



Further development of a phase space transport code with consistent sources and sinks

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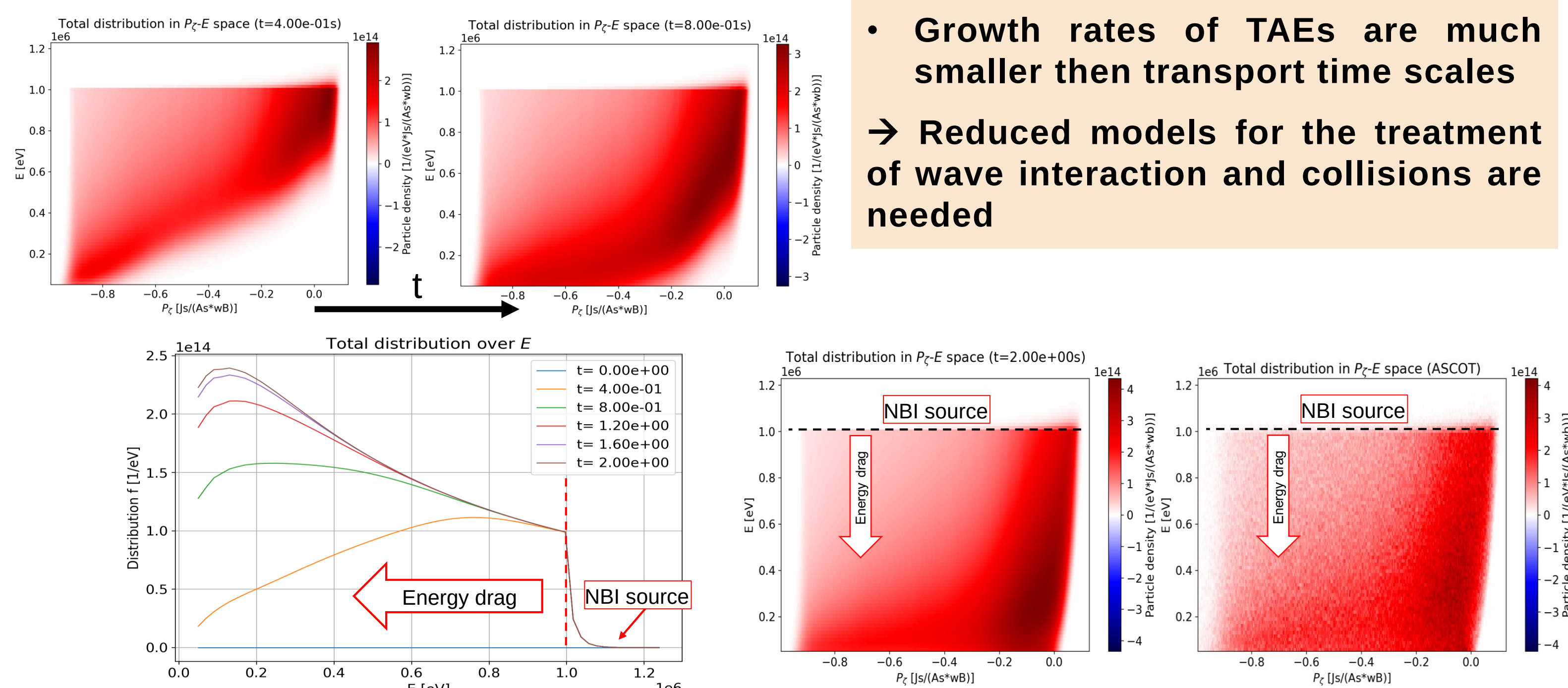
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ABSTRACT

The transport of supra-thermal particles in fusion plasmas can only be inadequately described with a local and 1-d radial-diffusive transport model due to the broad orbits and resonances that occur. A general gyrokinetic transport theory has been developed in recent years - the Phase Space Zonal Structure transport theory [1, 2, 3, 4], which extends the conventional transport equations into Phase Space and consistently evolves a (nonlinear) equilibrium referred to as the Zonal State. This new theory is currently implemented in a transport code (ATEP-3D)[5, 6] that has been developed to describe the fluxes in the three dimensional constants-of-motion (CoM) space as calculated from single particle trajectories determined from a pre-calculated perturbed Hamiltonian. Sources and sinks are modeled inside simulations as well, in order to properly simulate the evolution of the energetic particle (EP) distribution, and the eventual stationary distribution arising from the balance of sources and sinks in the presence of a spectrum of unstable modes. Comparison with well-established Monte-Carlo codes for the slowing down of EPs will be shown. Various models for describing the phase-space transport of particles due to their interaction with a spectrum of unstable modes are discussed.

NBI-SD (SHOT 53299):



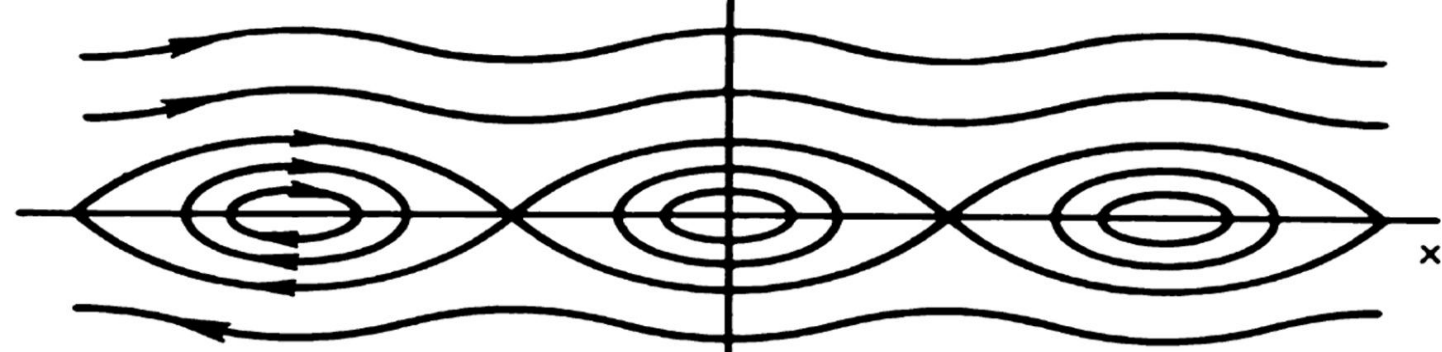
MOTIVATION

- **Multi scale problem in time**
- **Growth rates of TAEs are much smaller than transport time scales**
- **Reduced models for the treatment of wave interaction and collisions are needed**

THREE MAIN COMPONENTS OF THIS WORK

I: EP-TAE INTERACTION

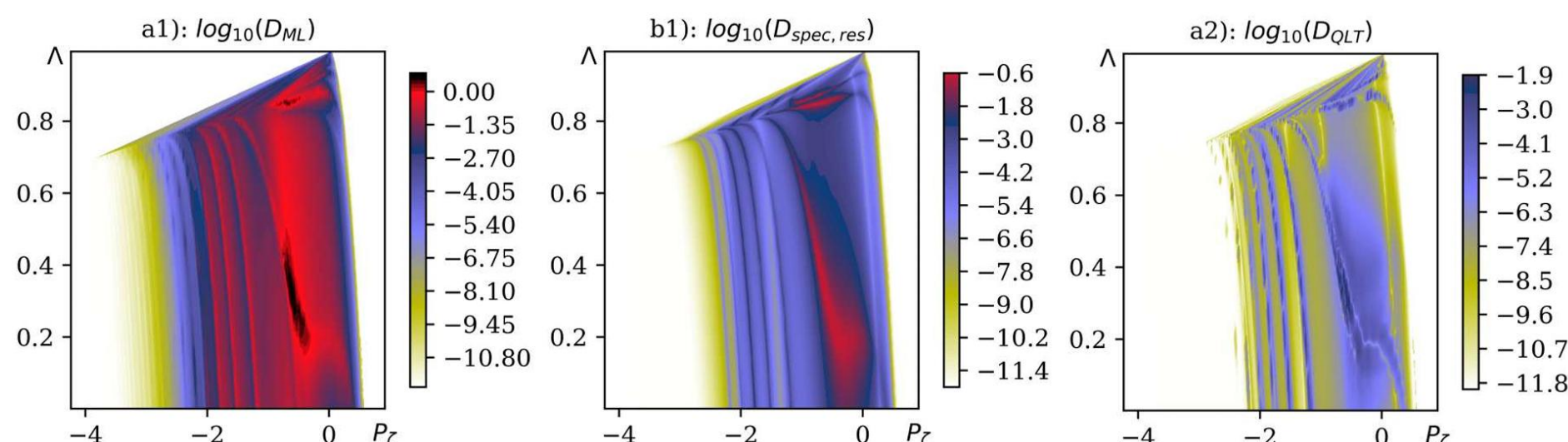
The interaction with a TAE changes the particle orbit and results in a displacement of the marker in CoM space. Their trajectories can be visualized via an island chain in velocity space:



- Different models to obtain **diffusion coefficients**:
 - QLT, ML and a model based on the autocorrelation time (here only P_z -transport):

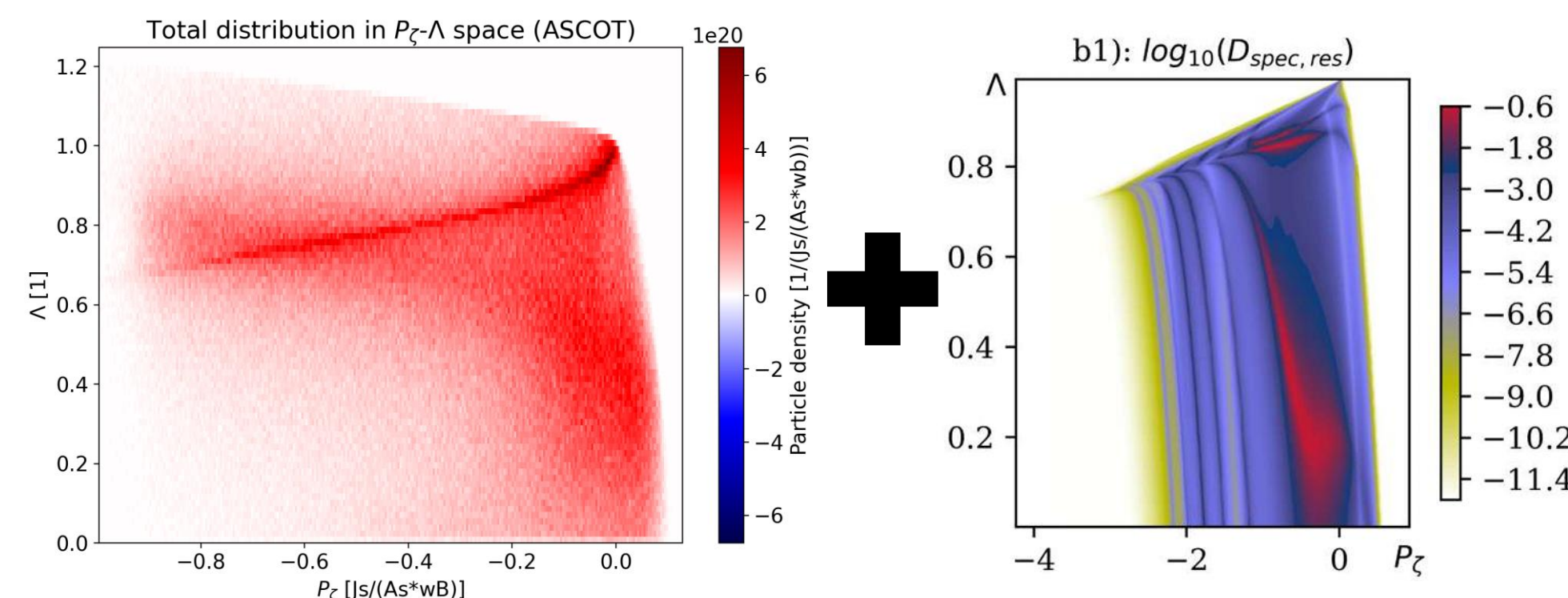
$$D_{QLT} = \left(\frac{dP_z}{dt} \right)^2 \tau_{ac} \quad D_{ML} = \frac{\Delta P_{z,max} \Delta P_{z,orbit} \omega_t}{4\pi}$$
$$D_{spec,res} = \sum_n \left(\sum_m \left(\frac{-m \Phi_n}{r} \right)^2 \sum_m \frac{|\gamma_n|}{(\omega_t |l_n - m|)^2 + \gamma_n^2} \right)$$

DIFF COEFF. FOR P_z



SD-DISTRIBUTION + DIFF. COEFF.

With the help of the calculated diffusion coefficients for the transport due to TAEs, the slowing down distribution resulting then from calculations of part II of this work can now be used as an initial distribution, and the time evolution can be calculated:



II: FVM SOLVER FOR COM-SPACE

Simulation of the transport of EPs in CoM-Space via a finite volumes method. The equation to discretize is a diffusion-advection equation in the three CoM-coordinates: Canonical Toroidal Momentum, Energy and Pitch.:

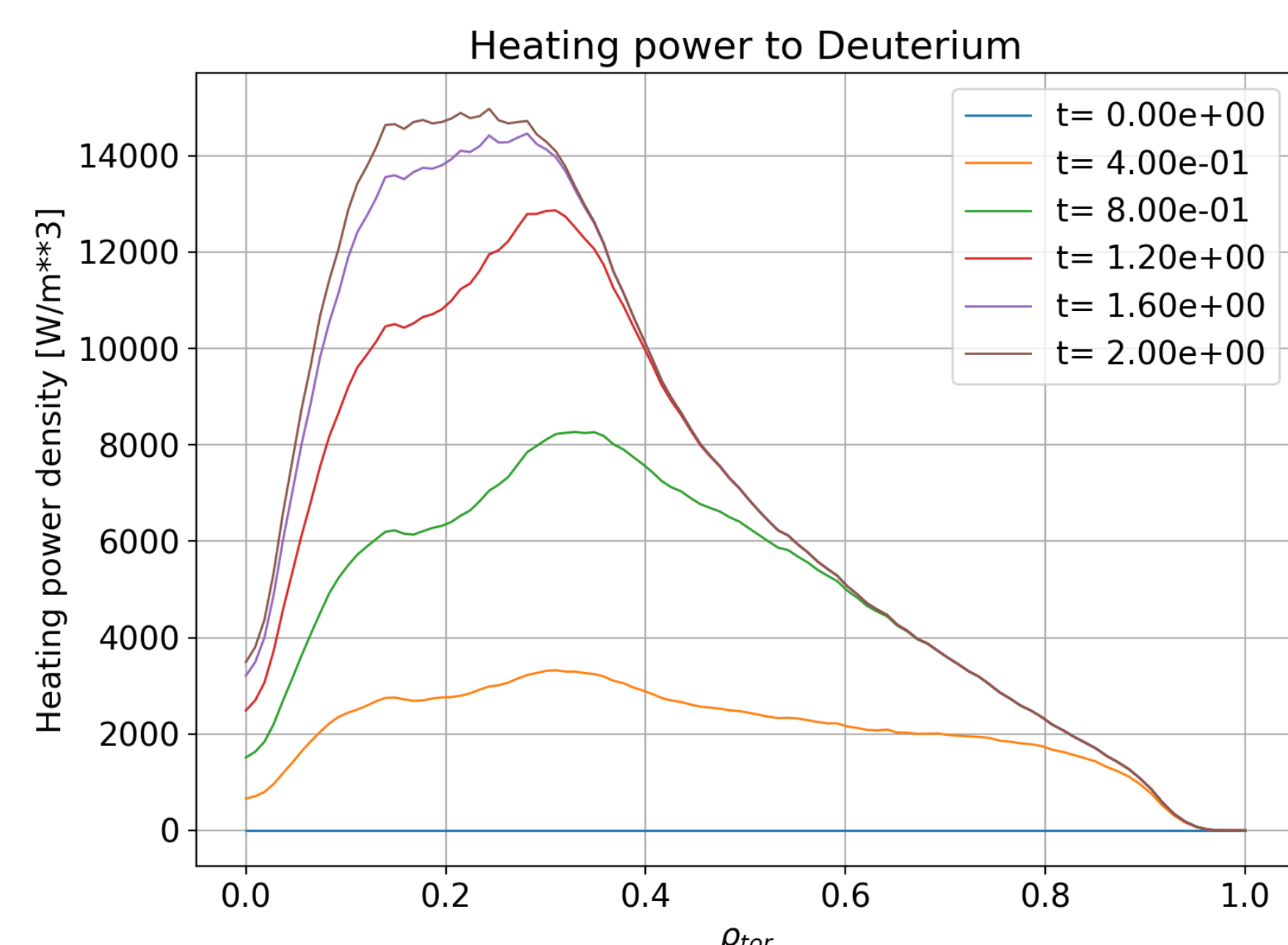
$$\frac{\partial f}{\partial t} = s + \gamma f + \frac{\partial}{\partial x^\alpha} (v^\alpha f) + \frac{\partial}{\partial x^\alpha} \left(D^{\alpha\beta} \frac{\partial}{\partial x^\beta} f \right) + \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial x^\beta} (D^{\alpha\beta} f)$$

- **Diffusion and dispersion discretized separately**
- **Time evolution with Crank-Nicolson Scheme**
 - Alternative mode: Steady-state solution
- **Separate mode for steady state solutions**
- **Efficient time evolution with PETSc**
- **Matrix preparation / field setup with Scipy/Numpy**

SLOWING-DOWN OF ITER NBI-EP

- A time evolution of the **slowing down of EPs** from ITERs NBI beams can be calculated
- The **source term “s”** in this case are the **NBI particles** and the sink are proper boundary conditions at the lower energy boundary (thermalization).
- Output can be represented as **densities over CoM-space or rho_tor**
- **Output of heating profiles to different species**

ENERGY PROFILE TO ^2H (SHOT 53299)



III: EP-BACKGROUND COLLISIONS

Collisional coefficients for the collision between a thermal background plasma consisting of multiple particle species and EPs have been derived. The derivation is based on methods used in [7], but the Fokker-Planck-Equation is formulated directly in CoM-space, where the coefficients are expected values of changes of CoM coordinates. The coefficients in this Fokker-Planck equation can be written in a compact form as follows:

$$v^\alpha = - \left(\frac{\langle \Delta P_z \rangle}{\langle \Delta E \rangle} \right), \quad D^{\alpha\beta} = \frac{1}{2} \begin{pmatrix} \langle \Delta P_z \Delta P_z \rangle & \langle \Delta P_z \Delta E \rangle & \langle \Delta P_z \Delta \Lambda \rangle \\ \langle \Delta E \Delta P_z \rangle & \langle \Delta E \Delta E \rangle & \langle \Delta E \Delta \Lambda \rangle \\ \langle \Delta \Lambda \Delta P_z \rangle & \langle \Delta \Lambda \Delta E \rangle & \langle \Delta \Lambda \Delta \Lambda \rangle \end{pmatrix}$$

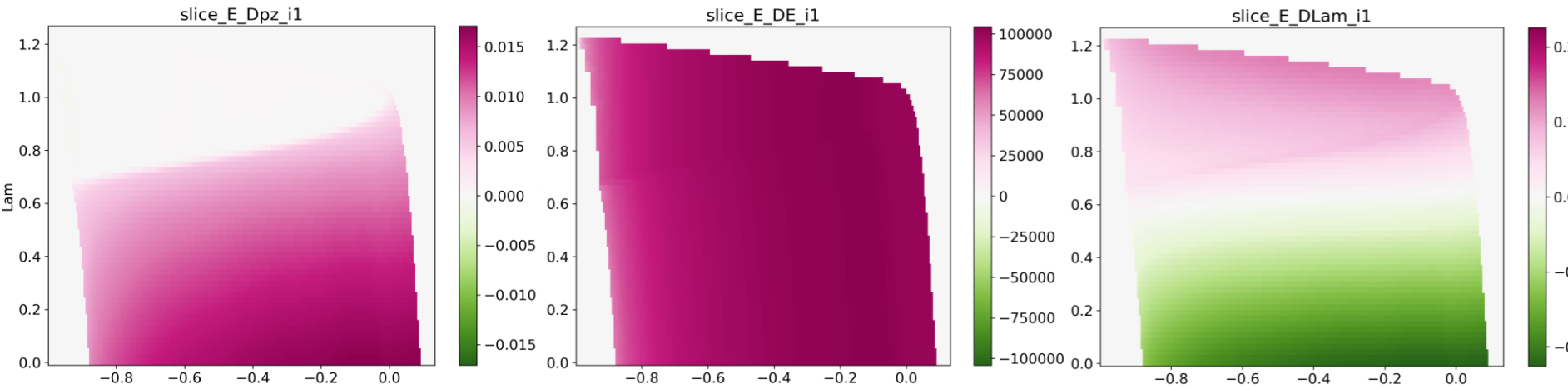
ADVECTION COEFFICIENTS

As an example of the collisional coefficients, the three derived advection coefficients can be seen in the following:

$$\langle \Delta P_z \rangle = \sum_b \Gamma_a \frac{m_a + m_b}{m_b} \frac{m_a g v_{\parallel}}{B v^3} \left[-\text{erf} \left(\frac{v}{v_{th}} \right) + \frac{v}{v_{th}} \frac{2}{\sqrt{\pi}} \exp \left(-\frac{v^2}{v_{th}^2} \right) \right]$$
$$\langle \Delta E \rangle = \sum_b m_a \Gamma_a \frac{1}{v} \left[\left(-\frac{m_a}{m_b} \right) \text{erf} \left(\frac{v}{v_{th}} \right) + \frac{m_a + m_b}{m_b} \frac{v}{v_{th}} \frac{2}{\sqrt{\pi}} \exp \left(-\frac{v^2}{v_{th}^2} \right) \right]$$
$$\langle \Delta \Lambda \rangle = \sum_b \Gamma_a \frac{B_0}{B v^5} (3 v_{\parallel}^2 - v^2) \left[\left(1 - \frac{1}{2} \frac{v_{th}^2}{v^2} \right) \text{erf} \left(\frac{v}{v_{th}} \right) + \frac{v_{th}}{v} \frac{1}{\sqrt{\pi}} \exp \left(-\frac{v^2}{v_{th}^2} \right) \right]$$

COLLISIONAL COEFFICIENTS IN HAGIS

- **Coefficients are implemented in HAGIS for marker**
- **Orbit averaging for all coefficients**; Averaging justified: large difference between collision times and orbit bounce frequency



PERFORMANCE – COMPARISON

- Computation of Coefficients + Time evolution takes **a few minutes** depending on the grid size
- **In between** large expensive models like **ASCOT4** and fast 1D models like **RABBIT**, while **still being able to perform computations in 3D-CoM-space**

CONCLUSION

- **FVM**: discretization of an advection-diffusion equation has been implemented
- **The time evolution with PETSc**: particle distribution function is evolved in time via a Crank-Nicolson-Scheme, or with a direct steady-state solver.
- **Collisional coefficients in CoM Space**: for the transport of energetic particles are derived
- **Heating-profiles**: A routine is implemented that tracks the energy deposition to various background particle species in order to obtain heating profiles.
- **Different diffusive models**: for the calculation of coefficients in CoM-space for the transport of EPs due to wave-particle interaction are compared.

CURRENT STATE AND FUTURE WORK

- **Where it stands**:
 - Slowing down distributions in CoM space can be efficiently calculated
 - Radial heating profiles as output
- **What is to be done**:
 - With the energy and density gradients the redistribution of the particle density due to interactions with TAE waves can be studied in CoM space.
 - The different diffusion coefficients need to be implemented in HAGIS so that the evolution of the single particle distribution can be studied
 - Momentum, and current profiles as output + comparison

1) Falessi M V and Zonca F 2019 Physics of Plasmas 26 022305 URL <https://doi.org/10.1063/1.5063874>

2) Zonca F, Chen L, Falessi M V and Qiu Z 2021 Journal of Physics: Conference Series 1785 012005 URL <https://dx.doi.org/10.1088/1742-6596/1785/1/012005>

3) Falessi M V, Chen L, Qiu Z and Zonca F 2023 New Journal of Physics 25 123035 URL <https://dx.doi.org/10.1088/1367-2630/ad127d>

4) F Zonca et al 2025 Proc. 30th IAEA FEC, Chengdu, China (2025)

5) Lauber P, Falessi M, Meng G, Hayward-Schneider T, Popa V A, Zonca F and Schneider M 2024 Nuclear Fusion 64 096010 URL <https://dx.doi.org/10.1088/1741-4326/ad6336>

6) Meng G, Lauber P, Lu Z, Bergmann A and Schneider M 2024 Nuclear Fusion URL <http://iopscience.iop.org/article/10.1088/1741-4326/ad5190>

7) Rosenbluth M., MacDonald W. and Judd D. 1957 Phys. Rev.107 1



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