



GRILLIX (work in progress topics):

- Status of TCV-X23
- Consistency verification
- Updates on FCI boundary approach

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EUROfusion



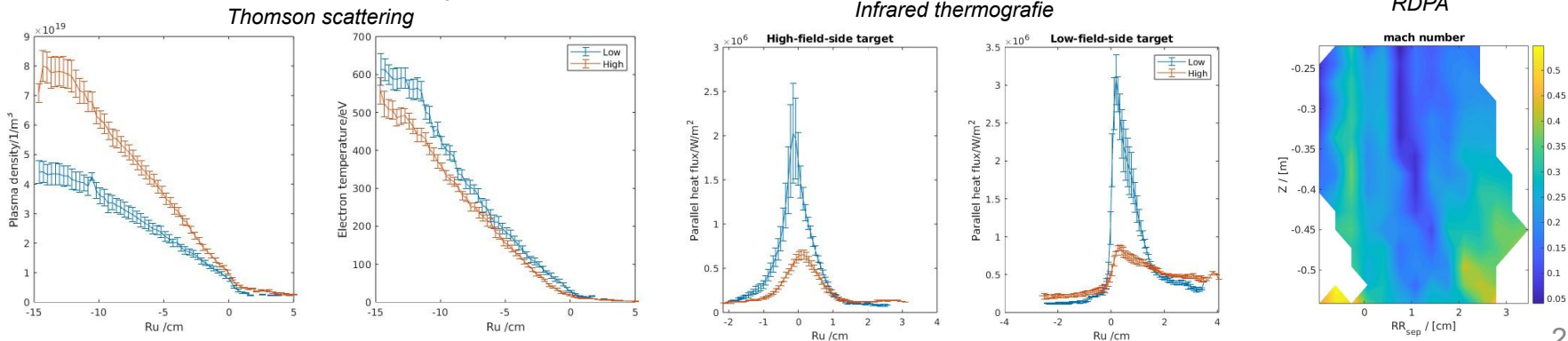
This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No. 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

Introduction

- Experimental dataset to validate edge/SOL turbulence codes
- Highly dissipative regime (detachment)
- In spirit of TCV-X21 validation effort
- Publicly accessible <https://github.com/SPCData/TCV-X23> , D. Mancini *et al* 2026 *PPCF* **68** 055029

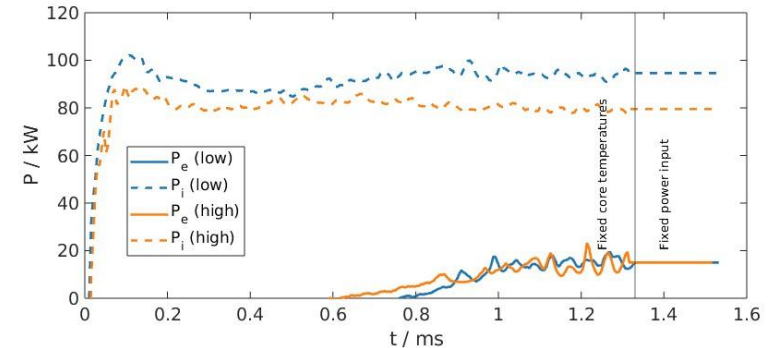
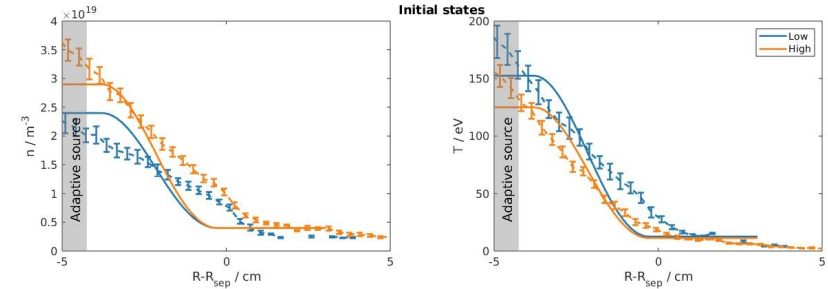
Dataset overview (user's view)

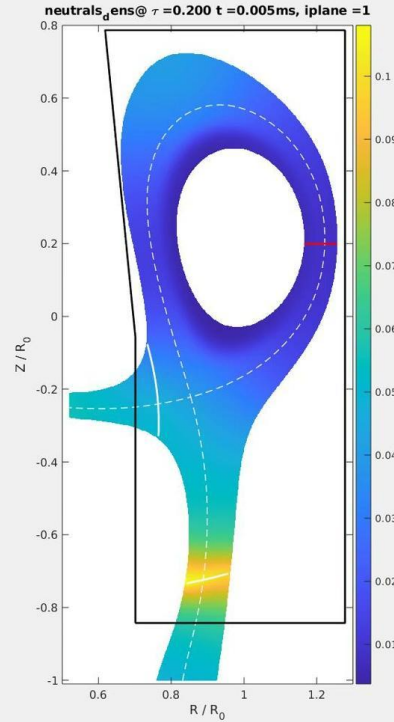
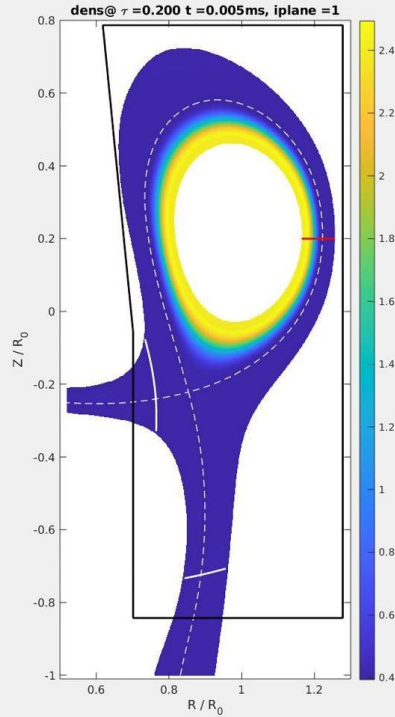
- Forward/reversed field and low/high density: Target heat flux reduction mostly in forward-high density case
- Single eqdsk file for magnetic field data
- jsat data missing in fwd-low
- GPI data not part of repository ?



Simulation setup

- Resolution: 16 planes, $h_{\perp} = 0.096\text{mm}$ ($1q_s$ @25eV) $\rightarrow$ 10.2 Million grid points in total, $\Delta t = 1.29\text{ns}$
- Initial state:
 - Only gradients in closed field line region
 - Homogeneous neutrals: 0.05(low) vs 0.17(high) 10^{19} m^{-3}
- Sourcing the system
 - Particles: Recycling coefficient $R=0.95$
Adaptive density source near inner boundary
 - Power:
 - $t < 1.35\text{ ms}$: Fixed temperatures at core
 - $t > 1.35\text{ ms}$: Fixed power input (Low: 110kW, High: 95kW)
- Model features
 - Full-f, electromagnetic
 - Free streaming limited heat fluxes
 - 3 moments neutrals + recycling, 1% Carbon impurity
 - Usual immersed boundaries

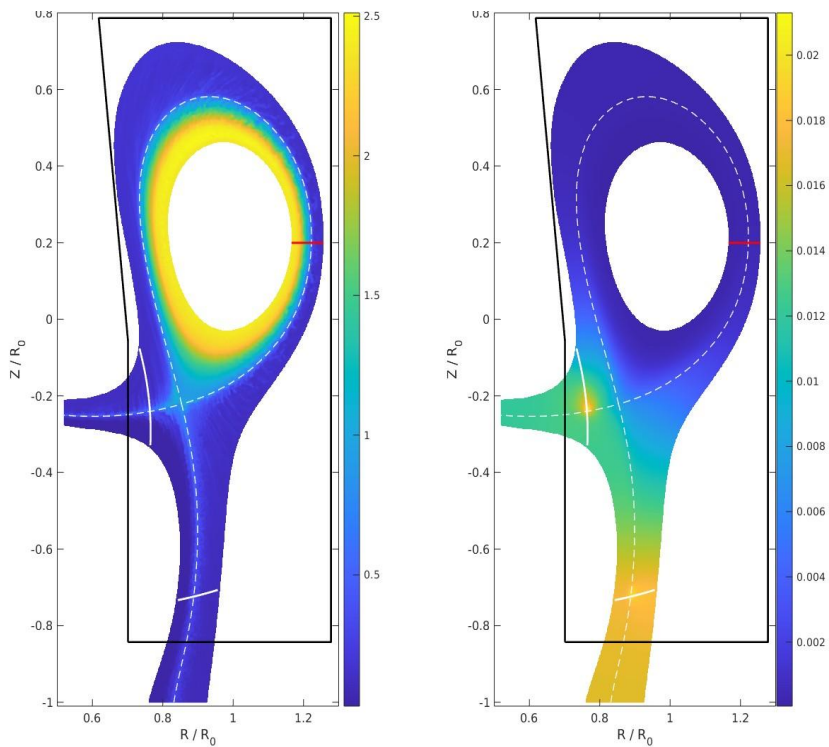




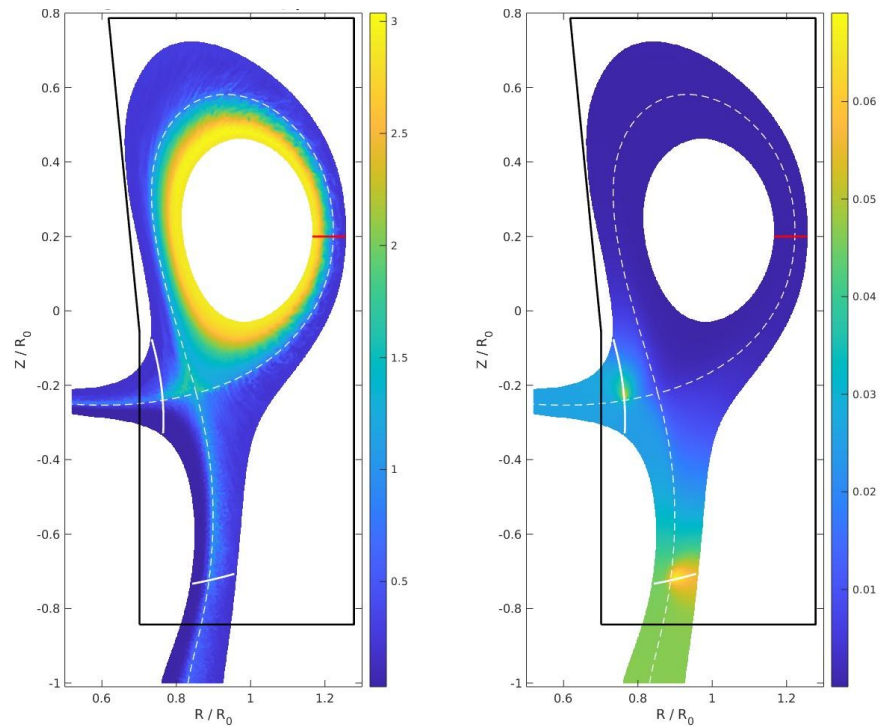
Computational resources

- 1 Pitagora node, 16 MPI x 16 OMP
- 1.97s / timestep
 ≈ 17 days / millisecond
- ≈ 105000 CPUh / millisecond

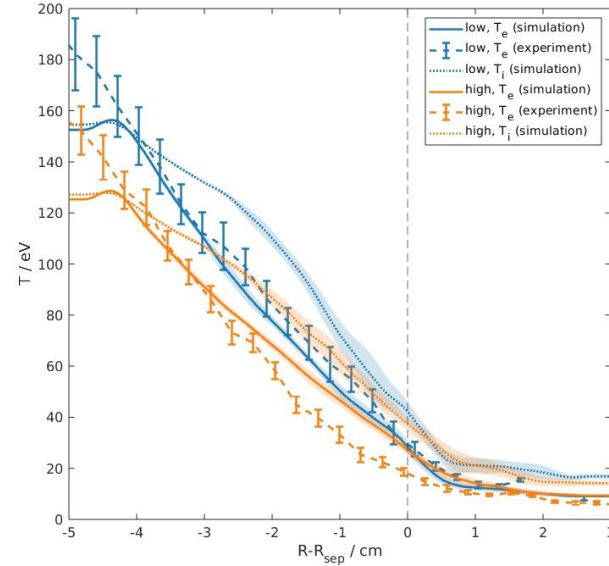
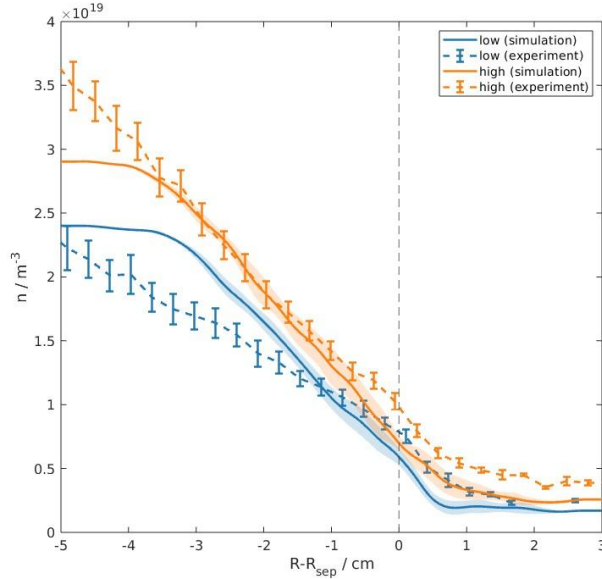
Low density



High density



TCV-X23: Outboard midplane profiles

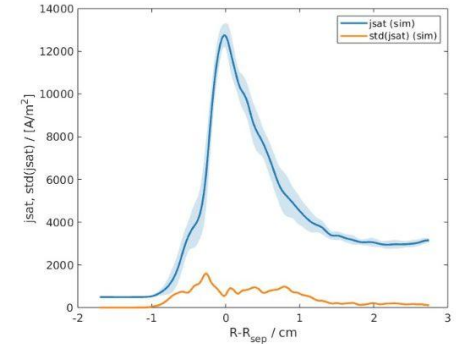
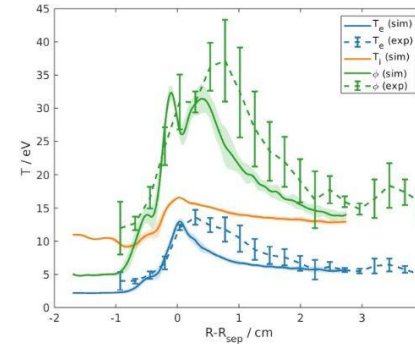
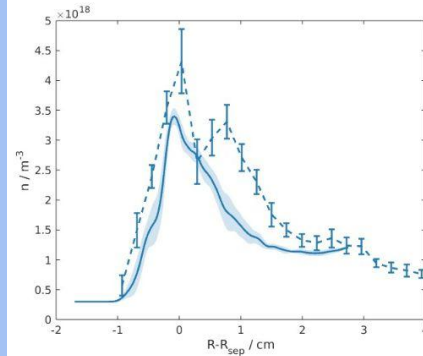


- Overall reasonable match with experiment
- Too low density in high simulation
- No pronounced differences around separatrix between low and high density cases
- Difficult to control separatrix values with given control knobs

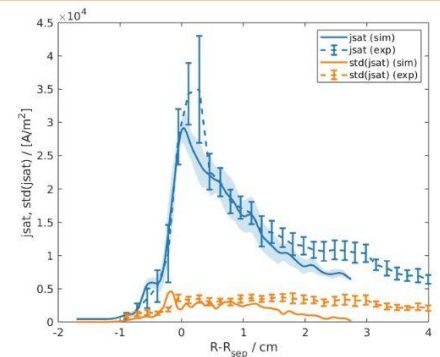
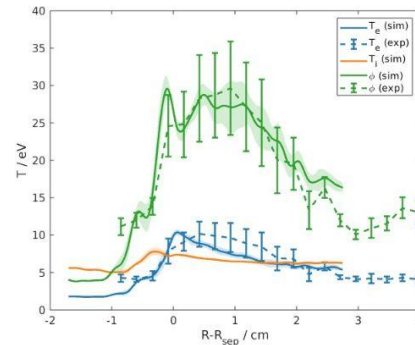
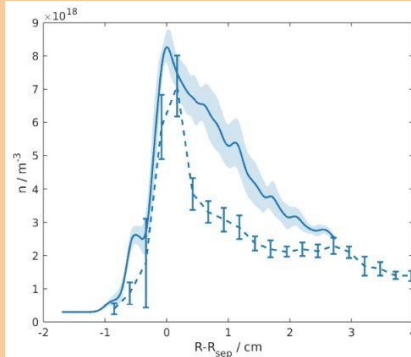
TCV-X23: Low field side target



Low density case:



High density case:

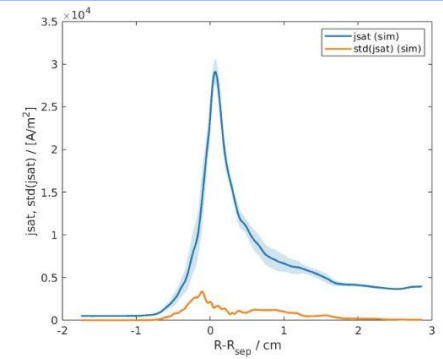
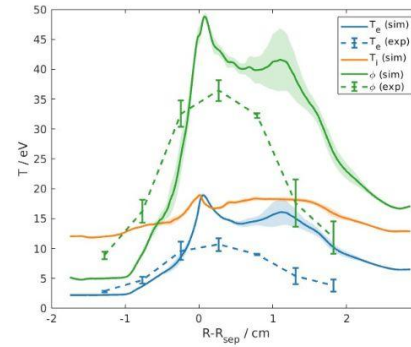
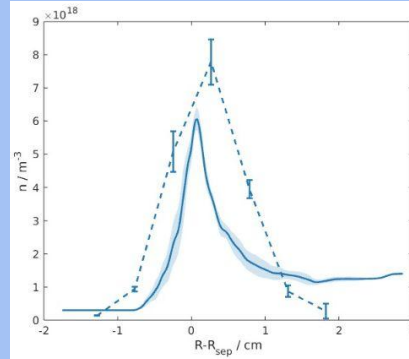


→ Pretty good agreement
with experiment

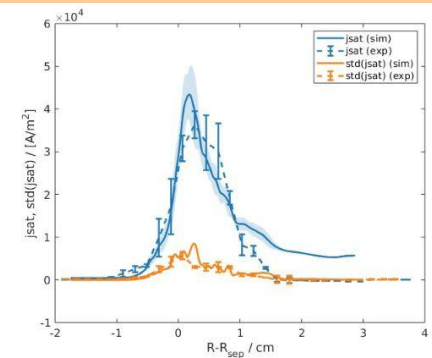
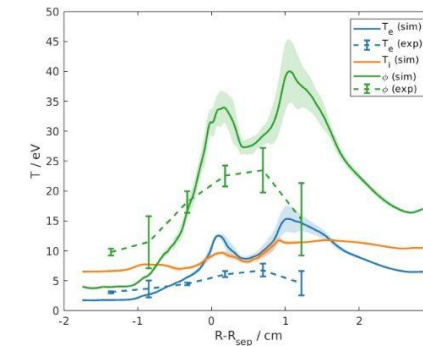
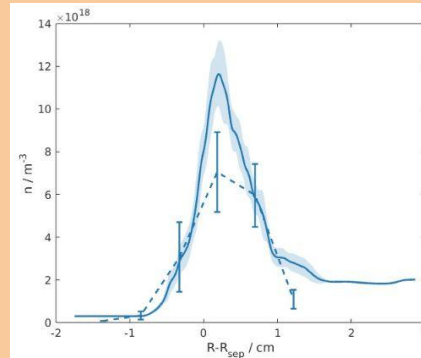
TCV-X23: High field side target



Low density case:



High density case:



→ Pretty good agreement
with experiment



TCV-X23: Target heat fluxes

Diagnosed target heat fluxes

- Parallel conductive and convective parts ($j_{||}=0$)

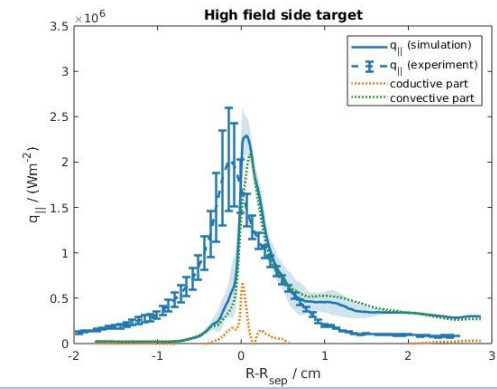
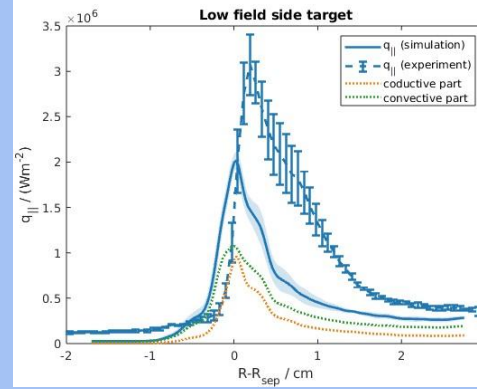
$$q_{||} = \underbrace{-\chi_{||,e}^{lim} \nabla_{||} T_e - \chi_{||,i}^{lim} \nabla_{||} T_i}_{\text{conductive}} + \underbrace{\frac{5}{2} (p_e + p_i) u_{||,i}}_{\text{convective}}$$

- No fluxes due to drifts and neutrals taken into account

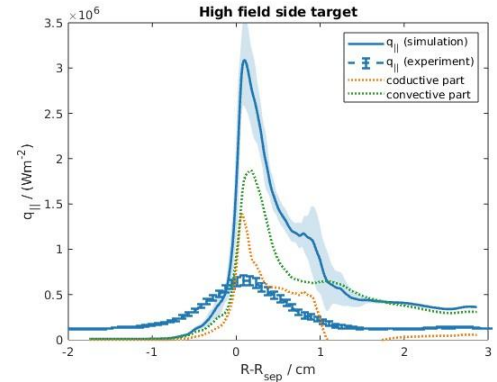
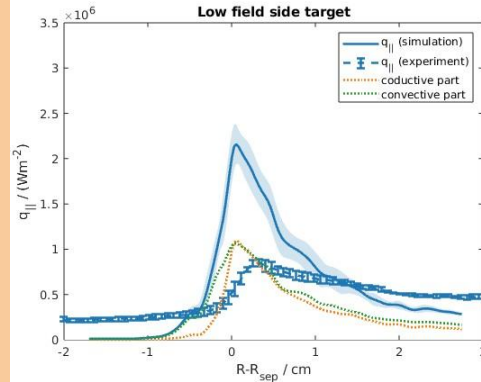
Observations:

- Good match in shape and magnitude for low density case
- No heat flux reduction in high density case

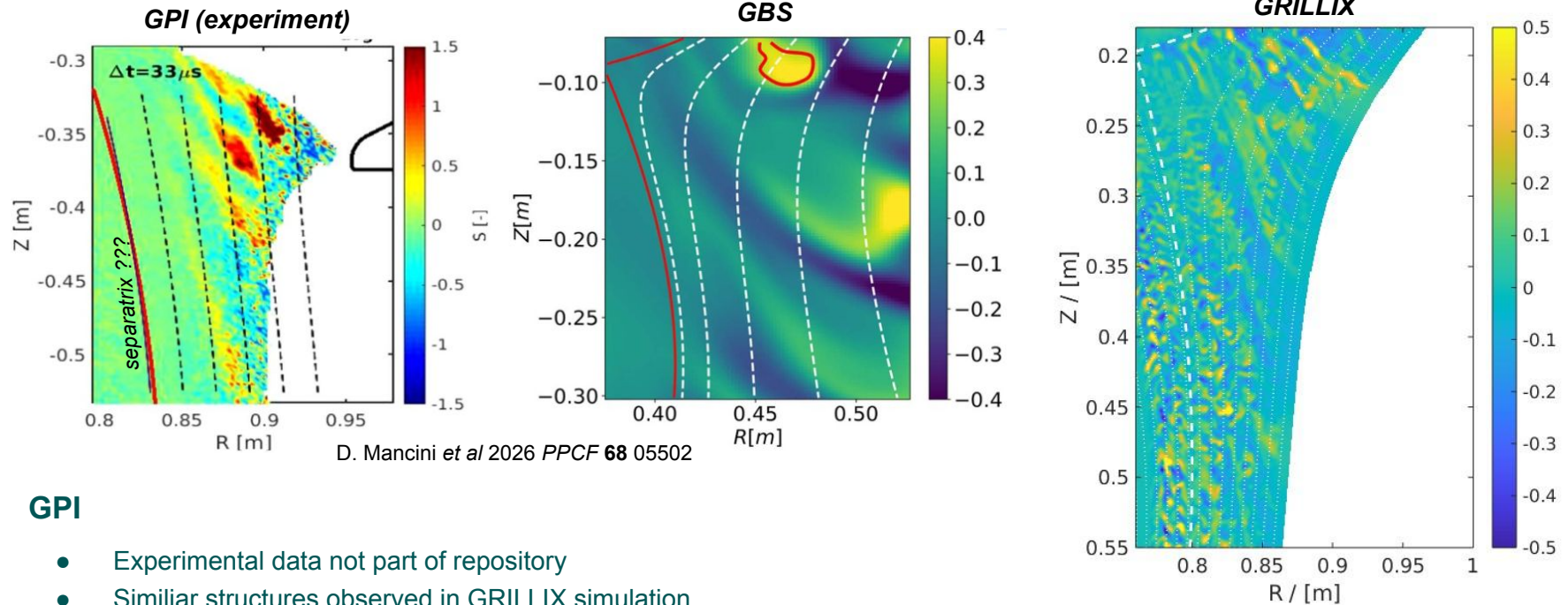
Low density case:



High density case:



TCV-X23: Further validation data



GPI

- Experimental data not part of repository
- Similar structures observed in GRILLIX simulation
- I am confused by the shown frame
- Blob tracking diagnostics would be cool

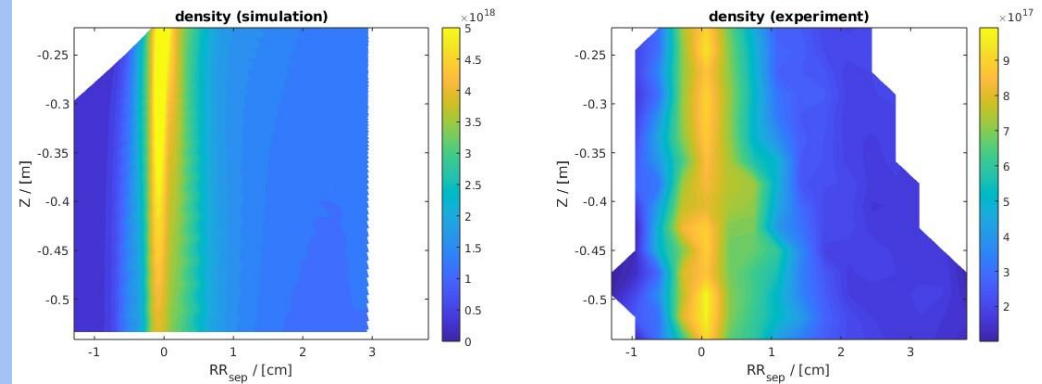
TCV-X23: Further validation data



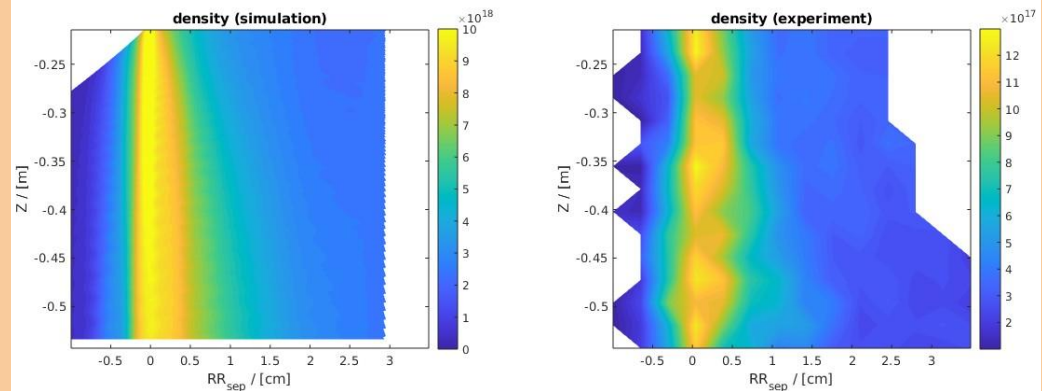
RDPA

- Shape looks ok
- Amplitude in many channels off by order of magnitude
- Experimental data strange, e.g. low case:
 - Thomson (OMP): $n_{\text{sep}} \sim 8 \cdot 10^{18}$
 - Langmuir probe (LFS): $n_{\text{sep}} \sim 4 \cdot 10^{18}$
 - RDPA: $n_{\text{sep}} \sim 1 \cdot 10^{18}$

Low density case:



High density case:



COMPASS validation (see P. Macha)



Compass validation

P. Macha 2026, Validation of an edge turbulence code in the COMPASS tokamak, PhD thesis, Czech Technical University in Prague

- Extensive validation with GBS/GRILLIX/DIFFFLOW
- Includes many higher order statistical quantities
- Very consistent overall agreement with experiment
- Discrepancies in radial electric field and divertor boundaries

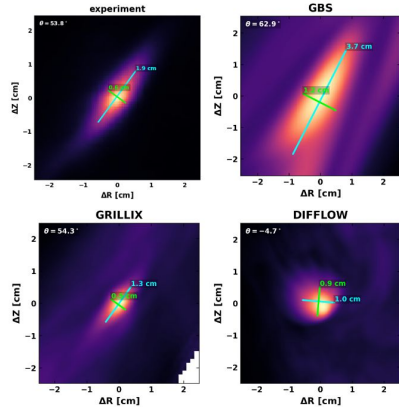


Figure 9.13: 2D conditionally averaged blobs from the experiment #15487, GBS, GRILLIX, and DIFFFLOW. The CA blobs are fitted with ellipses to determine their size and tilt.

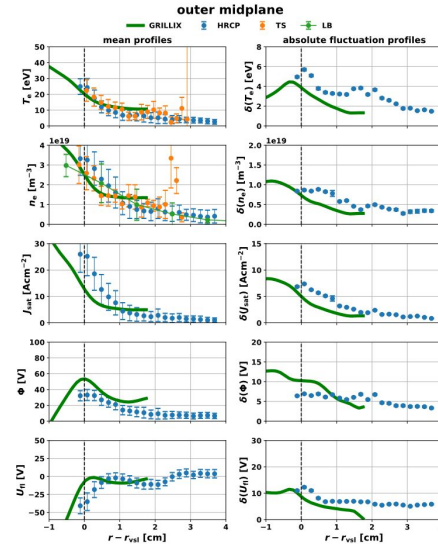
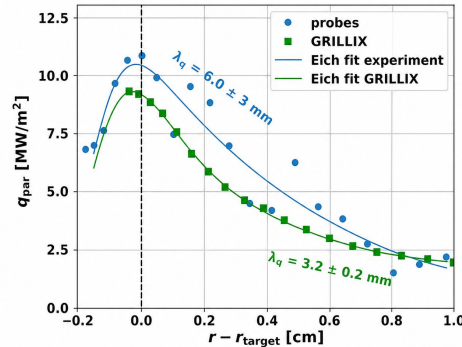


Figure 8.3: Radial profiles of the electron temperature, ion current density, plasma potential, and floating potential (left) mean values and (right) absolute fluctuations at the outer midplane.

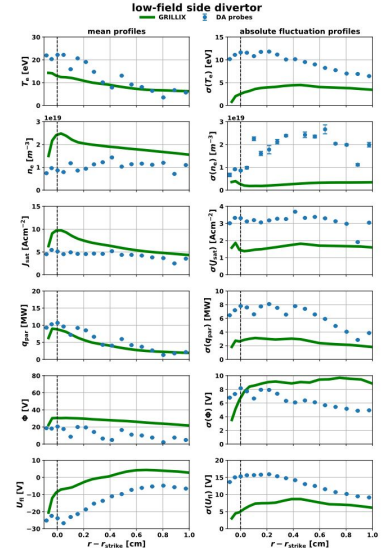


Figure 8.7: Radial profiles of the electron temperature, ion current density, plasma potential, and floating potential mean values (left column) and absolute fluctuations (right column) at the divertor.

‘The results reproduced the outer midplane measurements well and provided the best overall agreement in the advanced turbulence analysis, including temporal fluctuation properties, the two-region blob-regime classification, and the two-dimensional conditionally averaged blob morphology. In particular, GRILLIX most closely matched the experimentally relevant sheath-connected regime and reproduced the elongated, tilted blob structures observed experimentally. The remaining discrepancies in the divertor and radial electric fields pointed to boundary conditions and plasma-wall interaction modeling as the most likely limiting factors.’

Consistency verification:

Control volumes

- Volumes enclosed by adjacent flux surfaces V_ρ
- Integrate conservative equations, e.g. continuity equation

$$\oint_{V_\rho} \frac{\partial n}{\partial t} dV + \oint_{\partial V_\rho} \left(n \mathbf{v}_{E \times B} + n \mathbf{v}_{dia,e} + n v_{||,e} \tilde{\mathbf{b}}_\perp \right) \cdot d\mathbf{S} = \oint_{\partial V_\rho} \nu_\perp \nabla_\perp^2 n \cdot d\mathbf{S} + \oint_{V_\rho} S_n dV$$

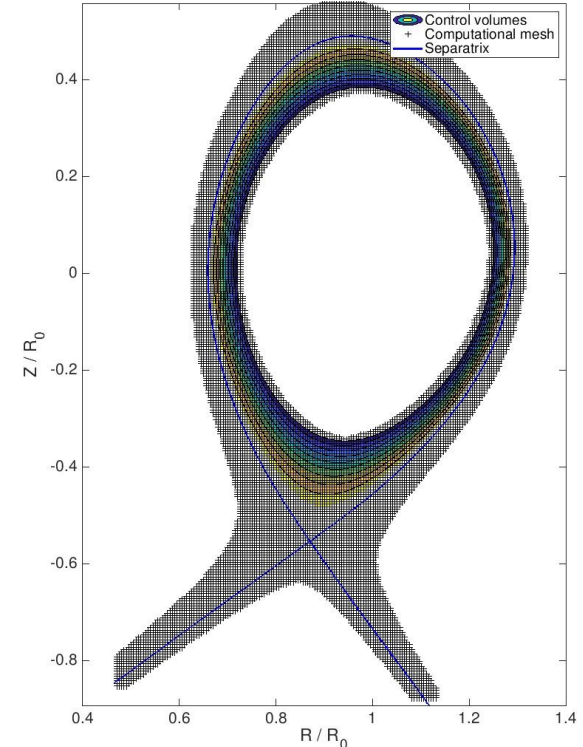
- Numerical considerations
 - Map variables from (R, φ, Z) mesh to polar (ρ, φ, θ) mesh
 - Volume integrals = flux surface integrals

$$\int_{V_i} f dV = V_{\rho_i} \langle f \rangle_{\rho_i} = \int_0^{2\pi} \int_0^{2\pi} \sqrt{g} f |_{\rho_i} d\theta d\varphi$$

- For drift fluxes map potential (avoid velocity components)

$$\oint_{\partial V_{\rho_i}} f \frac{\mathbf{B}}{B^2} \times \nabla \phi \cdot d\mathbf{S} = \int_0^{2\pi} \int_0^{2\pi} \left. \frac{f}{B^2} \frac{\partial \phi}{\partial \theta} \right|_{\rho_i + \frac{\Delta \rho}{2}} d\theta d\varphi - \int_0^{2\pi} \int_0^{2\pi} \left. \frac{f}{B^2} \frac{\partial \phi}{\partial \theta} \right|_{\rho_i - \frac{\Delta \rho}{2}} d\theta d\varphi$$

→ Reduces numerical pollution of diagnostics

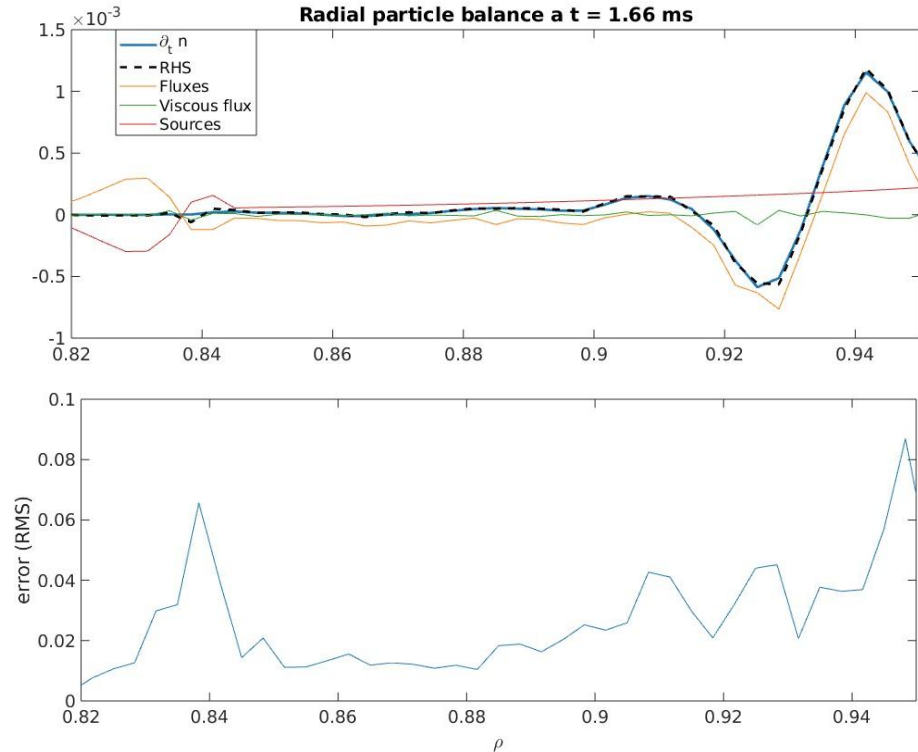


Consistency verification:



Radial particle balance

- $$\oint_{V_\rho} \frac{\partial n}{\partial t} dV = - \oint_{\partial V_\rho} \left(n \mathbf{v}_{E \times B} + n \mathbf{v}_{dia,e} + n v_{||,e} \tilde{\mathbf{b}}_\perp \right) \cdot d\mathbf{S} + \oint_{\partial V_\rho} \nu_\perp \nabla_\perp^5 n \cdot d\mathbf{S} + \oint_{V_\rho} S_n dV$$
- Radial particle balance consistent:
Root mean square error ~5%
- Deviations can be attributed to
 - Finite difference approach
 - Numerics in diagnostics





Consistency verification:

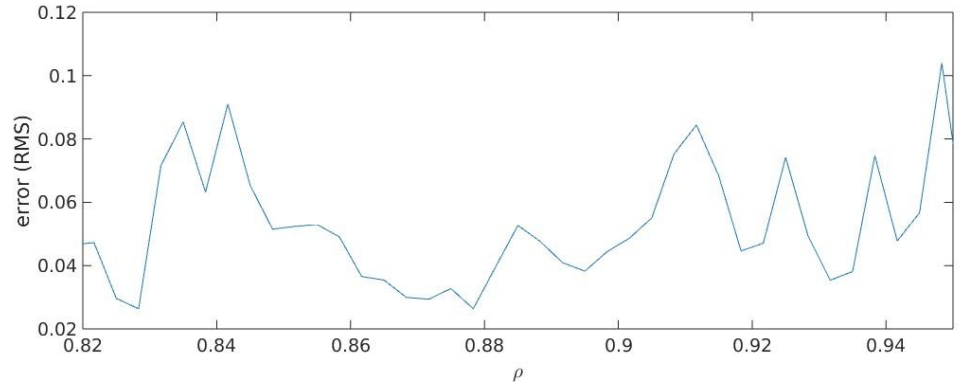
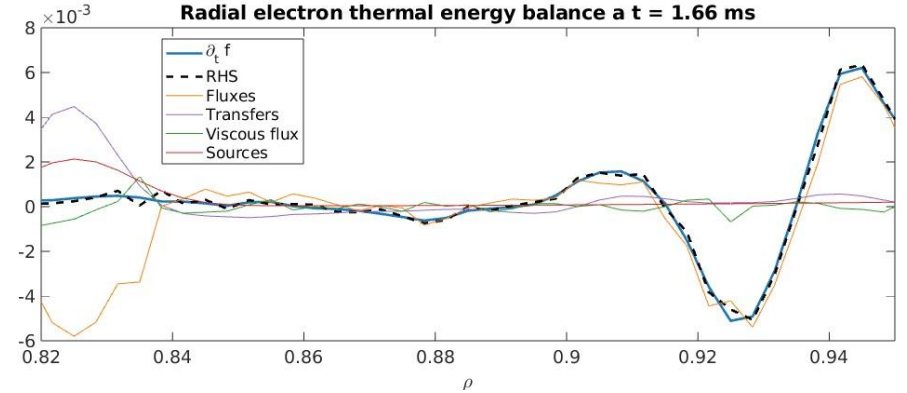
Radial electron thermal energy balance

- $$\oint_{V_\rho} \frac{3}{2} \frac{\partial p_e}{\partial t} dV = - \oint_{\partial V_\rho} \left[\frac{3}{2} p_e \mathbf{v}_{ExB} + \frac{5}{2} p_e \mathbf{v}_{dia,e} + \mathbf{q}_{e,\perp} + \left(\frac{5}{2} p_e v_{\parallel,e} + q_{\parallel} \right) \tilde{\mathbf{b}}_{\perp} \right] dV$$

$$\oint_{V_\rho} \left[-p_e \nabla \cdot \mathbf{v}_{ExB} + v_{\parallel,e} \nabla_{\parallel} p_e + \eta_{\parallel} j_{\parallel}^2 - 0.71 \frac{j_{\parallel}}{e} \nabla_{\parallel} T_e - Q_{\Delta} \right] dV$$

$$\oint_{\partial V_\rho} \frac{3}{2} \nu_{\perp} \nabla_{\perp}^2 p_e \cdot d\mathbf{S} + \oint_{V_\rho} S_{p_e} dV$$

- Radial balance consistent:
Root mean square error ~5%
- Deviations can be attributed to
 - Finite difference approach
 - Numerics in diagnostics
- No leakage due to numerical perpendicular diffusion



Consistency verification:



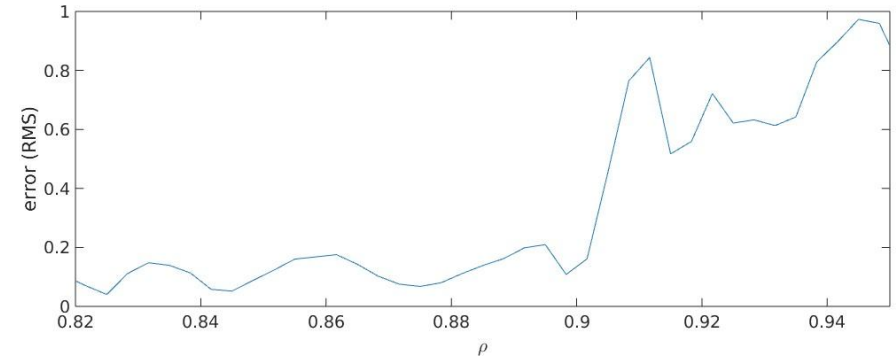
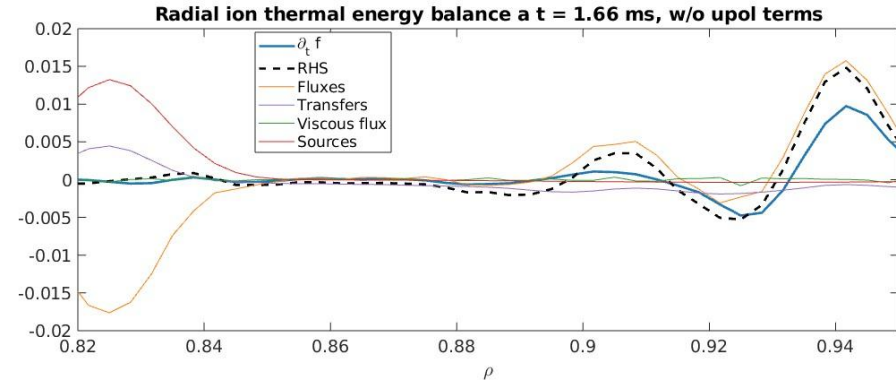
Radial ion thermal energy balance

- $$\oint_{V_\rho} \frac{3}{2} \frac{\partial p_i}{\partial t} dV = - \oint_{\partial V_\rho} \left[\frac{3}{2} p_i \mathbf{v}_{E \times B} + \frac{5}{2} \frac{c}{e B^2} \mathbf{B} \times \nabla (p_i T_i) + \left(\frac{5}{2} p_i v_{\parallel, i} + q_{\parallel, i} \right) \tilde{\mathbf{b}}_\perp + \cancel{\frac{5}{2} p_i \mathbf{u}_{pol}} \right]$$

$$\oint_{V_\rho} \left[-p_i \nabla \cdot \mathbf{v}_E + u_{\parallel} \nabla_{\parallel} p_i + Q_{\Delta} + \frac{G^2}{3\eta_i} + \cancel{\mathbf{u}_{pot} \cdot \nabla p_i} \right]$$

$$\oint_{\partial V_\rho} \frac{3}{2} \nu_{\perp} \nabla_{\perp}^5 p_i \cdot d\mathbf{S} + \oint_{V_\rho} \frac{3}{2} S_{p_i} dV$$

- No good match without upol terms



Consistency verification:



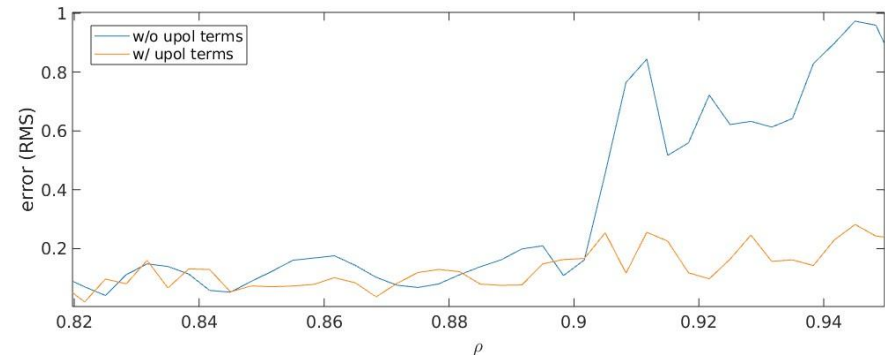
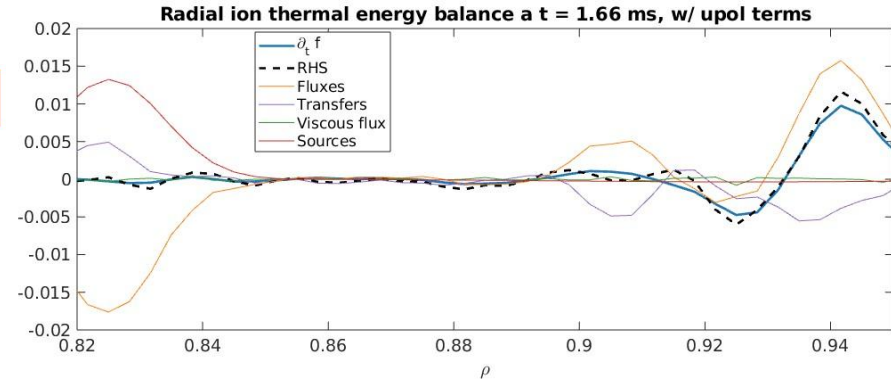
Radial ion thermal energy balance

$$\bullet \oint_{V_\rho} \frac{3}{2} \frac{\partial p_i}{\partial t} dV = - \oint_{\partial V_\rho} \left[\frac{3}{2} p_i \mathbf{v}_{ExB} + \frac{5}{2} \frac{c}{e B^2} \mathbf{B} \times \nabla (p_i T_i) + \left(\frac{5}{2} p_i v_{\parallel, i} + q_{\parallel, i} \right) \tilde{\mathbf{b}}_\perp + \frac{5}{2} p_i \mathbf{u}_{pol} \right]$$

$$\oint_{V_\rho} \left[-p_i \nabla \cdot \mathbf{v}_E + u_\parallel \nabla_\parallel p_i + Q_\Delta + \frac{G^2}{3\eta_i} + \mathbf{u}_{pol} \cdot \nabla p_i \right]$$

$$\oint_{\partial V_\rho} \frac{3}{2} \nu_\perp \nabla_\perp^2 p_i \cdot d\mathbf{S} + \oint_{V_\rho} \frac{3}{2} S_{p_i} dV$$

- Improved match with upol terms
- Deviation of ~15% attributed to
 - Finite difference approach
 - Numerics in diagnostics
 - Evaluation of upol cumbersome
 - Neglect of upol in advective derivative



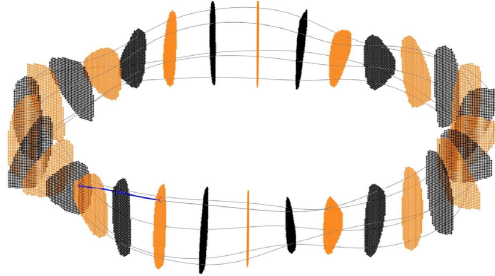
Boundary treatment

Immersed boundary approach

- Boundaries are problematic in FCI as generally not conformal with magnetic field
- Enforce boundary conditions at targets via source

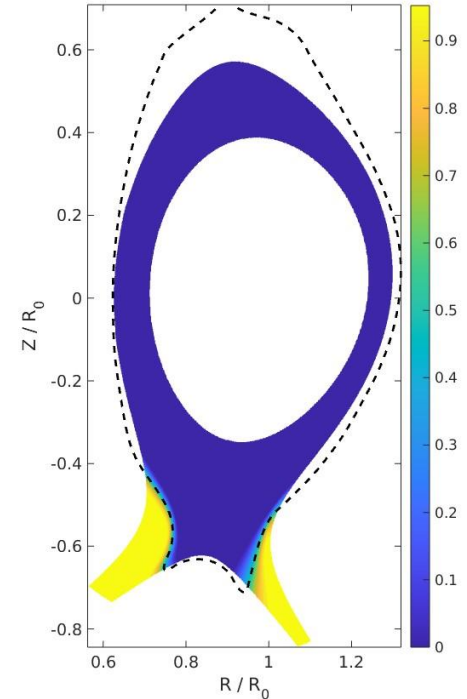
$$\partial_t f = F(t, \mathbf{x}, f) \rightarrow \partial_t f = (1 - \chi)F(t, \mathbf{x}, f) + \frac{\chi}{\epsilon} (f_{bnd} - f)$$

- Introduction of free parameters/function ϵ, χ
- Introduction of unphysical boundary layer
- Special complexity in GRILLIX due to usage of toroidally staggered grid



- Applied robustly in most FCI codes
 - GDB: B. Zhu *et al* 2018 *CPC* **232** 46
 - GRILLIX: A. Stegmeir *et al* 2023 *CPC* **290** 108801
 - FELTOR: M. Wiesenberger and M. Held 2024 *PPCF* **66** 065003

→ Simulated target profiles are still often in reasonable agreement with experiment



Taylor expansion approach

Taylor expansion

- Keep layer of guard points beyond target
- Assume boundary condition given in mixed form

$$\beta u|_X + \gamma \nabla_{\parallel} u|_X = \alpha$$

- Taylor expansion along magnetic field line around intersection point X

$$u_P \approx u_X + s_P \partial_s u + \frac{1}{2} s_P^2 \partial_s^2 u \quad P \in \{A, B, C\}$$

- Linear equation system with 4 unknowns

$$u_A = \frac{1}{2D} \{ (s_A - s_B)(s_A - s_C)(s_C - s_B)\alpha - [\beta s_A s_C - \gamma(s_A + s_C)](s_A - s_C)u_b + [\beta s_A s_B - \gamma(s_A + s_B)]$$

$$u_X = \frac{1}{2D} [s_B s_C (s_C - s_B)\alpha - \gamma(s_C^2 u_B - s_B^2 u_C)]$$

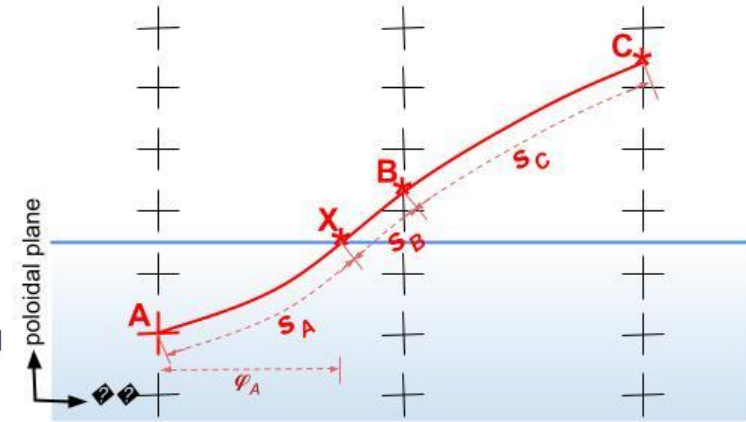
$$\partial_s u = \frac{1}{2D} [(s_B^2 - s_C^2)\alpha + \beta(s_C^2 u_B - s_B^2 u_C)]$$

$$\partial_s^2 u = \frac{1}{D} [(s_C - s_B)\alpha - (\beta s_C - \gamma)u_B + (\beta s_B - \gamma)u_C]$$

- Determinant

$$D = \frac{1}{2} [\beta s_B s_C - \gamma(s_B + s_C)](s_C - s_B)$$

→ Singularities, e.g. for $\gamma=0$: System degenerate at $s_B=0$ or $s_C=0$

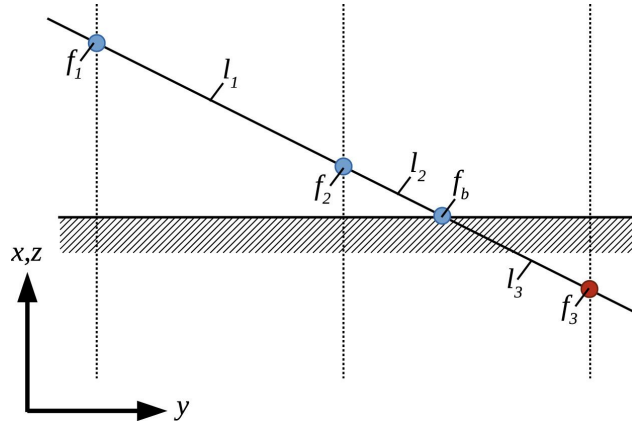


Depth parameter: $\delta = \frac{\varphi_A}{\Delta\varphi}$

Taylor expansion approach

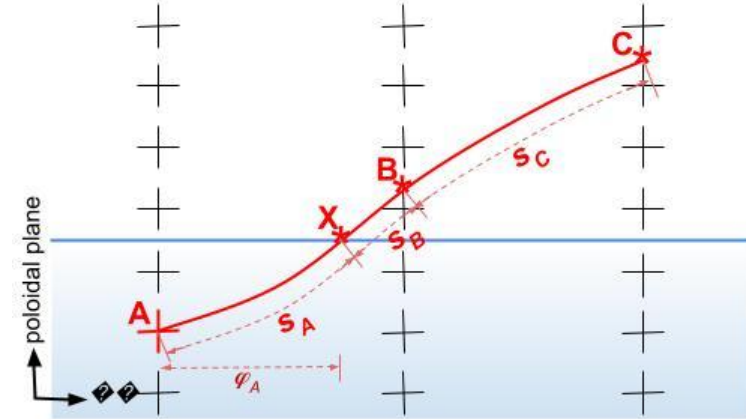
Difference to leg value fill approach

P. Hill *et al* 2017 *CPC* **213** 9



A field line leaving the boundary. f_2 is located on a grid point, while f_1, f_3 are located on intersection points with the adjacent perpendicular planes, and f_b is located on the intersection with the boundary. l_1, l_2, l_3 are distances along the field lines between the four points above.

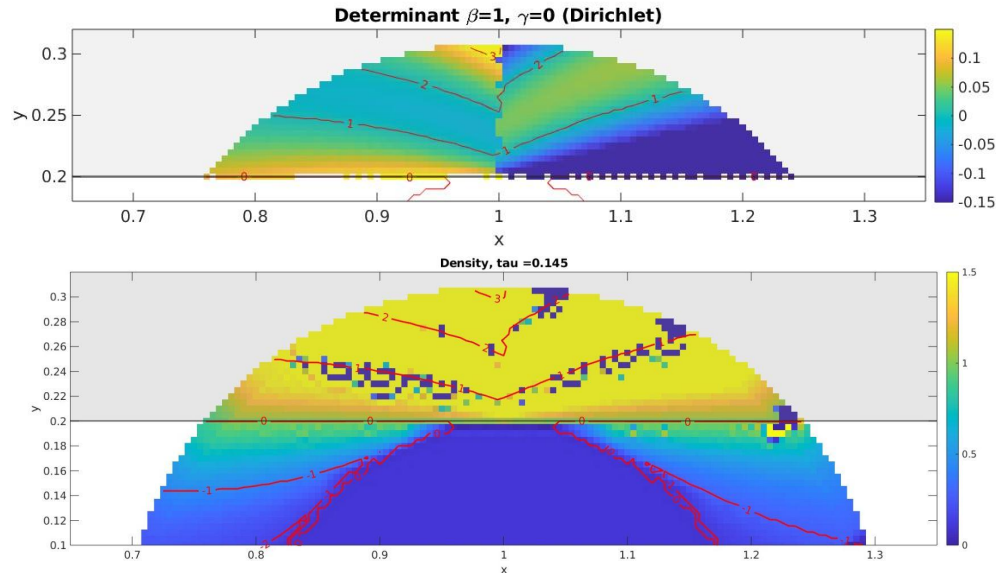
- + No interpolation to obtain guard value f_3
- Adaption of numerical scheme at evaluation of parallel operators (special treatment of target connected points at location of f_2)



Taylor expansion approach

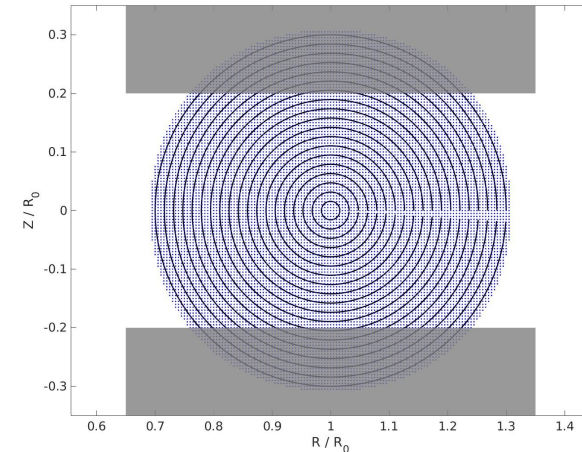
Singularities

- Determinant: $D = \frac{1}{2} [\beta s_B s_C - \gamma (s_B + s_C)] (s_C - s_B)$
- Solution explodes around integer depth layers $\delta = \frac{\varphi A}{\Delta \varphi}$



Example

- Circular toroidal flux surfaces
- Continuous limiters at $y = \pm 0.2$
- Safety factor: $q_0 = 1, q_{\text{LCFS}} = 1.49$
- Parallel heat equation with Dirichlet boundaries



Taylor expansion approach

Remedy

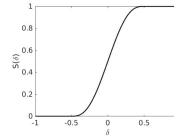
- Around location $\delta=1$ skip one plane (and reduce order to 1)

$$\rightarrow u_A^{skip} = \frac{1}{D^{skip}} [-\alpha (s_A - s_C) + \beta (s_A - \gamma) u_C],$$

$$D^{skip} = \beta s_C - \gamma$$

- Smooth transition from original to skipped expression

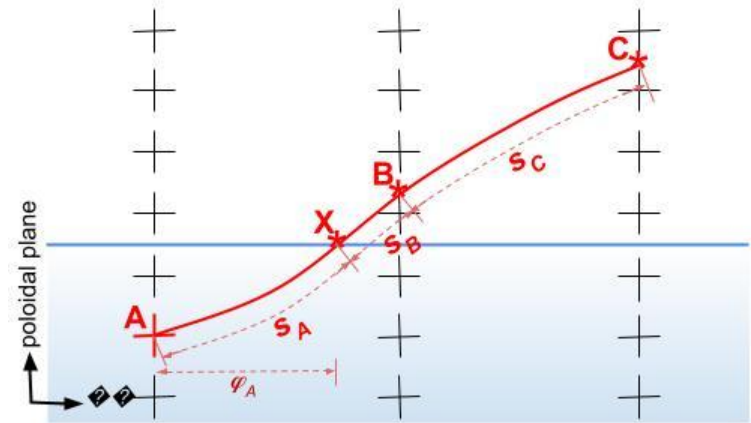
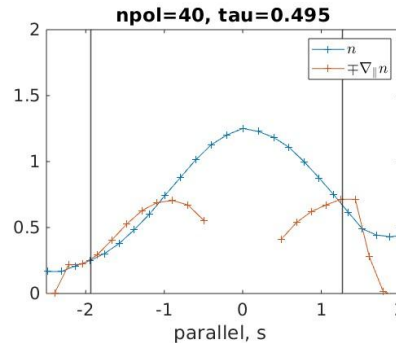
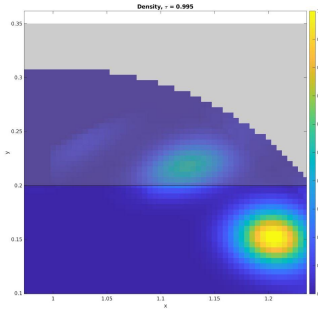
$$u_A \rightarrow [1 - S(\delta - 1)] u_A + S(\delta - 1) u_A^{skip}$$



Test: Blob diffusion with mixed boundary conditions

$$\partial_t n = \nabla \cdot (\mathbf{b} \nabla_{\parallel} n),$$

$$n(t=0, \mathbf{x}) = \delta(\varphi) \exp\left(-\frac{(x-1.25)^2 + (y-0.05)^2}{0.03^2}\right) \quad n|_{target} = \mp \nabla_{\parallel} n|_{target}$$



- Smooth solutions: No corrugations/explosions
- Boundary condition is accurately fulfilled



Taylor expansion approach

Integrated test

Simplified 3D transport model

$$\frac{\partial}{\partial t} n = -\nabla \cdot n (v_{\parallel} \mathbf{b} + \mathbf{v}_E + \mathbf{v}_e^{\text{mag}}) + D_{\perp} \nabla_{\perp}^2 n + D_{\parallel} \nabla_{\parallel}^2 n$$

$$\frac{\partial}{\partial t} (n v_{\parallel}) = -\nabla \cdot [n v_{\parallel} (\mathbf{v}_E + v_{\parallel} \mathbf{b} + \mathbf{v}_e^{\text{mag}})] - \nabla_{\parallel} (p_e + p_i) + D_{\perp} \nabla_{\perp}^2 (n v_{\parallel}) + D_{\parallel} \nabla_{\parallel}^2 (n v_{\parallel})$$

$$\frac{3}{2} \frac{\partial p_e}{\partial t} = -\nabla \cdot \left(\frac{3}{2} p_e \mathbf{v}_E + \frac{5}{2} p_e \mathbf{v}_e^{\text{mag}} \right) - p_e \nabla \cdot \mathbf{v}_E - \nabla \cdot \left(\frac{3}{2} v_{\parallel} p_e \mathbf{b} \right) - p_e \nabla \cdot v_{\parallel} \mathbf{b} + \nabla \cdot (\kappa_e^{\parallel} \mathbf{b} \nabla_{\parallel} T_e) + \frac{0.71}{e} T_e \nabla \cdot j_{\parallel} \mathbf{b} + \frac{\eta_{\parallel}^0}{T_e^{3/2}} j_{\parallel} + D_{\perp} \nabla_{\perp}^2 p_e + D_{\parallel} \nabla_{\parallel}^2 p_e$$

$$\frac{3}{2} \frac{\partial p_i}{\partial t} = -\nabla \cdot \left(\frac{3}{2} p_i \mathbf{v}_E + \frac{5}{2} p_i \mathbf{v}_i^{\text{mag}} \right) - p_i \nabla \cdot \mathbf{v}_E - \nabla \cdot \left(\frac{3}{2} v_{\parallel} p_i \mathbf{b} \right) - p_i \nabla \cdot v_{\parallel} \mathbf{b} + \nabla \cdot (\kappa_i^{\parallel} \mathbf{b} \nabla_{\parallel} T_i) + \frac{5}{2e} T_i \nabla \cdot j_{\parallel} \mathbf{b} + \frac{5}{2e} T_i \nabla \cdot [(p_e + p_i) \nabla \times \frac{\mathbf{b}}{B}] + D_{\perp} \nabla_{\perp}^2 p_i + D_{\parallel} \nabla_{\parallel}^2 p_i$$

$$\frac{\partial \omega}{\partial t} = c \nabla \cdot \left[(p_e + p_i) \nabla \times \frac{\mathbf{b}}{B} \right] + \nabla \cdot j_{\parallel} \mathbf{b} + \mu_{\perp} \nabla_{\perp}^2 \omega + \mu_{\parallel} \mathbf{b} \nabla_{\parallel}^2 \omega$$

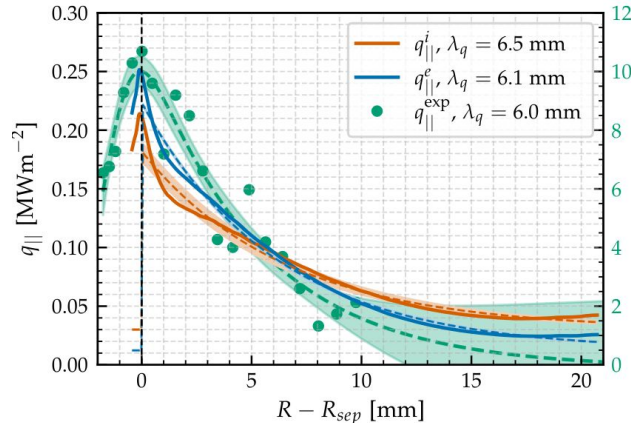
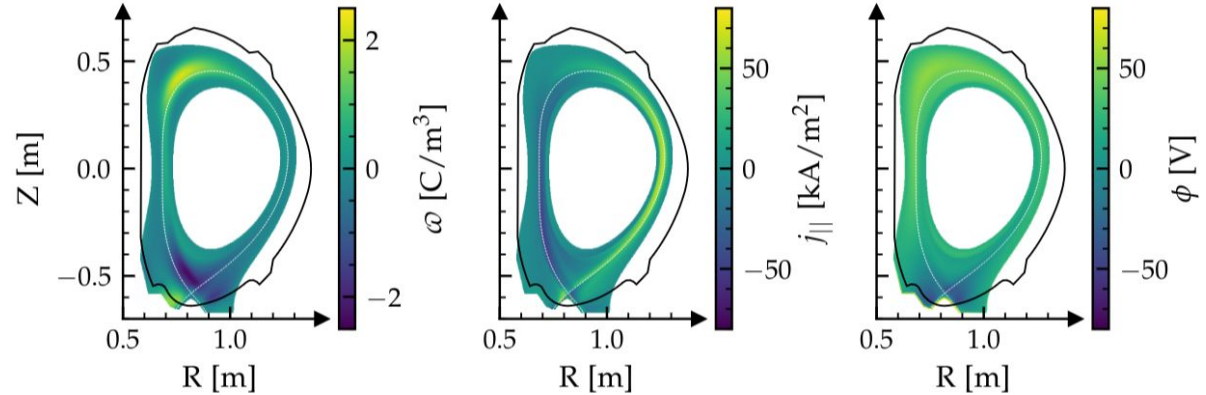
$$\left[\frac{1}{\lambda_{\phi}} \frac{\partial \phi}{\partial t} \right] = \nabla \cdot \left[\frac{m_i n c^2}{B^2} \left(\nabla_{\perp} \phi + \frac{\nabla_{\perp} p_i}{en} \right) \right] - \omega$$

$$\left[\frac{1}{\lambda_j} + \frac{m_e}{e} \right] \frac{\partial j_{\parallel}}{\partial t} = -\frac{\eta_{\parallel}^0}{T_e^{3/2}} j_{\parallel} - \nabla_{\parallel} \phi + \frac{1}{en} \nabla_{\parallel} p_e + \frac{0.71}{e} \nabla_{\parallel} T_e$$

→ Relaxation parameters λ introduced to suppress 3d drift wave turbulence

T. Tork *et al* 2026 *CPP* **290** 0:e70157

E. Gomes 2026 Master Thesis, TUM



So far the new boundary scheme seems to work reliably

Conclusions

TCV-X23

- Simulations are running and results so far in good overall agreement
- No reduction of heat flux observed yet
 - Increase fuelling rate/recycling coefficient for limited time frame
 - Landau-fluid heat flux
 - Run longer until full saturation
- RDPA and GPI data?

Consistency checks

- Particle and energy conservation diagnostics per flux surfaces established
- Good particle and electron thermal energy balance
- Quantities involving polarisation velocity tricky

Boundary approach

- Taylor based method aiming for more accuracy
- Numerical pitfalls resolved
- Promising results in simplified systems obtained
- Quantitative assessment and comparison to immersed approach to be done
- Unclear if it will work in turbulent environment and stellarators