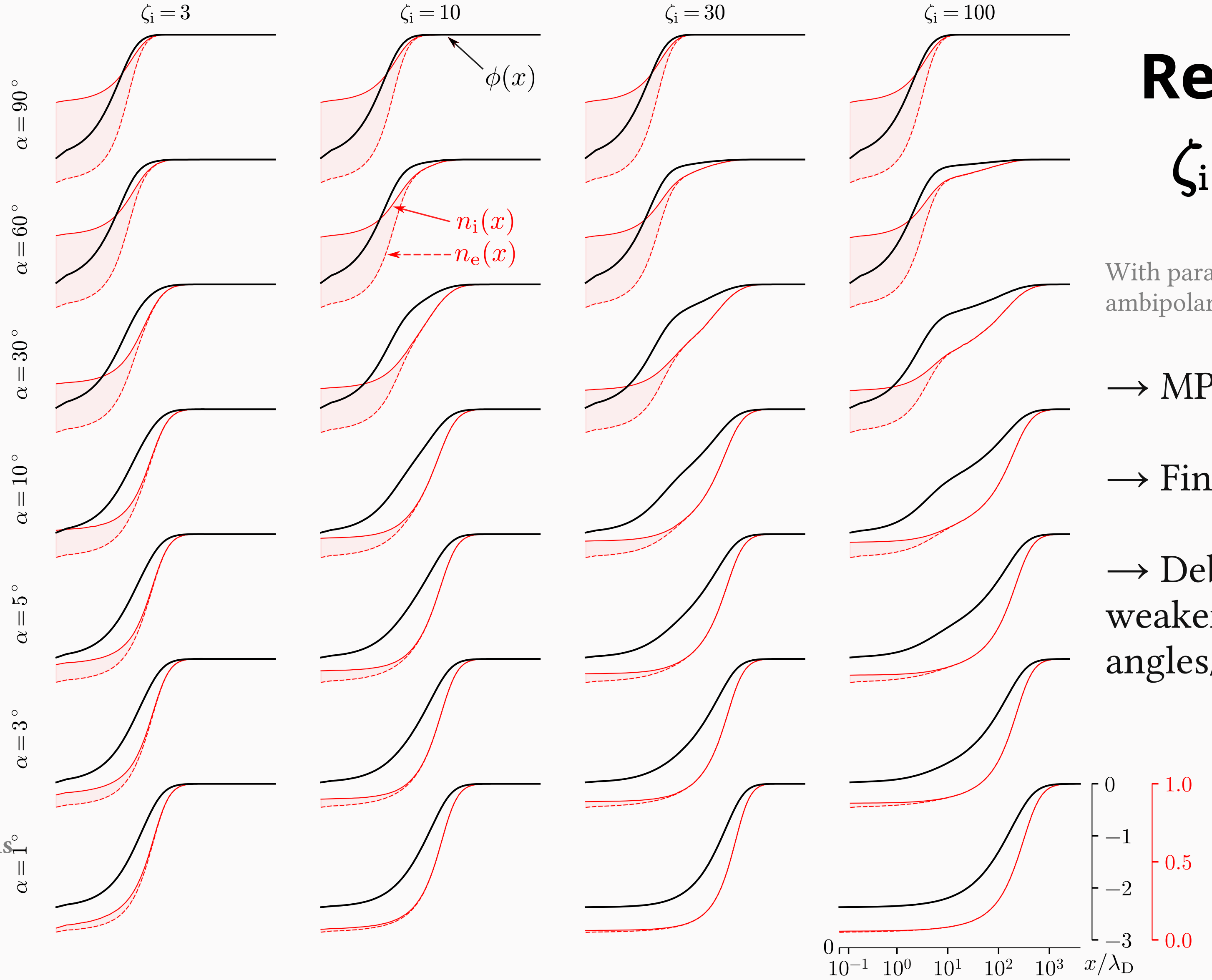
$$n u_x \frac{\partial \boldsymbol{u}}{\partial x} = n\left(-\tau \frac{\mathrm{d}\phi}{\mathrm{d}x} \boldsymbol{e_x} + \omega \boldsymbol{u} \times \boldsymbol{b}\right) - \nabla \cdot \mathsf{P}$$

(!) Not exact due to finite velocity integral resolution and finite root finder tolerance



Adapated from: T. Stucker, *Implementation of Collisional Effects in a Kinetic Magnetized Plasma Sheath Code*, Masters' thesis, École Polytechnique Fédérale de Lausanne, 2026.

Results: α and ζ_i behaviour

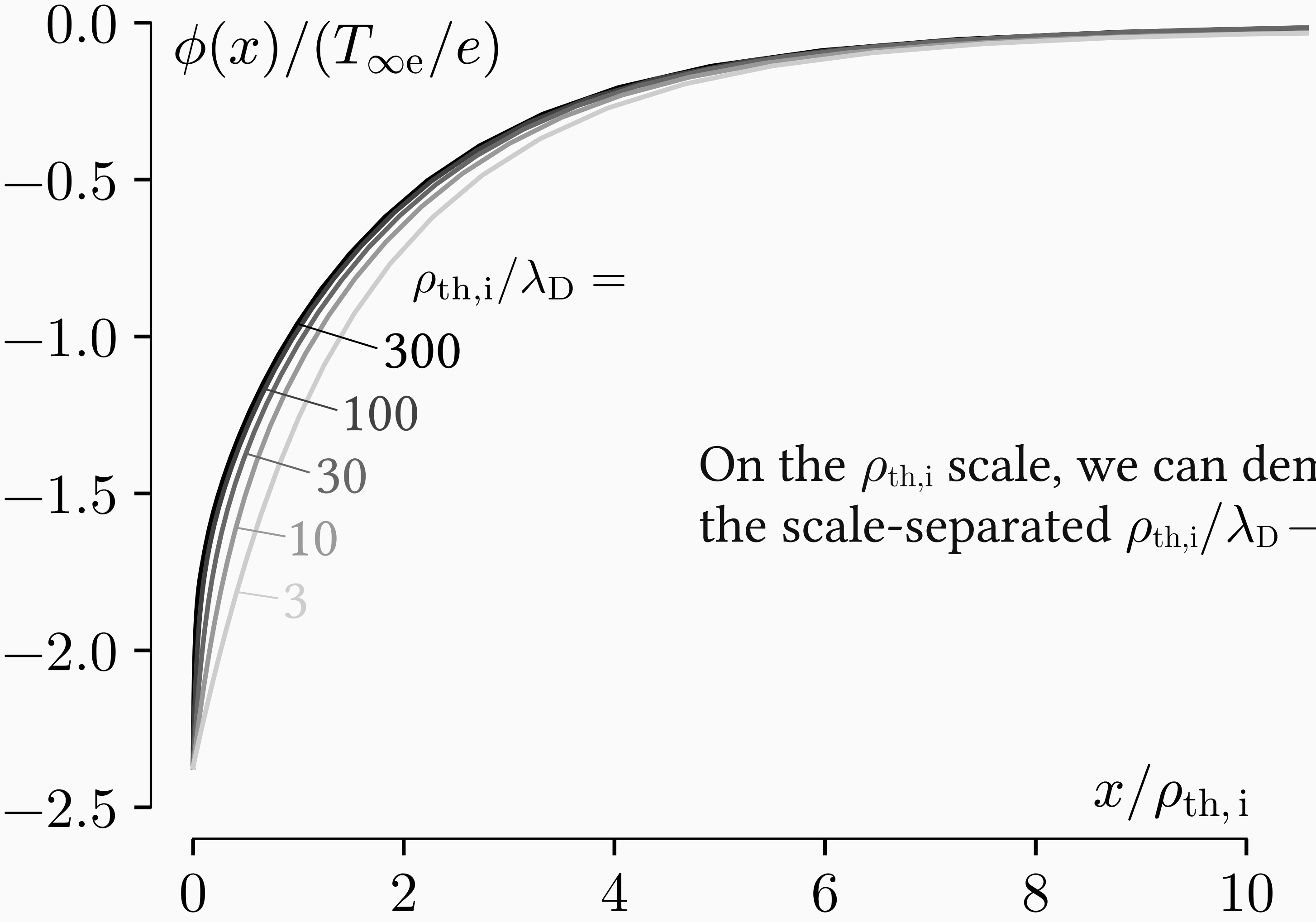


With parallel streaming electrons, ambipolar

$\rightarrow$ MPS scales as $\zeta_\mathrm{i} \sin(\alpha)$

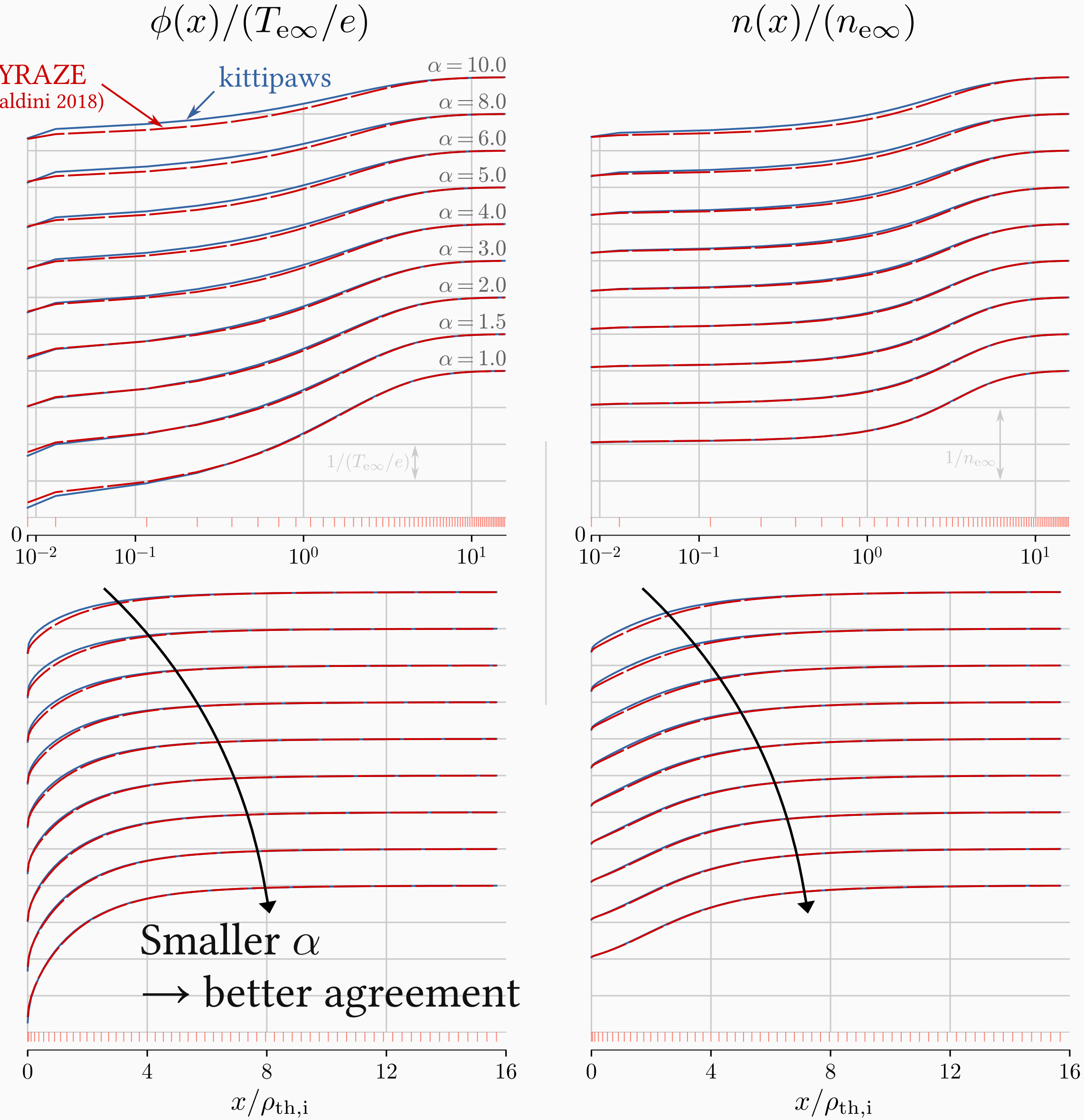
$\rightarrow$ Finite $\zeta_\mathrm{i} = \rho_{\mathrm{th,i}}/\lambda_\mathrm{D}$ effects

$\rightarrow$ Debye sheath tends to weaken at shallow angles/large gyroradius



On the $\rho_{\mathrm{th,i}}$ scale, we can demonstrate the scale-separated $\rho_{\mathrm{th,i}}/\lambda_{\mathrm{D}} \to \infty$ limit

α = 5°, parallel streaming electrons, ambipolar



Validation: recovering Geraldini's approach for the MPS

- GYRAZE[1] is valid in the small α and scale-separated ($\zeta_i \to \infty$) limit
- Quasineutral ⟶ "stiff" system !! ⟶ difficult to converge
- Still, very good agreement, except on total drop $\phi_{\mathrm{MPS}}(0)$
  ⟶ divergence[2,3]: $\phi_{\mathrm{MPS}}(0) \sim \ln(\alpha)$
  and[1] $\phi_{\mathrm{MPS}}(x) - \phi_{\mathrm{MPS}}(0) \sim \sqrt{x}$!!
  ⟶ finite tolerance: 0.1% rel ℓ_∞ err

[1]: A. Geraldini, F. I. Parra, and F. Militello, *Plasma Physics and Controlled Fusion* 60, 125002 (2018).
[2]: R. Chodura, *The Physics of Fluids* 25, 1628 (1982).
[3]: P. C. Stangeby, *Nuclear Fusion* 52, 83012 (2012).

Ongoing: FLR electrons with large ρ_e / λ_D

Difficulties in density deposition…

At the location where electrons reflect…

$\longrightarrow$ v_x vanishes

$\longrightarrow$ singularity in density deposition

$$n(x) = \int \mathrm{d}^3\boldsymbol{v}_\infty \, f_\infty(\boldsymbol{v}_\infty) \frac{v_{\infty x}}{v_x(x;\boldsymbol{v}_\infty)}$$

$\longrightarrow$ problems when $\partial_x \phi$ is small: "spreads out" the singularities

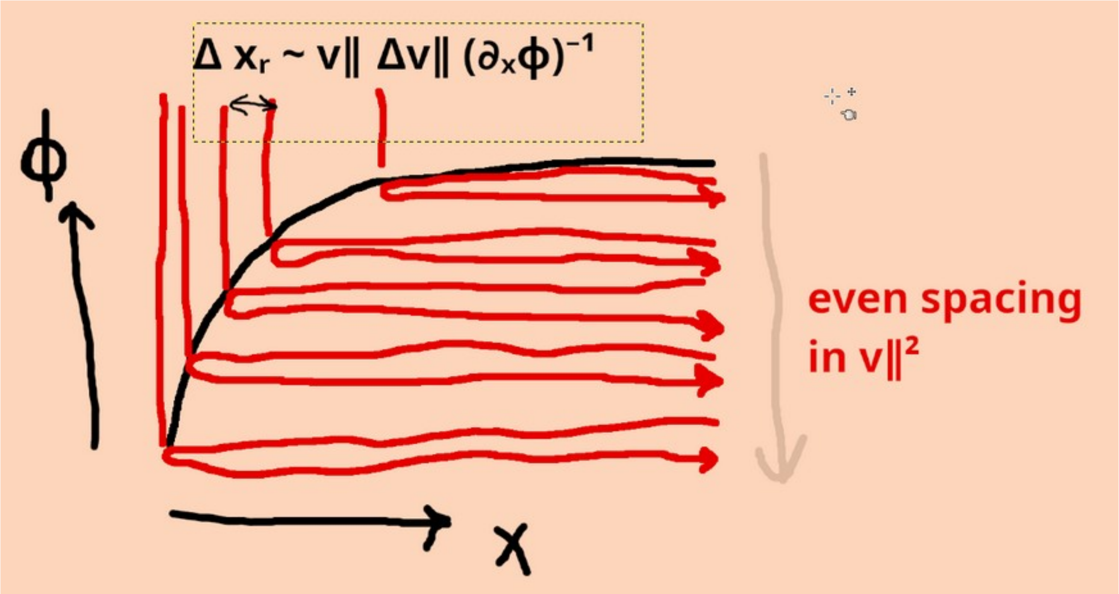
$\Rightarrow$ Requires fine velocity grid

$\Rightarrow$ Helped by using $(v_\parallel, \lambda, \theta)$ coordinates

Difficulties in convergence…

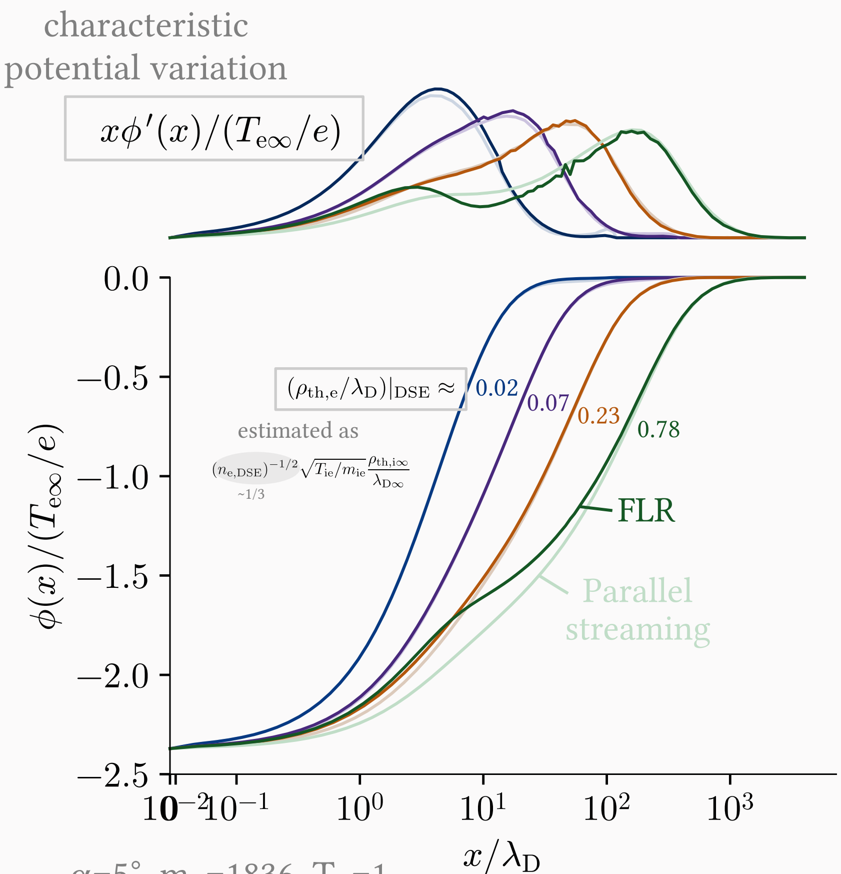
"Wave-like" behaviour in iterations once ρ_e / λ_D becomes large [video]

$\Rightarrow$ "Adiabatic" electron profiles update



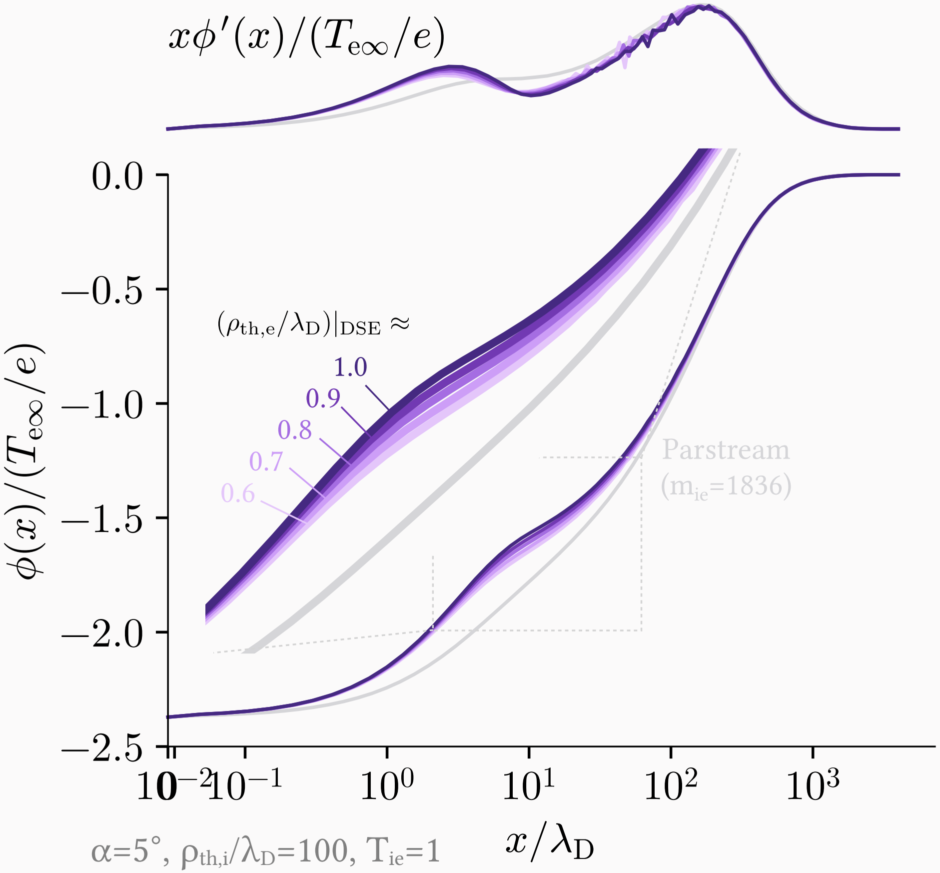
Ongoing: FLR electrons with large ρ_e / λ_D

… via changing λ_D (fixed mass ratio)

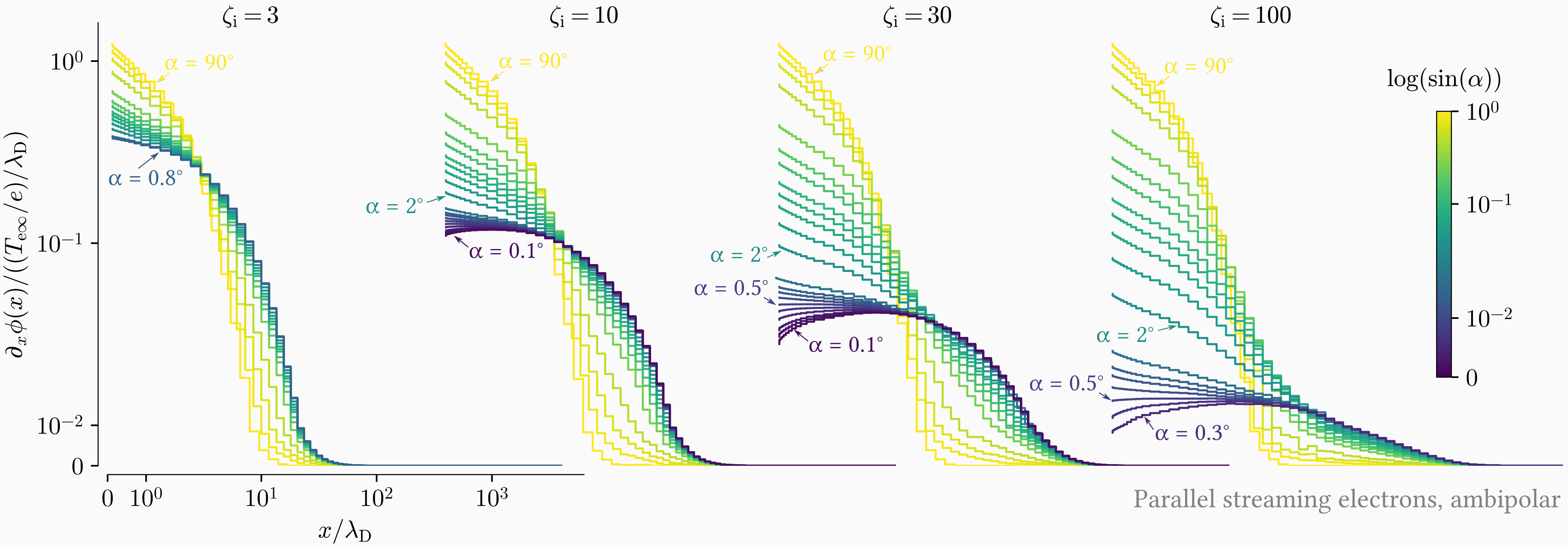


α=5°, m_{ie}=1836, T_{ie}=1

… via changing mass ratio



α=5°, $\rho_{th,i}/\lambda_D$=100, T_{ie}=1



Ongoing: critical angle studies

Keep pushing α downwards
→ Electric field $-\partial\phi$ weakens
and is expected to invert at α^*

Adiabatic electrons[1]: $\alpha^* \sim O(\sqrt{(m_e/m_i)}) \sim 4.7°$ for ^{1}H plasma
Accounting for electron reflection[2]: $\alpha^* \sim O(m_e/m_i) \sim 0.5°$
Accounting for finite ρ_e[3]: $\alpha^* \sim O(\sqrt{(\rho_{e,DSE}/\lambda_D)}\ \sqrt{(m_e/m_i)}) \sim 2°$

subtle !! this is a rough est. for our params

But these do not account for finite $\zeta_i = \rho_{th,i} / \lambda_D$!

[1]: P. C. Stangeby, *Nuclear Fusion* 52, 83012 (2012).
[2]: R. J. Ewart, F. I Parra, and A. Geraldini, *Plasma Physics and Controlled Fusion* 64, 15010 (2021).
[3]: Geraldini, A., Ewart, R. J., Brunner, S., & Parra, F. I. (2025). Characteristics of monotonic sheaths near a wall with grazing magnetic incidence. *arXiv preprint* arXiv:2508.09067.

EPFL

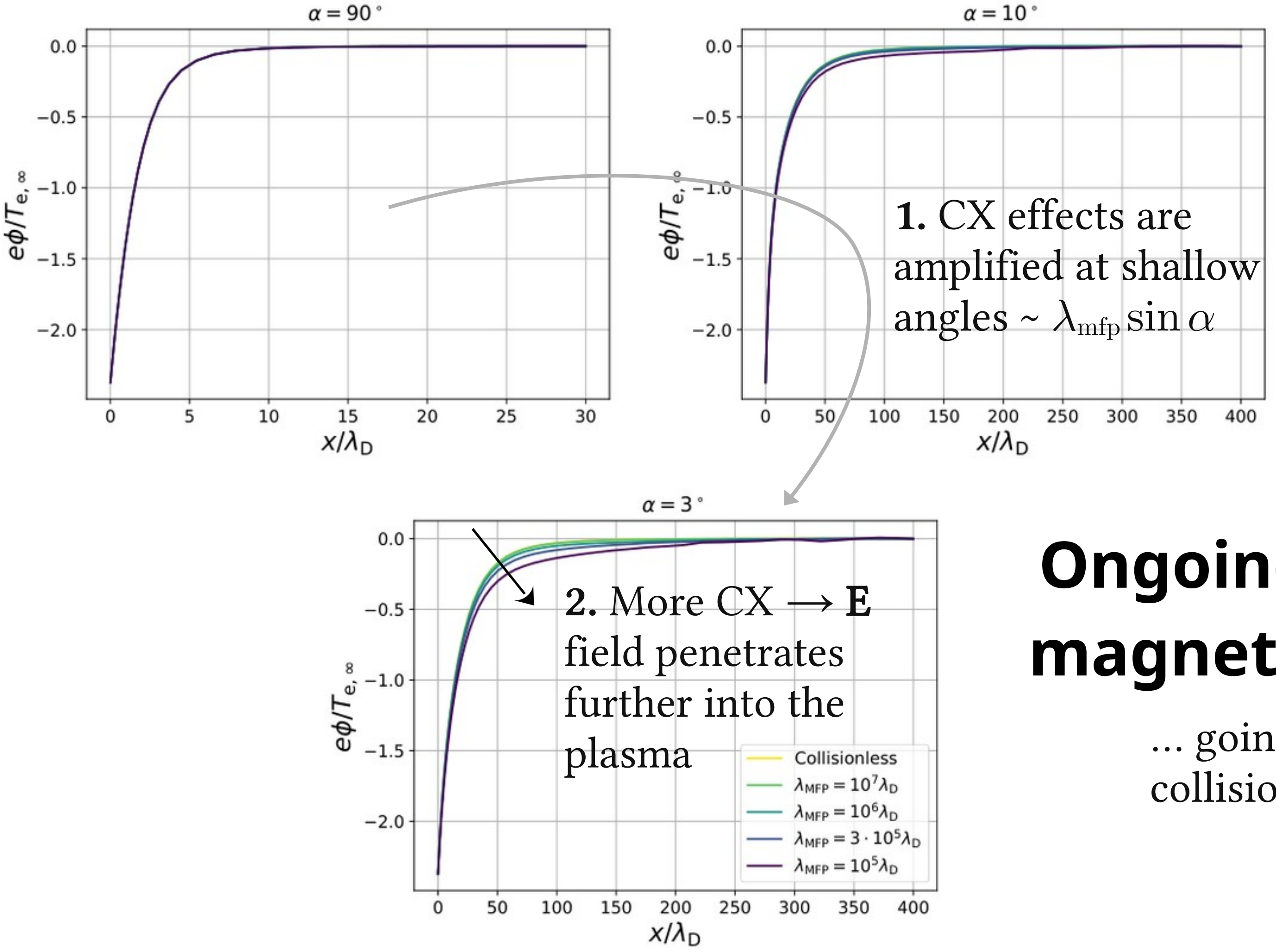


Figure 17: Potential profiles for $\alpha \in \{90°, 10°, 3°\}$ and in different collisionality regimes: $\lambda_{MFP} \in \{\infty, 10^7\lambda_D, 10^6\lambda_D, 3 \cdot 10^5\lambda_D, 10^5\lambda_D\}$.

Ongoing: CX in the magnetized sheath

… going towards the collisional presheath

T. Stucker, *Implementation of Collisional Effects in a Kinetic Magnetized Plasma Sheath Code*, Masters' thesis, École Polytechnique Fédérale de Lausanne, 2026.

Conclusions and outlook

What works:

- Software engineering:
  - Fast, semi-analytical solver for evaluating particle trajectories
  - Adaptive, parallelized velocity grid
- Physics: solutions of magnetized (pre-)sheath, over a wide range of
  - length scales $\rho_e \lesssim \lambda_D \lesssim \rho_i$
    - verified again GYRAZE in the $\lambda_D/\rho_i \to 0$ limit
  - incidence angle α (down to ~0.01°),
  - λ_{CX} in the magnetized sheath,
  - temperature and mass ratios…

What's in progress:

- Publication on the main code
- CX pre-sheath[1]: obtain $f_{i\infty}$ self-consistently
- Simulation database
  - Provide ambipolar potential drops $j(\phi_0)$ required in edge gyrokinetic simulations
  - Investigate critical angle and sheath inversion
  - Plasma-facing components: ion energy-angle distributions etc…

[1]: T. Stucker, *Implementation of Collisional Effects in a Kinetic Magnetized Plasma Sheath Code*, Masters' thesis, École Polytechnique Fédérale de Lausanne, 2026.

Swiss Plasma Center

Conclusions and outlook

What's planned:

- In view of 2D simulations:
  - Adapt trajectory solver to 2D, or adopt numerical integration shemes
  - (Re-)write some Python modules in Chapel[3a,3b]
- Benchmark full magnetized sheath results with PIC codes[2]

Thank you !

[2]: D. Tskhakaya, PPCF, 114001 (2017).
[3a]: Bradford L. Chamberlain. Chapel. In Pavan Balaji, editor, *Programming Models for Parallel Computing*, chapter 6, pages 129–159. MIT Press, November 2015.
[3b]: The Chapel Contributors, *The Chapel Programming Language*, online: https://chapel-lang.org/, https://github.com/chapel-lang/chapel/, retrieved March 2026