Ion-temperature-gradient instability in the asymptotic transit-effect regime

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[1] E.A. Frieman & L. Chen 1982 *Phys. Fluids* **25** 502-508
[2] H. Yamada *et al* 2005 *Nucl. Fusion* **45** 1684

Motivation - 1

- Energy confinement limited by microturbulence
- Microturbulence sensitive to geometric details
  - Predicted by gyrokinetic framework[1]
  - Confirmed by need of "renormalisation factor" in ISS04-scaling[2]

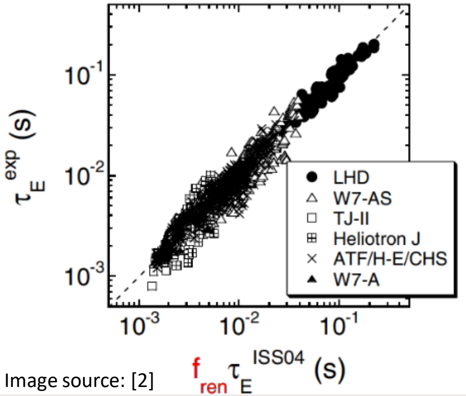


Image source: [2]

- Could be exploited to optimise for new low-turbulence devices
- Iterative direct gyrokinetic simulation of heat flux prohibitive expensive

Motivation - 2

- Gyrokinetic codes: versatile number crunchers → all physics retained as $\mathcal{O}(1)$ effects

- Necessary?
- Leverage model reduction: retain essentials to get fast and accurate(?) alternatives to facilitate geometric surveys of transport

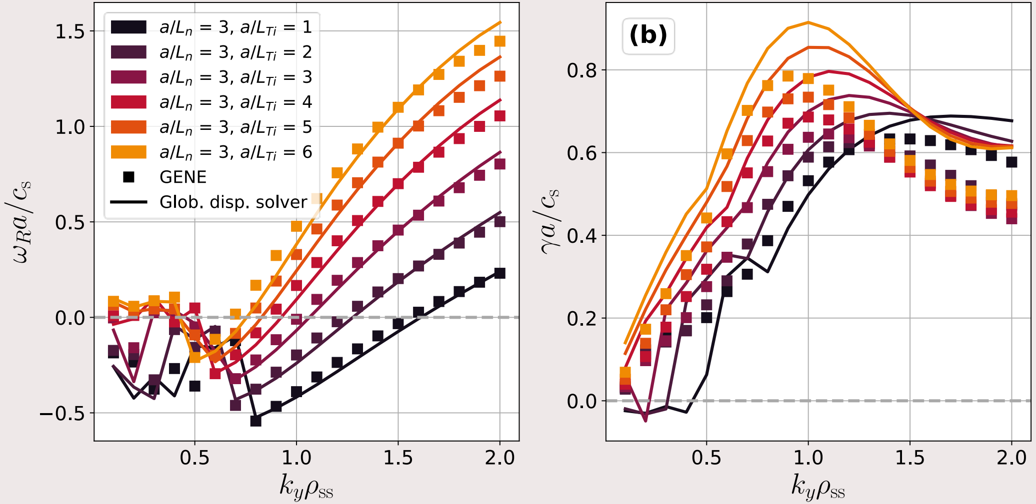
Motivation - 3

- Majority of transport in present-day devices attributed to ITG/TEM[3]
- Both predominantly toroidally driven electrostatic instabilities

- However different (assumed) regimes:
  - ITG: $\frac{\omega_{ti}}{\omega} \sim \delta \ll 1$
  - TEM: $\frac{\omega_{be}}{\omega} \sim \frac{1}{\delta} \gg 1$

Motivation - 4

- Previous work: order-of-magnitude differences in ion/electron streaming used to obtain variational dispersion relation[4]
- Closely follows ITG-TEM eigenfrequency predictions from GENE
- Growth rate overestimation → lack of ion-transit effects
- Disadvantage: inconsistent solver → relies on ϕ guess



Approach - 1

- Goal: self-consistent reduced dispersion relation model
- Strategy: explore simplest scenario adiabatic-electron ITG
- Linearised Vlasov-Poisson

$$v_\parallel \nabla_\parallel \hat{g}_s - i(\omega - \omega_{ds})\hat{g}_s = -\frac{iq_s}{T_s}\hat{\phi} J_0(k_\perp \rho_s)(\omega - \omega_{\star s}^T) F_{Ms}$$

$$\sum_s \frac{q_s n_s \hat{\phi}}{T_s} = \sum_s q_s \int \hat{g}_s J_0(k_\perp \rho_s)\, \boldsymbol{d^3 v_s}$$

- Solve in appropriate $\frac{\omega_{ti}}{\omega} \ll 1$ regime → asymptotically expand in weak-streaming effects

Approach - 2

⚠️ Streaming effects retained asymptotically → no transit-resonances ⚠️

- Lowest-order dispersion relation

$$\left(1+\frac{T_i}{T_e}\right)\hat{\phi}(l) = \int J_0 \left[1 - \frac{v_\parallel}{\omega-\omega_{di}}\nabla_\parallel \frac{v_\parallel}{\omega-\omega_{di}}\nabla_\parallel\right]\mathcal{F}_{Mi}\frac{\omega-\omega_{*i}^T}{\omega-\omega_{di}}J_0\hat{\phi}(l)d^3\boldsymbol{v_i}$$

- Local toroidal ITG eigenmode eqn + finite-transit correction
- Effect formally $\mathcal{O}\left(\left(\frac{\omega_{ti}}{\omega}\right)^2\right)$, but determines admissible eigenmodes to $\mathcal{O}(1)$ → necessary to maintain

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Approach - 3

- Dispersion relation can be written in diffusion-advection form

$$h_\parallel^{(2)}(\omega,l)\nabla_\parallel^2\hat{\phi}(l) + h_\parallel^{(1)}(\omega,l)\nabla_\parallel\hat{\phi}(l) + \left(h_{\text{tor}}^{\text{loc}}(\omega,l) - h_\parallel^{(0)}(\omega,l)\right)\hat{\phi}(l) = 0$$

$$h_\parallel^{(2)} = \int \mathcal{F}_{Mi}\, v_\parallel^2 \frac{\omega - \omega_{*i}^T}{(\omega-\omega_{di})^3} J_0^2 d^3\boldsymbol{v_i}$$

$$h_\parallel^{(1)} = \int \mathcal{F}_{Mi} \frac{\omega - \omega_{*i}^T}{\omega-\omega_{di}} J_0 v_\parallel \left(\frac{v_\parallel}{\omega-\omega_{di}} \nabla_\parallel \frac{J_{0s}}{\omega-\omega_{di}} + \nabla_\parallel \frac{v_\parallel J_{0s}}{(\omega-\omega_{di})^2} \right) d^3\boldsymbol{v_i}$$

$$h_{tor}^{loc} = 1 + \frac{T_i}{T_e} - \int \mathcal{F}_{Mi} \frac{\omega - \omega_{*i}^T}{\omega-\omega_{di}} J_0^2 d^3\boldsymbol{v_i}$$

$$h_\parallel^{(0)} = \int \mathcal{F}_{Mi} \frac{\omega - \omega_{*i}^T}{\omega-\omega_{di}} J_0 v_\parallel \nabla_\parallel \left(\frac{v_\parallel}{\omega-\omega_{di}} \right) \nabla_\parallel \frac{J_0}{\omega-\omega_{di}} d^3\boldsymbol{v}_i$$

Approach - 4

- RAW model: 11 resonant-type integrals
- Ordering
  - $h_\parallel^{(2)}\nabla_\parallel^2\hat{\phi} \sim \hat{\phi}\left(\frac{v_{Ti}}{L_\phi \omega}\right)^2$
  - $h_\parallel^{(1)}\nabla_\parallel\hat{\phi} \sim \hat{\phi}\frac{v_{Ti}^2}{L_\phi L_{\text{geo}}\omega^2}$
  - $h_{tor}^{loc}\hat{\phi} \sim \hat{\phi}$
  - $h_\parallel^{(0)}\hat{\phi} \sim \hat{\phi}\left(\frac{v_{Ti}}{L_{\text{geo}}\omega}\right)^2$
- Reduction 11→2 integrals admissible if $L_\phi \ll L_{\text{geo}}$

Tokamak

$$L_{\text{geo}} \sim \frac{1}{qR}$$ ⚖️

Stellarator

$$L_{\text{geo}} \sim \frac{N_{\text{fp}}}{qR}$$ ⚠️

[5] A. Hirose *et al.* 1995 Phys. Plasmas **2** 859-875
[6] H.-S. Xie & B. Li 2016 Phys. Plasmas **23** 082513

Approach - 5

- Two dispersion relation variants; retaining ($L_{\mathrm{geo}} \sim L_\phi$) and neglecting ($L_{\mathrm{geo}} \gg L_\phi$) geometric variation
- Compared vs two alternative heuristic models $\Rightarrow$Need a prior ϕ‼
  - $D_{\mathrm{global}}^{\mathrm{ad-hoc}}(\omega) = \int h_{tor}^{loc}(\omega, l)\hat{\phi}(l)^2 \frac{dl}{B} = 0$
  - $D_{\mathrm{semi-loc}}(\omega) = 1 + \frac{T_i}{T_e} - \int \mathcal{F}_{Mi} \frac{\omega - \omega_{*i}^T}{\omega - \langle\omega_{di}\rangle - \sqrt{\langle k_\parallel^2\rangle}\, v_\parallel} J_0^2\left(\frac{\sqrt{\langle(k_\perp\rho_{Ti})^2\rangle} v_\perp}{v_{Ti}}\right) d^3\boldsymbol{v_i} = 0$
- Semi-local model based on operator norms[5,6]

$$\langle\{\omega_{di}, (k_\perp\rho_{Ti})^2\}\rangle = \frac{\int \{\omega_{di}, (k_\perp\rho_{Ti})^2\}\hat{\phi}^2 dl}{\int \hat{\phi}^2 dl} \qquad \langle k_\parallel^2\rangle = \frac{\int \left(\nabla_\parallel\hat{\phi}\right)^2 dl}{\int \hat{\phi}^2 dl}$$

Approach 6

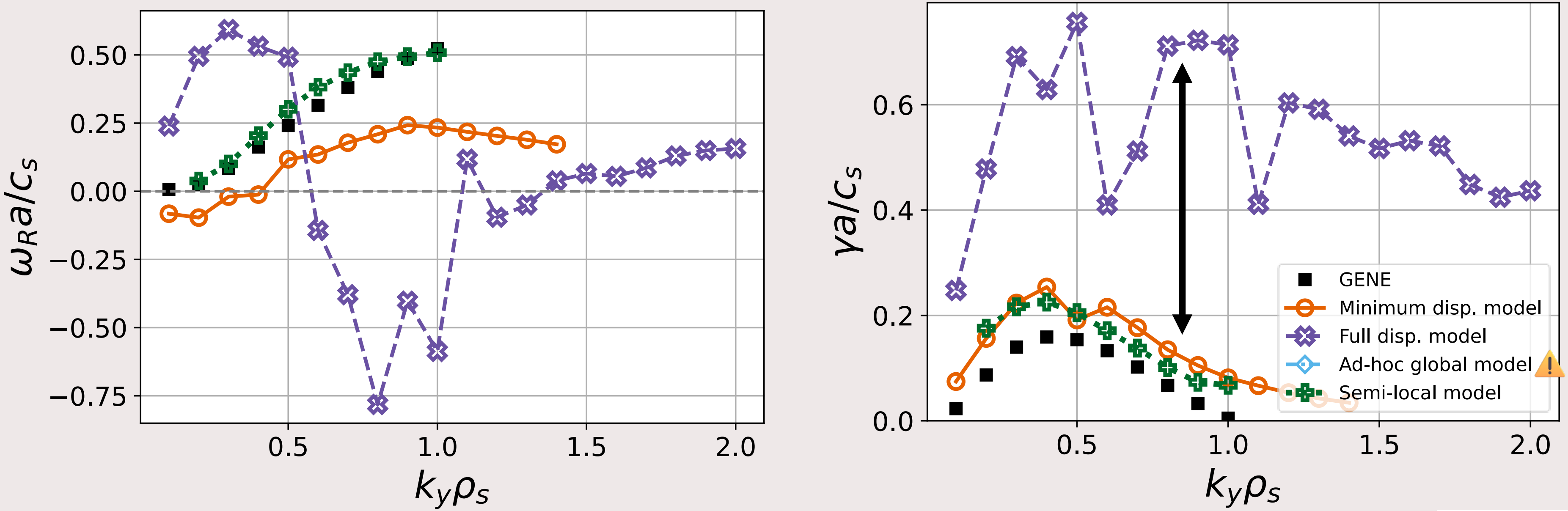
- Compare eigenfrequency and eigenmode predictions of model(s) with GENE flux-tube simulations
- Heuristic models: use $\hat{\phi}_{GENE}$ for mode structure guess → lower-bound on error
- Asymptotic model $(\omega, \hat{\phi})$ determined self-consistently

- Test cases:
  - Dispersion spectra of CBC, LHD (inwards), W7-X (high-mirror)
  - Geometry sensitivity: $\hat{s}$-scan in $s\!-\!\alpha$

$\gamma(L_{\mathrm{geo}} \sim L_\phi) \gg \gamma(L_{\mathrm{geo}} \ll L_\phi)$

Results - 1

- ITG dispersion in LHD; $^a/_{L_n} = 2, ^a/_{L_T} = 4$



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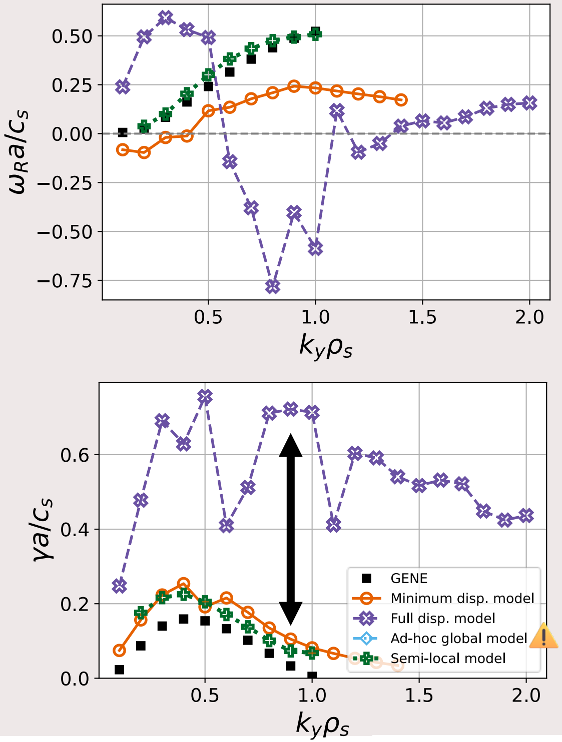
$\gamma(L_{\text{geo}} \sim L_\phi) \gg \gamma(L_{\text{geo}} \ll L_\phi)$

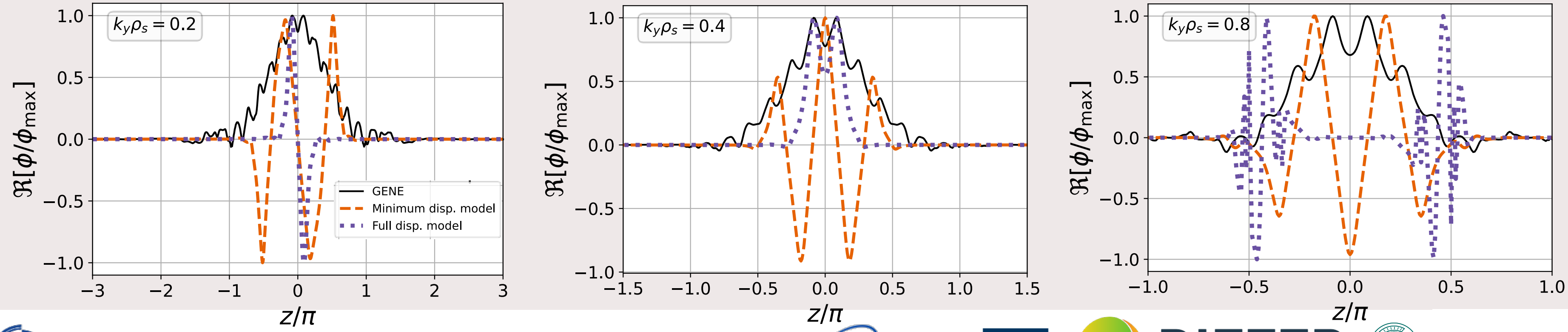
Results - 2

- ITG dispersion in LHD; $a/L_n = 2, a/L_T = 4$
- Semi-local model → closest match to GENE
- Asymptotic model w/o geometric variation → comparable γ predictions, though mismatch in k_y-stabilisation threshold
- ω_R predictions → larger discrepancy but following qualitative trends
- Inclusion of $\nabla_\parallel$geo terms → significant shift of eigenfrequency spectrum



Results - 3

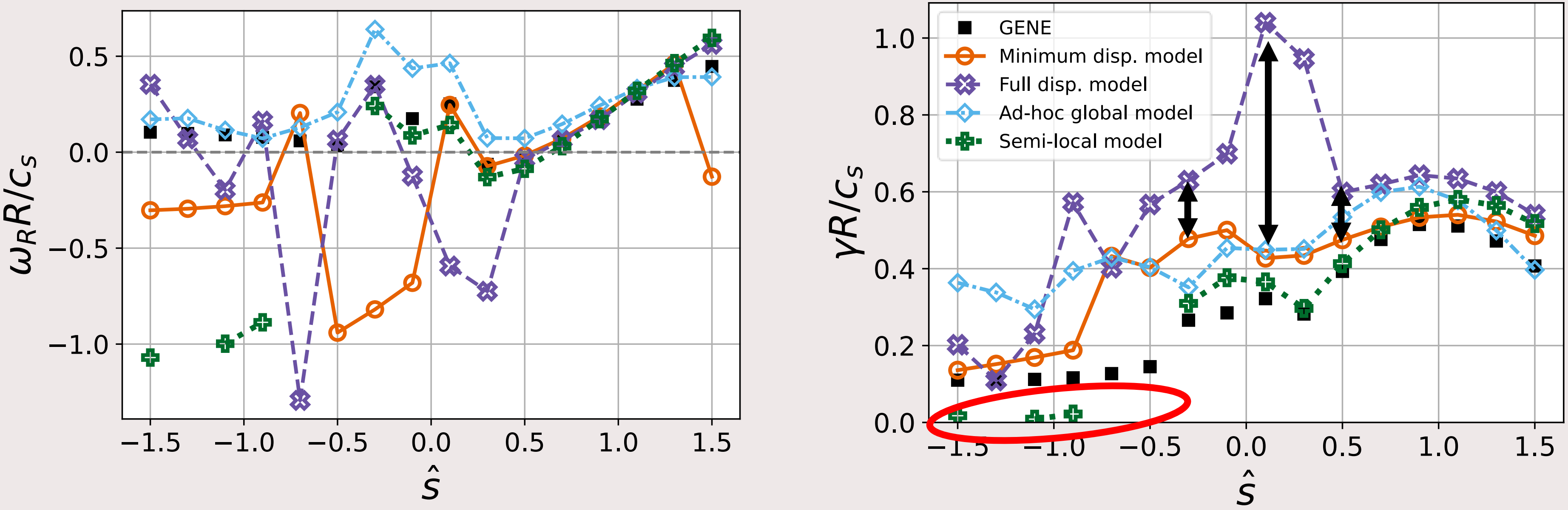
- ITG dispersion in LHD; $^{a}/_{L_n} = 2, ^{a}/_{L_T} = 4$
- Eigenfunctions → minimal model closely capture envelope width, though miss finer ripple-like modulation
- Not improved by including geometric variation → additional squeezing of potential widths



$\gamma(L_{\text{geo}} \sim L_\phi) \geqslant \gamma(L_{\text{geo}} \ll L_\phi)$

Results - 4

- Shear-stabilisation of ITG in $s-\alpha$; ${}^{R}/_{L_n} = 7.86, {}^{R}/_{L_T} = 15.71$

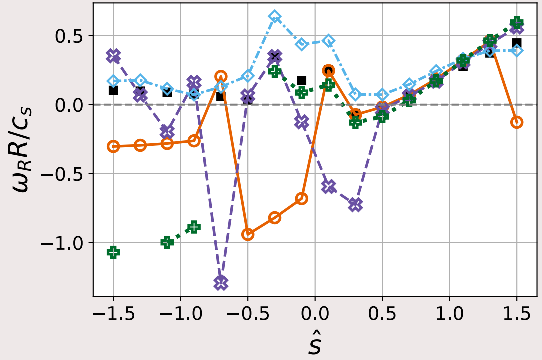


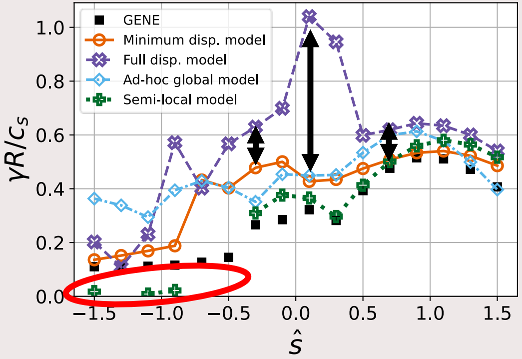
Tokamak: $L_{\mathrm{geo}} \sim \frac{1}{qR}$

Stellarator: $L_{\mathrm{geo}} \sim \frac{N_{\mathrm{fp}}}{qR}$

Results - 5

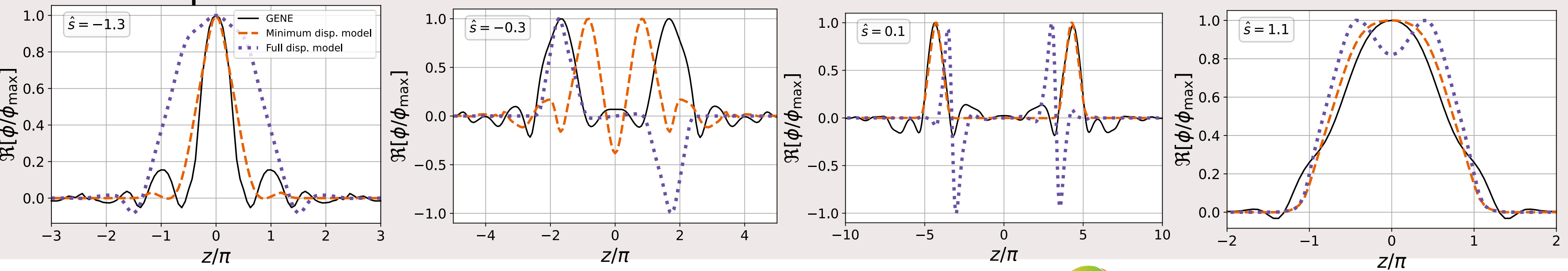
- Geometry effects on ITG; ${}^{R}/_{L_n} = 7.86, {}^{R}/_{L_T} = 15.71$
- Ad-hoc global model provides predictions → overestimate growth rates below $\hat{s} < -0.5$
- Semi-local model closest match down to $\hat{s} < -0.5$, subsequently strong discrepancy ⚠️
- Additional destabilisation by geometry variation present, but more mild compared to LHD
- Minimal model → closest agreement to GENE above $\hat{s} > 0.1$





Results - 6

- Geometry effects on ITG; $^R/_{L_n} = 7.86, {}^R/_{L_T} = 15.71$
- Eigenfunctions: strongly localized near OMP for large $|\hat{s}|$ vs extended at small $|\hat{s}|$
- Minimal model most accurately represents GENE eigenfunctions, with exception of low-to-intermediate negative shear → geometry variation provides most accurate width + localisation



Results - 7

- Why eigenfrequency-eigenmode degradation by $\nabla_{\parallel}\mathrm{geo}$ inclusion?
- Supposedly $\mathcal{O}\left(\left(\frac{\omega_{ti}}{\omega}\right)^2\right)$ corrections……
- Asymptotic expansion parameter ${}^{\omega_{ti}}/_{\omega}$ → matter of convenience
- Underlying physics → (im)balance between parallel/perpendicular streaming

$$\frac{\partial g_s}{\partial t} + v_{\parallel}\nabla_{\parallel} g_s + \boldsymbol{v}_{\boldsymbol{di}} \cdot \nabla_{\perp} g_s = \frac{q_i}{T_i}\left\langle\frac{\partial\phi}{\partial t}\right\rangle_R F_{Mi} - \langle\boldsymbol{v}_{\boldsymbol{E}\times\boldsymbol{B}}\rangle \cdot \nabla F_{Mi}$$

Results - 7

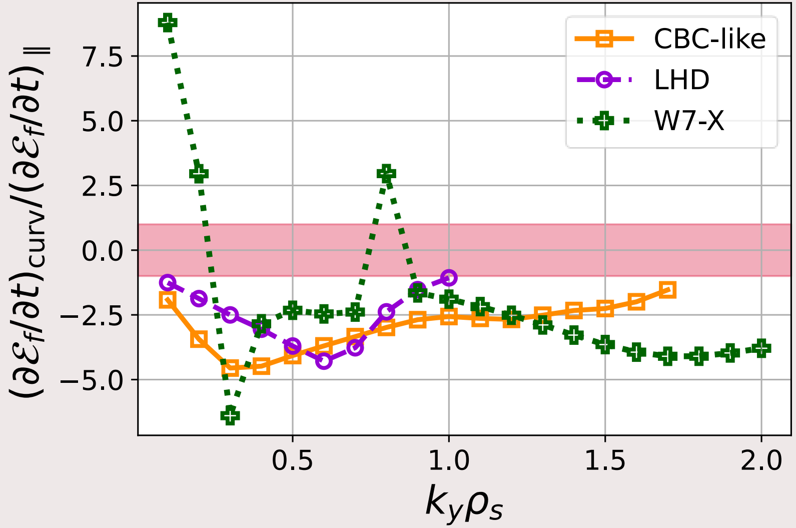
- Asymptotic expansion parameter ${}^{\omega_{ti}}/_{\omega}$ → matter of convenience
- Underlying physics → (im)balance between parallel/perpendicular streaming

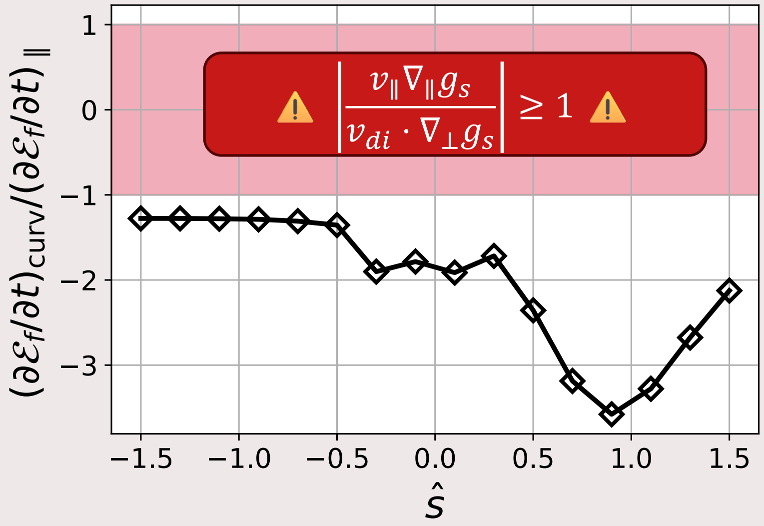
$$\frac{\partial g_s}{\partial t} + v_\parallel \nabla_\parallel g_s + \boldsymbol{v}_{\boldsymbol{di}} \cdot \nabla_\perp g_s = \overbrace{\frac{q_i}{T_i}\left\langle \frac{\partial \phi}{\partial t} \right\rangle_R F_{Mi} - \langle \boldsymbol{v}_{\boldsymbol{E\times B}} \rangle \cdot \nabla F_{Mi}}^{\text{Field-equilibrium coupling}}$$

- Neglect of parallel streaming justifiable if toroidal drives dominates $\frac{v_\parallel \nabla_\parallel g_s}{\boldsymbol{v}_{\boldsymbol{di}} \cdot \nabla_\perp g_s} \ll 1$
- Impossible to satisfy globally along the field; $\boldsymbol{v}_{\boldsymbol{di}} \approx 0$ near reversals of drift ⚠️

Results - 8

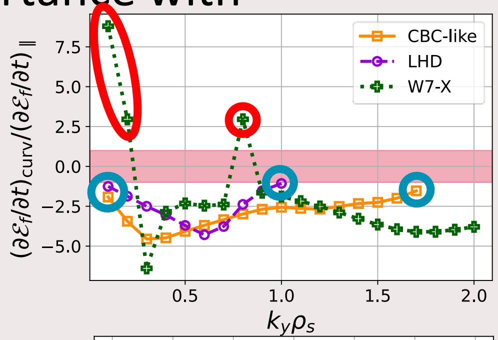
- Parallel vs toroidal effects → investigated relative importance with GENE's free-energy diagnostic[7]

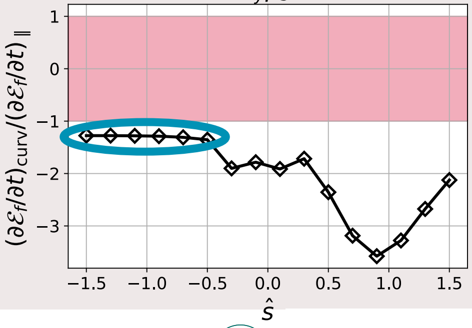




Results - 8

- Parallel vs toroidal effects → investigated relative importance with GENE's free-energy diagnostic[7]
- Generally → toroidal drive exceeds parallel Landau damping
- Toroidal effects ~ 2−5 × stronger → not convincingly dominant
- Close to marginal stability → near 1:1 ratio ⚠️
- Peculiarities W7-X: simultaneous toroidal and slab drive





Conclusions & outlook - 1

M.RM-DISP-REL

- Self-consistent adiabatic-electron ITG dispersion relation model constructed based on asymptotically small transit-effect limit
- Two model variants considered → with and without geometric variations corrections
- Tested versus heuristic dispersion models ($\hat{\phi} \rightarrow \phi_{GENE}$) and high-fidelity flux-tube simulations
- Minimal model provides best agreement with GENE; extended model overpredicts growth rates & hyper-localizes eigenmodes
- Eigenfrequencies predicted to greater accuracy than eigenmodes

Conclusions & outlook - 2

M.RM-DISP-REL

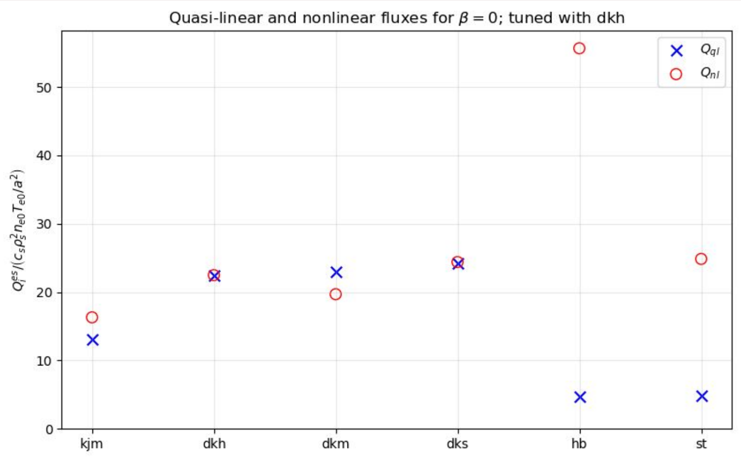
- Applicability of asymptotic-transit limit scrutinized → toroidal effects do not strongly dominate globally
- Greater impact eigenmodes → expansion terms dictate ϕ to $\mathcal{O}(1)$ ⚠️
- ➔ Adiabatic-electron scenarios: better off with high-fidelity tools

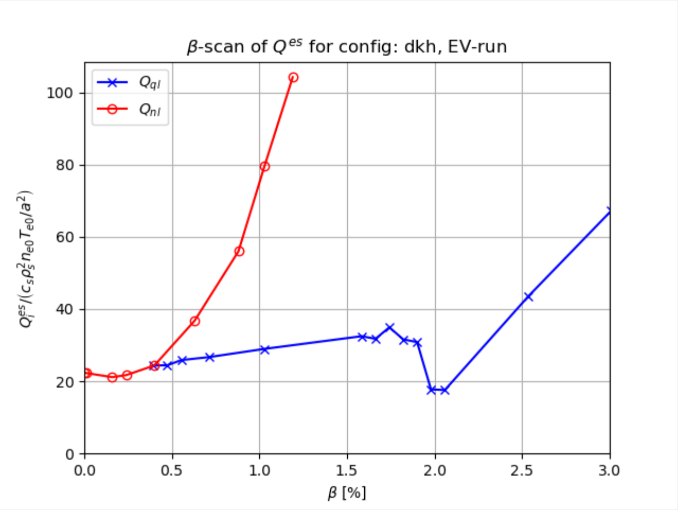
- Model improvement → reintroduce trapped-electron physics
  - Opposite $\frac{\omega_{be}}{\omega} \gg 1$ asymptotic
  - Bounce-average account for parallel streaming on $\mathcal{O}(f_t)$ in dispersion relation
  - Opportunity for accuracy vs speed competition in kinetic-electron scenario

Additional update

M.RM-QL-FLUXES

- Aside from ITG/TEM transport losses due to KBM important at finite-β
- First tests of QL model: cross-configuration (ES) and within configuration (dialing β)





Additional update

M.RM-QL-FLUXES

- Aside from ITG/TEM transport losses due to KBM important at finite-β
- First tests of QL model: cross-configuration (ES) and within configuration (dialing β)
- @ fixed gradient intra-device configuration effects captured, but inter-device effects poorly represented
- @ fixed configuration → transition from electrostatic-electromagnetic turbulence near (st)KBM onset poorly represented
- Conclusion: QL modelling for stellarators does not have widespread applicability, though may yield informative predictions in limited regions of parameter space