

Updates on k-epsilon turbulence models with applications on SolEdge HDG

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01/10/2026 TSVVB workshop

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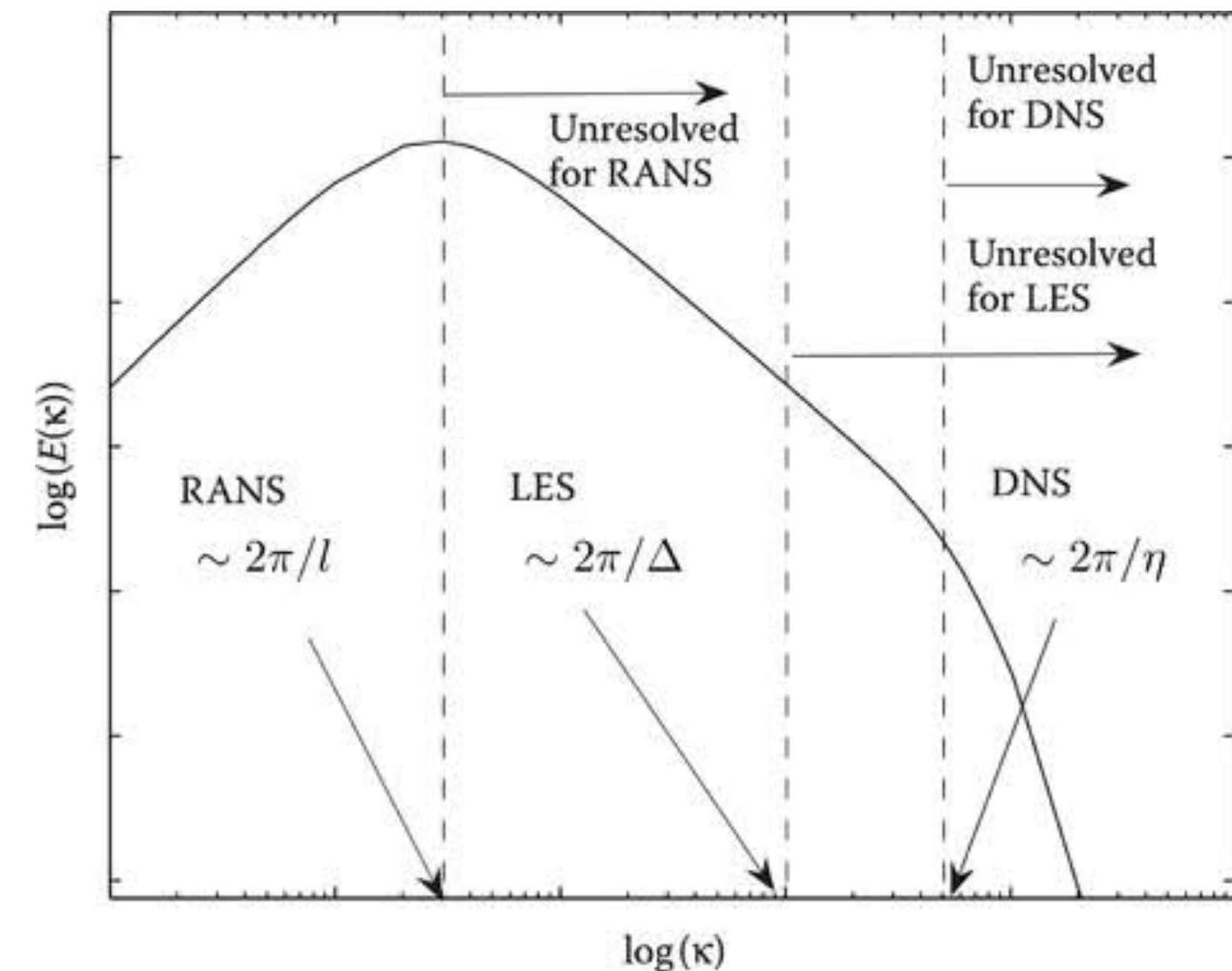
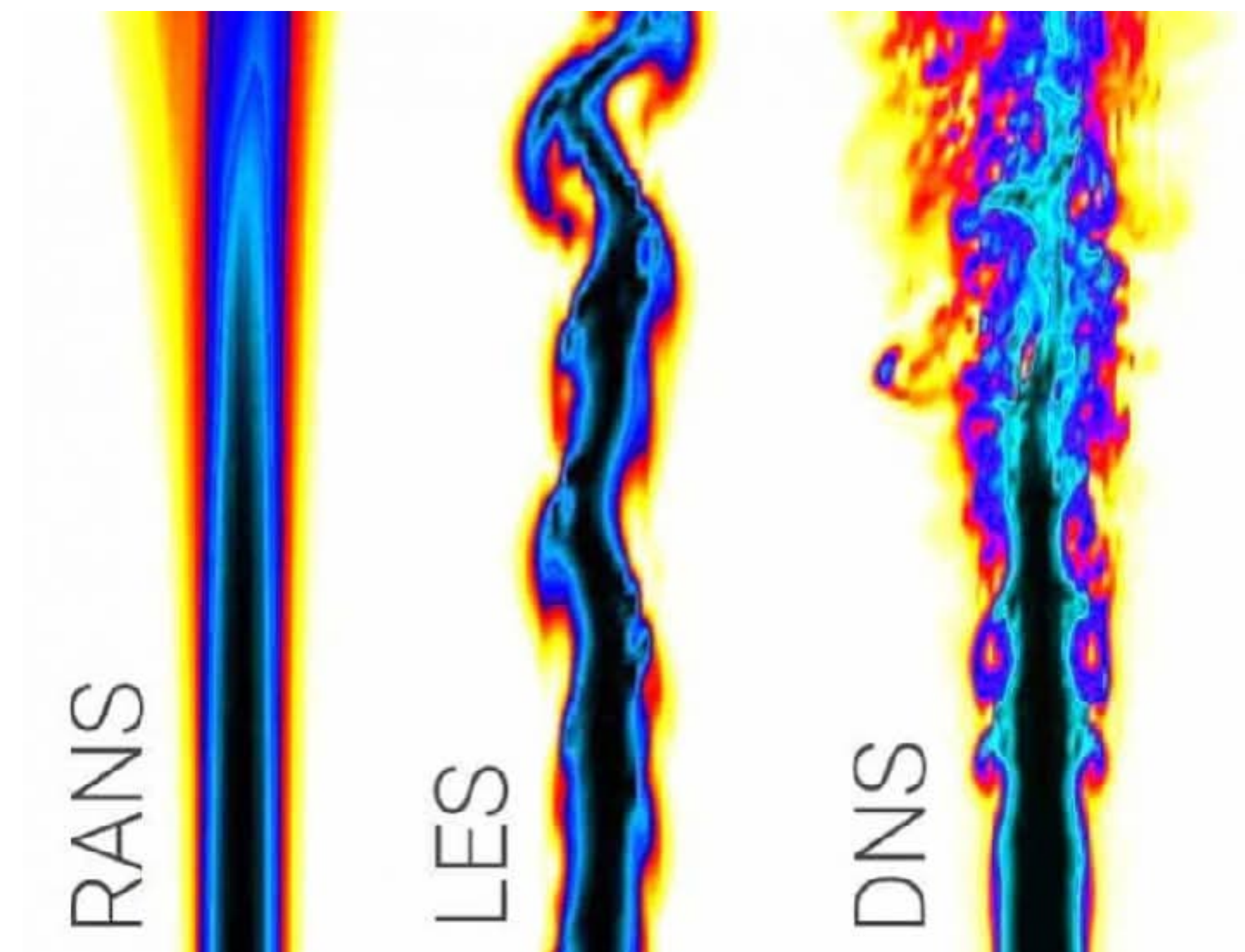
- SolEdge HDG overview
- Latest progress (I. Kudashev) : core transport and neutral flux limiter
- TCV-X23 simulations and k- ϵ turbulence models

SolEdge-HDG code

Reduced models

RANS for plasmas (mean field)

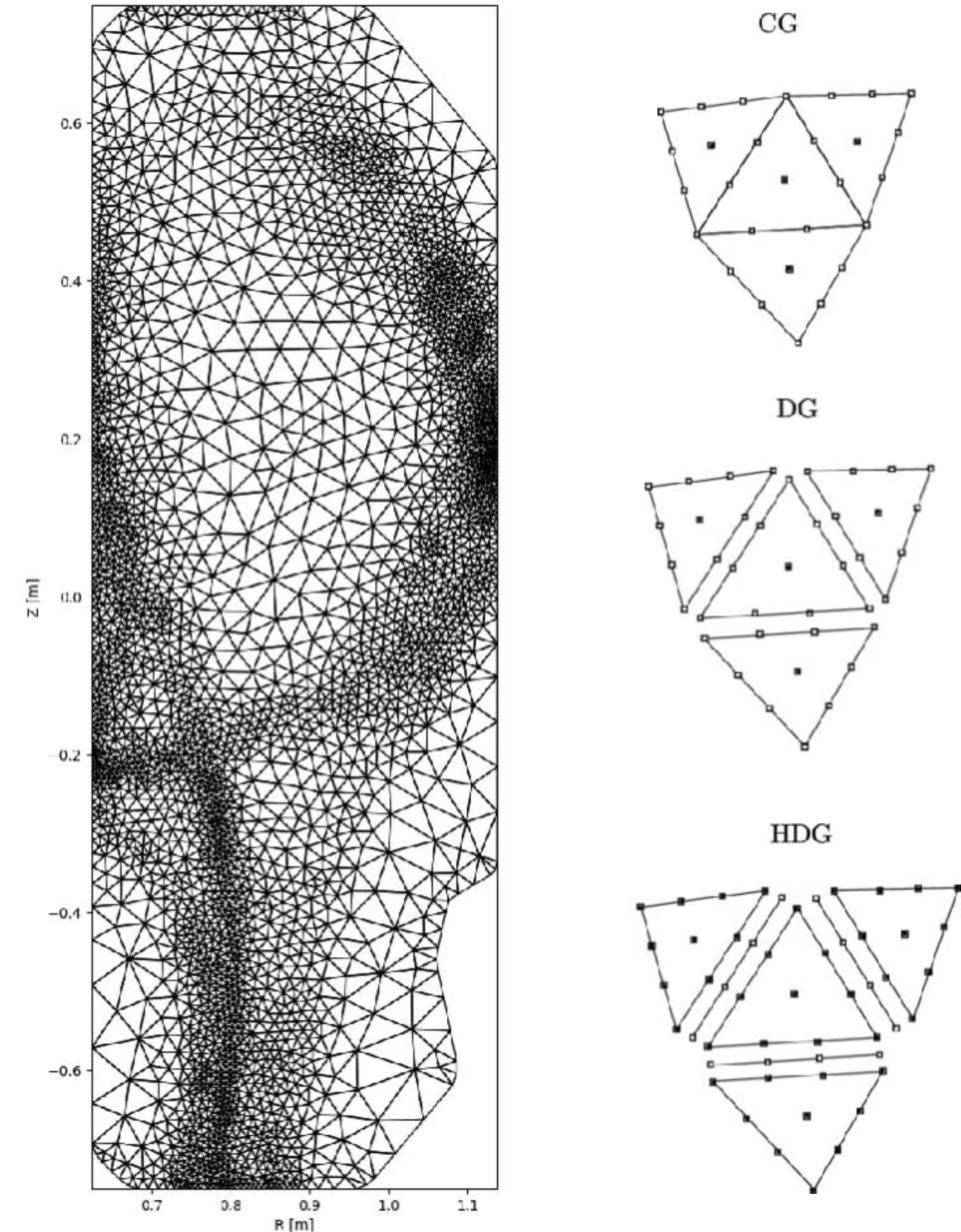
- RANS : only large scales are resolved
- $\overline{n'u'} = -D \nabla \bar{n}$
- Reduces the computations
- But we need to model the turbulence
- Mean field transport codes for tokamaks : SOLPS-ITER, SolEdge-HDG, SolEdge-3x, ...



SolEdge-HDG

Mean field / transport / RANS model

- Solves the averaged fluid Braginskii equations
- 2D simulation of the poloidal cross-section
- High order finite element method with non-aligned mesh : precise wall description
- Hybridizable Discontinuous Galerkin:
 - Reduced number of global DoF
 - Parallelizable
 - Flux stabilization
- Mesh adaptivity
- No drift velocity yet ...



Mesh refinement of the TCV geometry

SolEdge-HDG

Fluid Braginskii equations

Diffusive turbulence transport

$D = \mu = \chi = \text{const}$

$$\text{Continuity: } \frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{u}_{\parallel}) - \nabla \cdot (D \nabla_{\perp} n) = S_n$$

$$\text{Neutral density: } \frac{\partial n_n}{\partial t} - \nabla \cdot (D_n \nabla_{\perp} n_n) = S_{n_n}$$

Total momentum

$$\Gamma = (m_i + m_e) n u_{\parallel}$$

$$\frac{\partial \Gamma}{\partial t} + \nabla \cdot (\Gamma \mathbf{u}_{\parallel}) - \nabla \cdot (\mu \nabla_{\perp} \Gamma) + \nabla_{\parallel} (p_i + p_e) = S_{\Gamma}$$

Energies

$$\mathcal{E}_s = \frac{3}{2} k_b T_s + \frac{1}{2} m_s u^2$$

$$s = \begin{cases} \text{ion} \\ \text{electron} \end{cases}$$

$$\frac{\partial \mathcal{E}_i}{\partial t} - \nabla \cdot \mathbf{q}_i + n q u_{\parallel} E_{\parallel} + \frac{3}{2} \frac{p_e - p_i}{\tau_{ie}} = S_{\mathcal{E}_i}$$

$$\frac{\partial \mathcal{E}_e}{\partial t} - \nabla \cdot \mathbf{q}_e - n q u_{\parallel} E_{\parallel} - \frac{3}{2} \frac{p_e - p_i}{\tau_{ie}} = S_{\mathcal{E}_e}$$

$$\mathbf{q}_s = \frac{3}{2} k_b (T_s D \nabla_{\perp} n + n \chi \nabla_{\perp} T_s) + \frac{1}{2} m_s (u^2 D \nabla_{\perp} n + \mu n \nabla_{\perp} u^2) + (p_s - \mathcal{E}_s) \mathbf{u}_{\parallel} + k_{\parallel s} T_s^{5/2} \nabla_{\parallel} T_s$$

SolEdge-HDG

Boundary conditions

Continuity: Neumann

Neutrals: $D_{n_n} \nabla_{\perp} n_n = R \overset{\text{Recycling}}{(-D \nabla_{\perp} n + n \mathbf{u}_{\parallel})} + \Gamma_{\text{puff}}$

Momentum: Bohm condition

$$u \geq c_s \text{ if } \mathbf{b} \cdot \mathbf{n} > 0$$

$$u \leq c_s \text{ if } \mathbf{b} \cdot \mathbf{n} < 0$$

$$c_s = \sqrt{\frac{T_e + T_i}{m_i}}$$

Energies

$$\mathcal{E}_s = \frac{3}{2} k_b T_s + \frac{1}{2} m_s u^2$$

Heat flux

$$\mathbf{q}_i \cdot \mathbf{n} = \underbrace{\left(\gamma_i u p_i + \frac{1}{2} n u^3 \right)}_{\text{Sheath flux}} \mathbf{b} \cdot \mathbf{n}$$

$$\mathbf{q}_e \cdot \mathbf{n} = \gamma_e u p_e \mathbf{b} \cdot \mathbf{n}$$

$$\mathbf{q}_s = \frac{3}{2} k_b (T_s D \nabla_{\perp} n + n \chi \nabla_{\perp} T_s) + \frac{1}{2} m_s (u^2 D \nabla_{\perp} n + \mu n \nabla_{\perp} u^2) + (p_s - \mathcal{E}_s) \mathbf{u}_{\parallel} + k_{\parallel s} T_s^{5/2} \nabla_{\parallel} T_s$$

SolEdge-HDG slides on Bohm-GyroBohm and flux limiter

On behalf of SolEdge-HDG team

summary slides for Y. Kunimoto



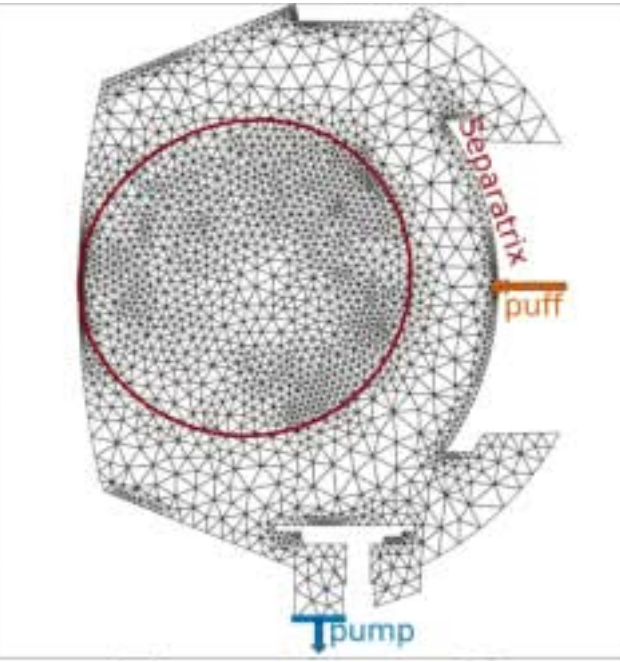
This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

Physical drivers for code development

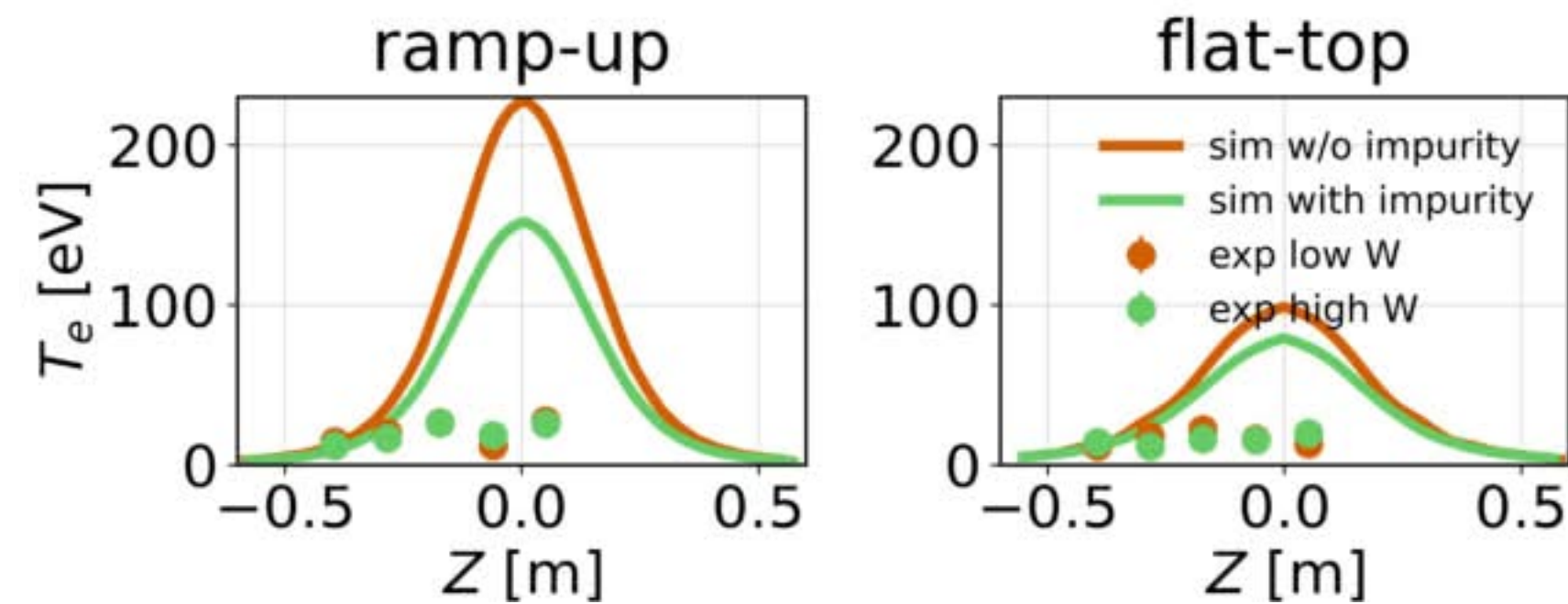
- Previous simulations of WEST limited discharges have shown:

Same transport coefficients in SOL and core can result in **compromise between midplane and limiter profiles** $\Rightarrow$ **core transport model required**

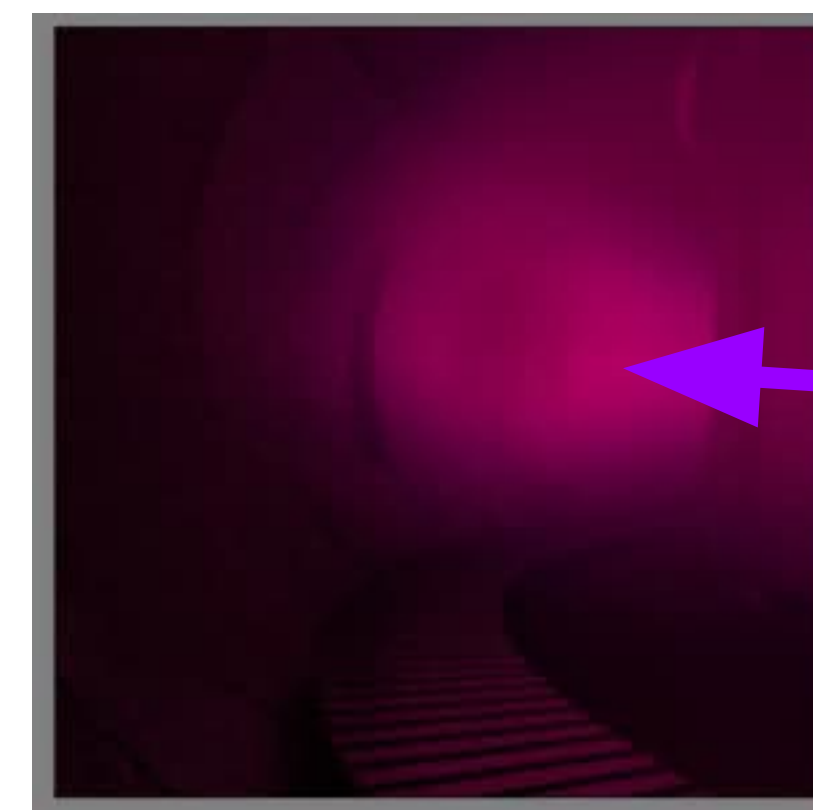
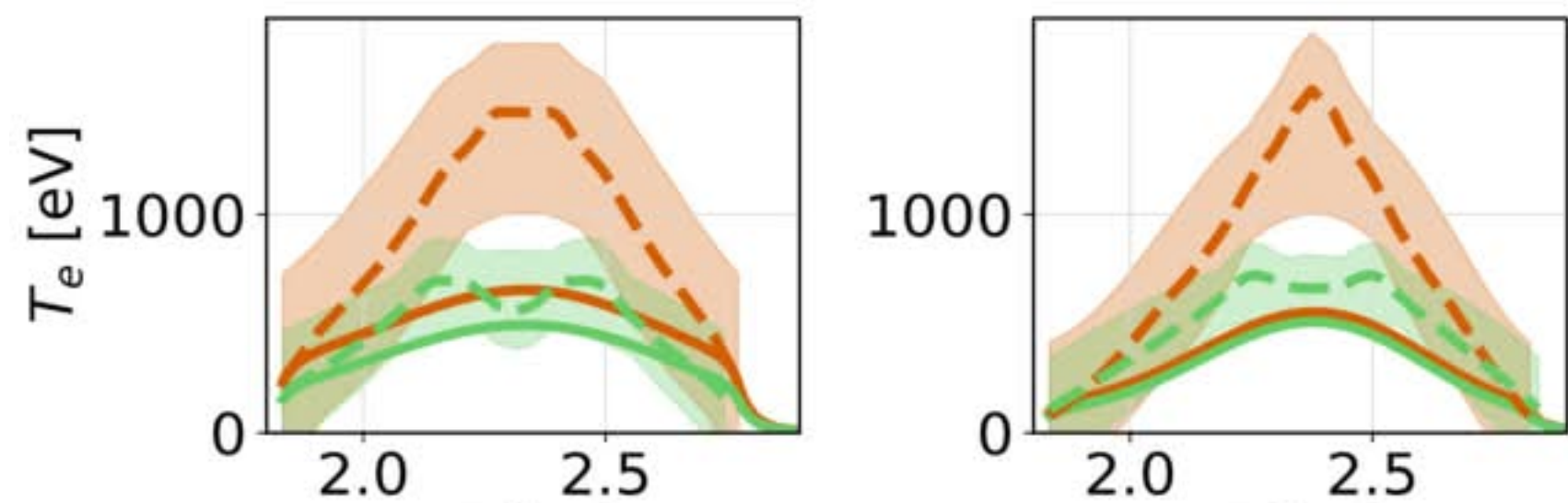
Simple diffusive fluid transport model **allows too deep neutrals propagation** $\Rightarrow$ **neutral model modifications required**



HFS
limiter



midplane



at the edge
 $T \sim 15$ eV

inside
core
 $T \sim 300$ eV

simulation



experiment

Visible camera images

| SolEdge-HDG team

core transport model and flux limiter



model predicts unphysically large fluxes

$$\partial_t n_n - \nabla \cdot (D_{nn} \nabla n_n) = S_{nn} \quad D_{nn} = \frac{eT_i[\text{eV}]}{m_i (n (\langle \sigma v \rangle_{cx} + \langle \sigma v \rangle_{iz}) + n_n \langle \sigma v \rangle_{nn})}$$

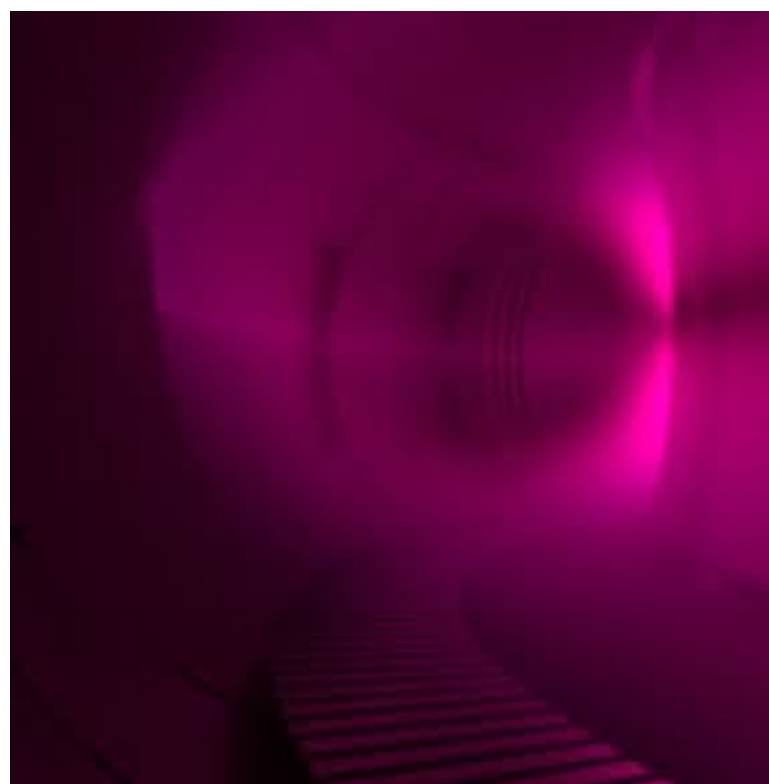
flux limiter [Van Uytven W., NF22] is applied
based on free streaming flux ($\alpha \approx 0.15$)

$$D_{nn} \Rightarrow D_{\text{eff}} = D_{nn} \left[1 + \frac{D_{nn} |\nabla n_n|}{\alpha n_n c_s} \right]^{-1}$$

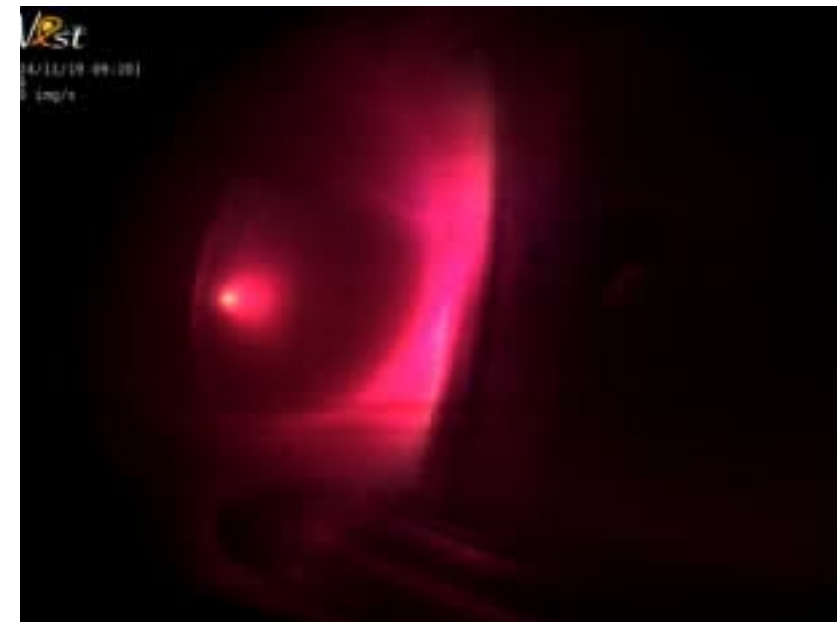
without limiter



with limiter



experiment



flux limiter significantly improves ionization
source distribution

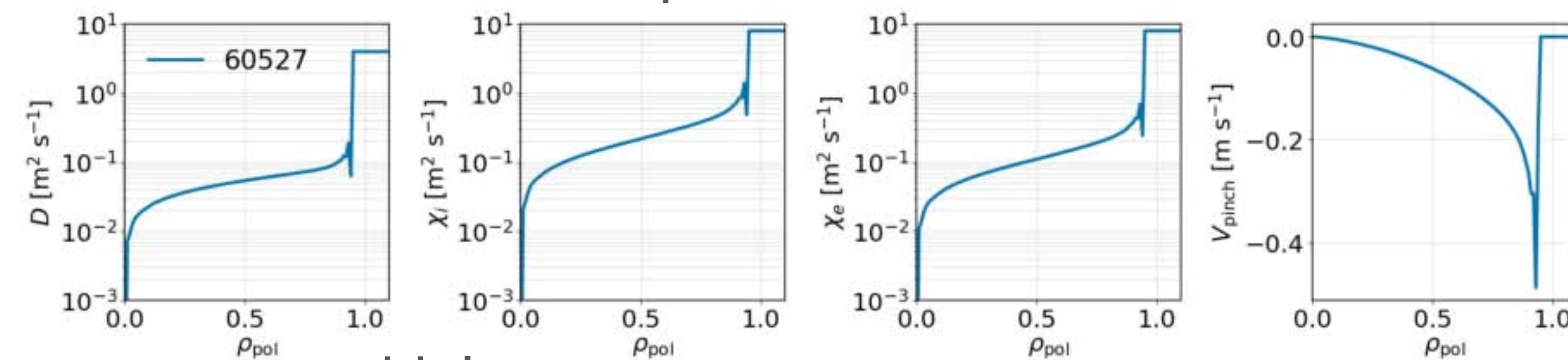
flux-surface average machinery is added:
Bohm-gyroBohm[Erba, NF98] **pinch** [Militello Aslp, NF22]

$$\chi_i = C_{Bi} \chi_B + C_{gBi} \chi_{gB} \quad D = C_n \left(1 - 0.7 \frac{r}{a} \right) \frac{\chi_i \chi_e}{\chi_i + \chi_e} \quad v = C_v D \frac{r}{a^2}$$

$$\chi_e = C_{Be} \chi_B + C_{gBe} \chi_{gB} \quad \mu = \chi_i \quad \chi_B = \chi_0 q^2 L_{pe,*}^{-1} \Delta_e \quad \chi_0 = \frac{k_B T_e}{e B_T}$$

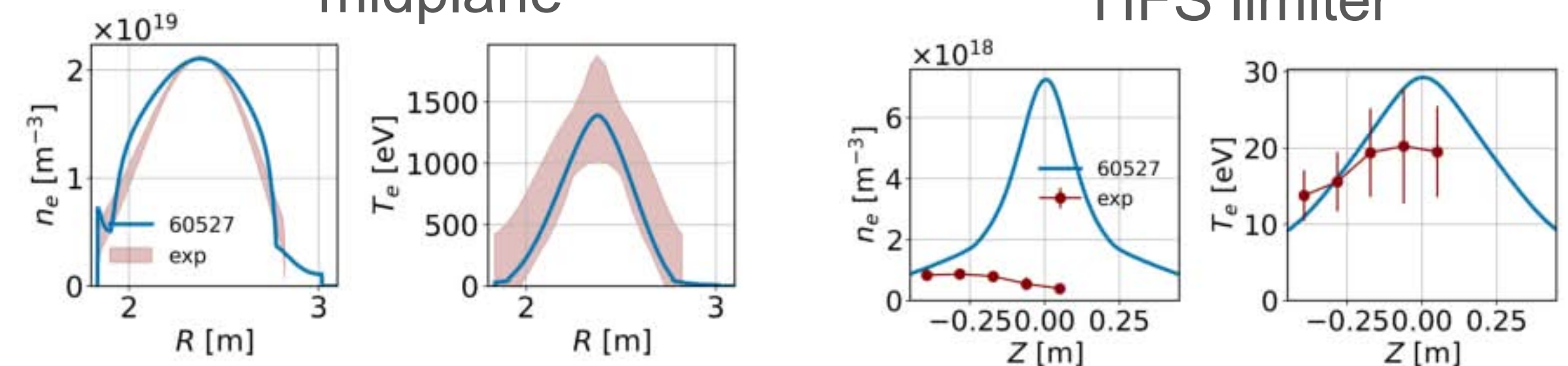
$$\chi_{gB} = \chi_0 \rho_* L_{Te,*}^{-1} \Delta_e = \left| \frac{T_e(\rho = 0.8) - T_e(\rho = 1)}{T_e(\rho = 1)} \right|$$

transport coefficients



midplane

HFS limiter



BgB+pinch allows better comparison both
in core and at the limiter

Open question on fluid neutrals



Flux streaming crucially depends on T_n guess:

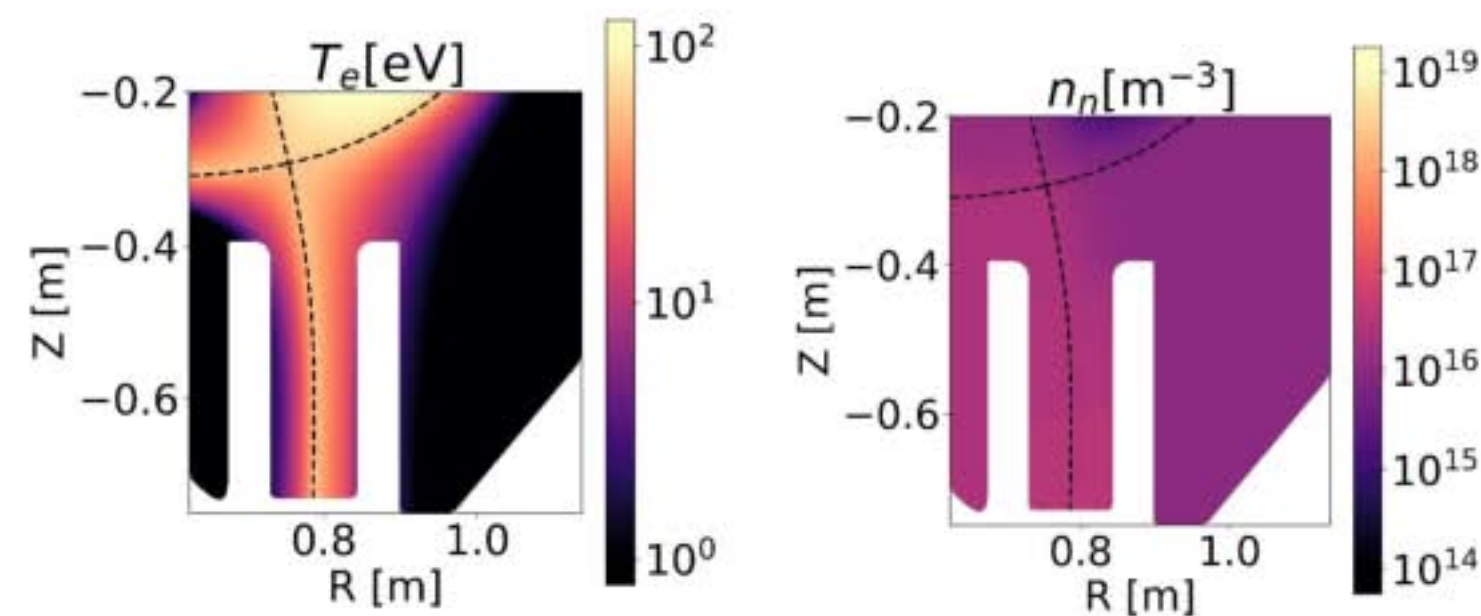
$$D_{nn} \Rightarrow D_{\text{eff}} = D_{nn} \left[1 + \frac{D_{nn} |\nabla n_n|}{\alpha n_n c_s} \right]^{-1}$$

$$c_s = c_s(T_n)$$

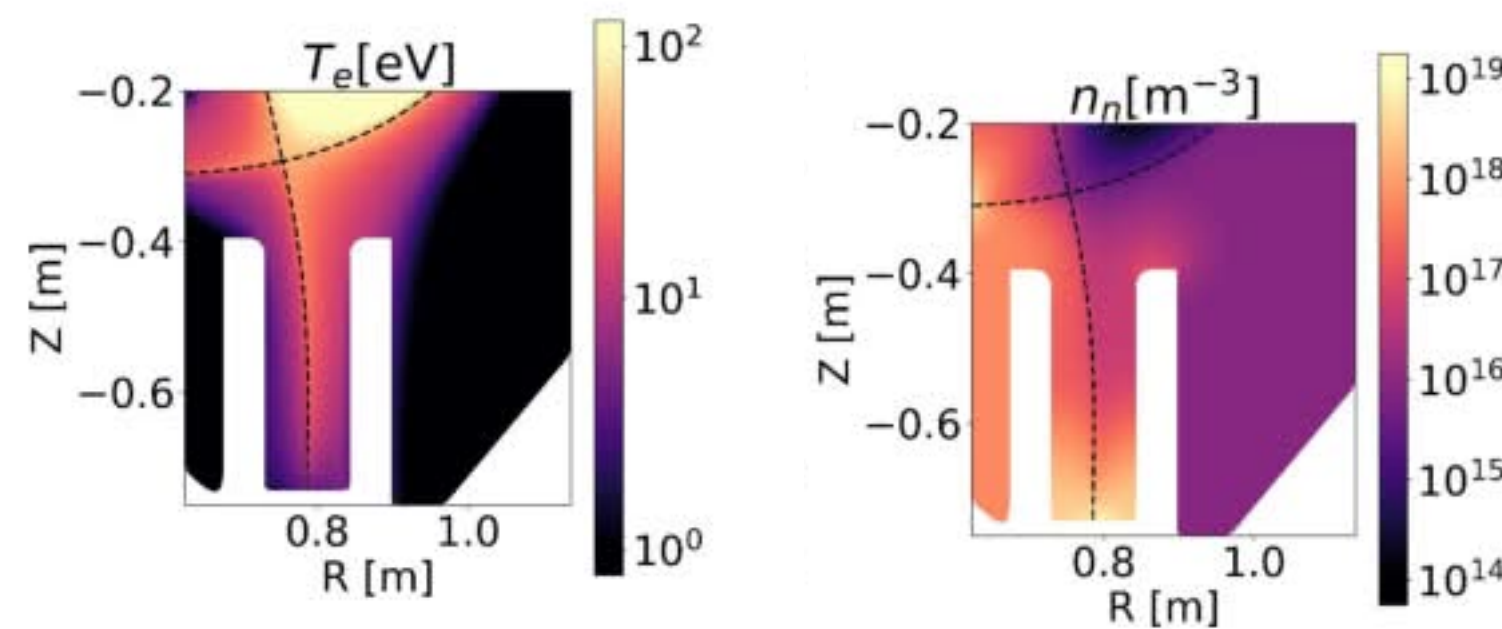
usually $T_n = T_i$ assumed, but

- low n_e : $T_i \gg T_n$, so Franck-Condon (2-3 eV) is a better guess
- high n_e : $T_i \approx T_n$, but usually already detached <10 eV, $T_n = \text{const}$ around 3 eV may be ok

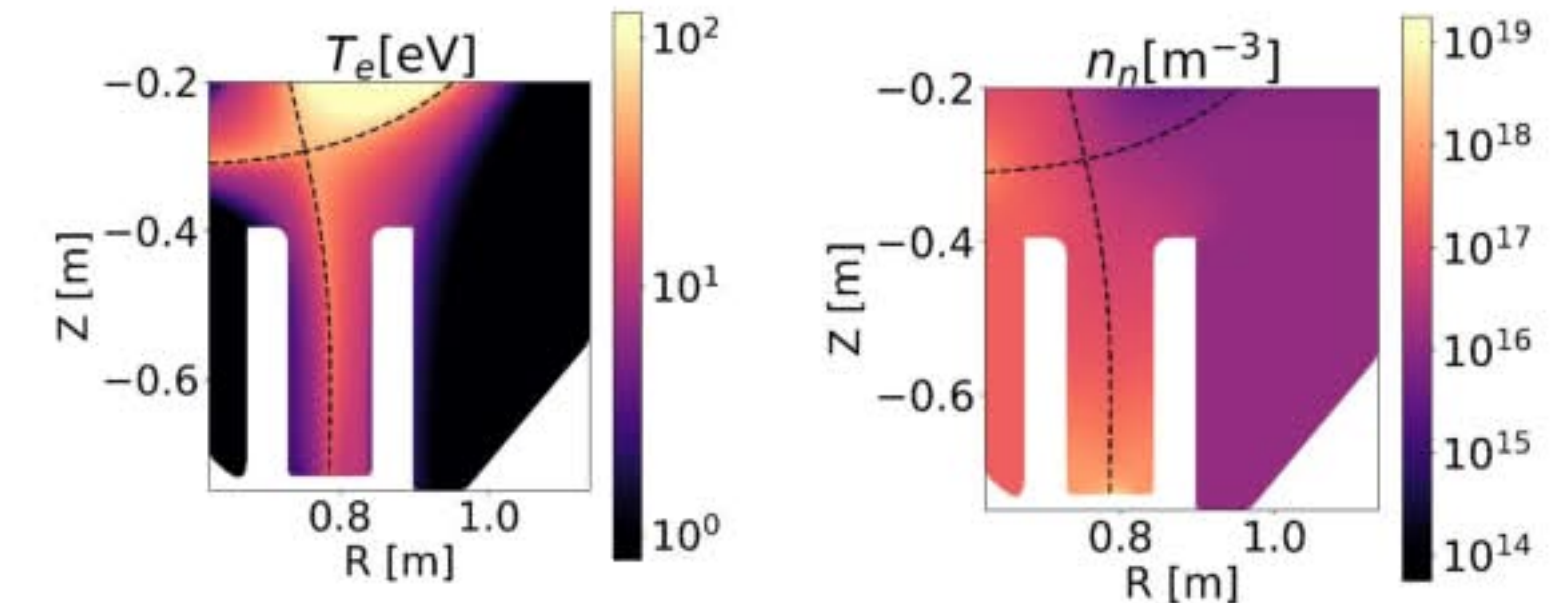
no limiter



$c_s = c_s(3\text{eV})$



$c_s = c_s(T_i)$



Without flux limiter plasma is completely attached (contrary to experiment)
The choice between T_i and fixed T_n has to be calibrated with experiment

k - ε turbulence model

K-ε turbulence model

S. Baschetti et al. 2021

Production term

$$\begin{aligned} \frac{\partial k}{\partial t} + \nabla \cdot (k \mathbf{u}_{\parallel}) - \nabla \cdot (D_k \nabla_{\perp} k) &= \gamma k - \varepsilon - \frac{k^2 \gamma}{k_{\max}} \\ \frac{\partial \varepsilon}{\partial t} + \nabla \cdot (\varepsilon \mathbf{u}_{\parallel}) - \nabla \cdot (D_{\varepsilon} \nabla_{\perp} \varepsilon) &= \gamma \varepsilon - V \varepsilon^2 k^{-3/2} \end{aligned}$$

Transport terms
Local dynamics

$$\text{Eddy viscosity } D_{k\varepsilon} = C \frac{k^2}{\varepsilon}$$

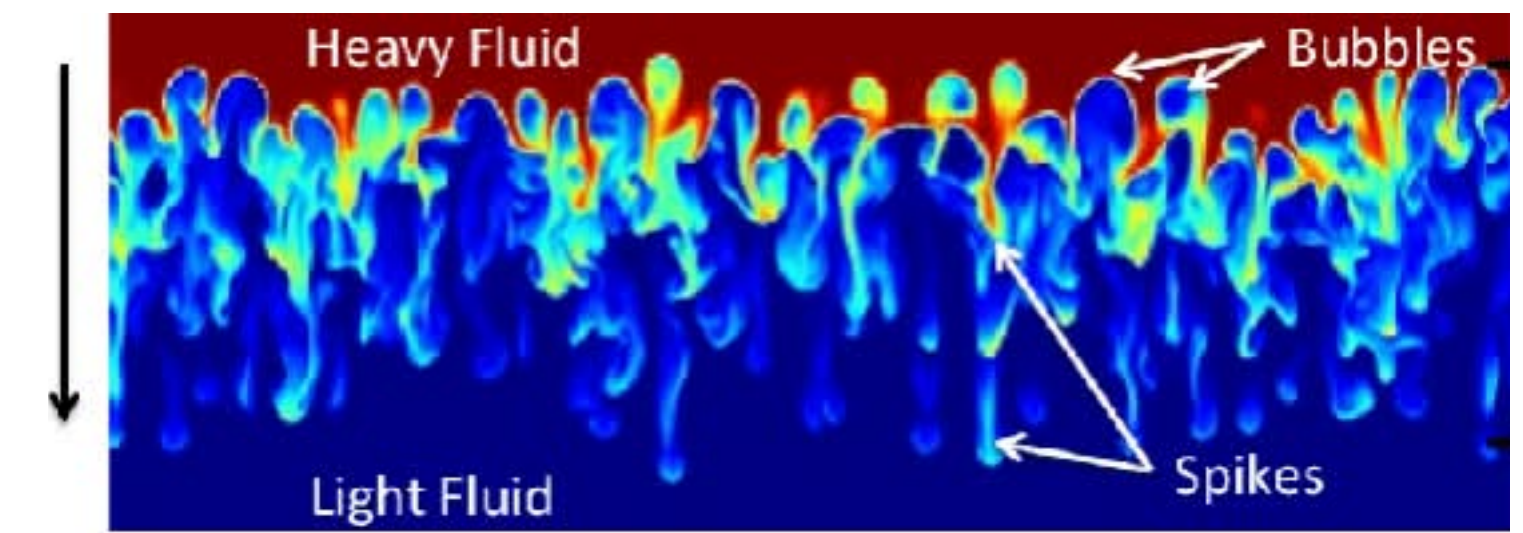
$$\varepsilon \propto l k^{3/2}$$

Simple plasma-turbulence coupling:
 $D = \chi_i = \chi_e = \mu = D_{k\varepsilon}$ with $C = 1$

Control parameter $V = c_s \frac{\rho_i}{R} \alpha_s \sqrt{\tau_{\parallel} \gamma}$
 closed using the scaling law of the
 SOL width : $\lambda_{\text{SOL}} = \alpha_s q \rho_i$

Growth rate : production term

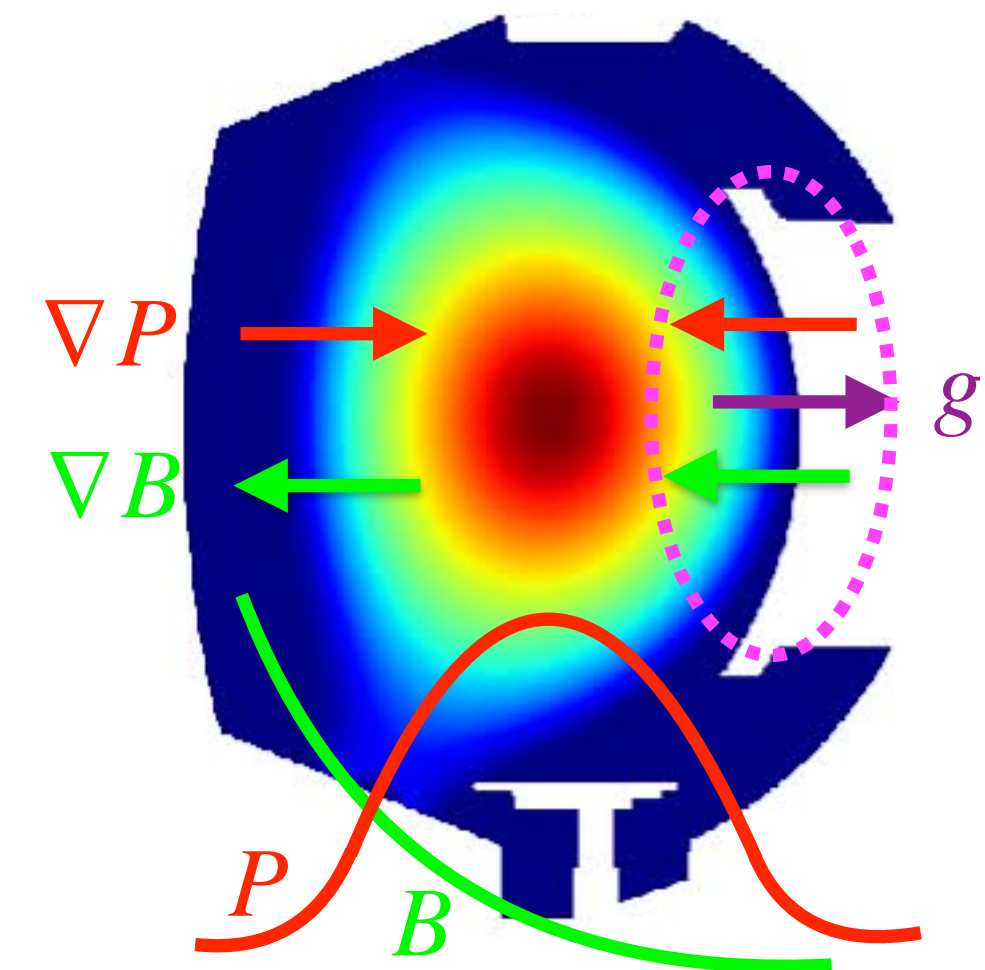
Interchange instability



Analogy with the Rayleigh-Taylor instability
Schwarzkopf, J.D., et al., 2016

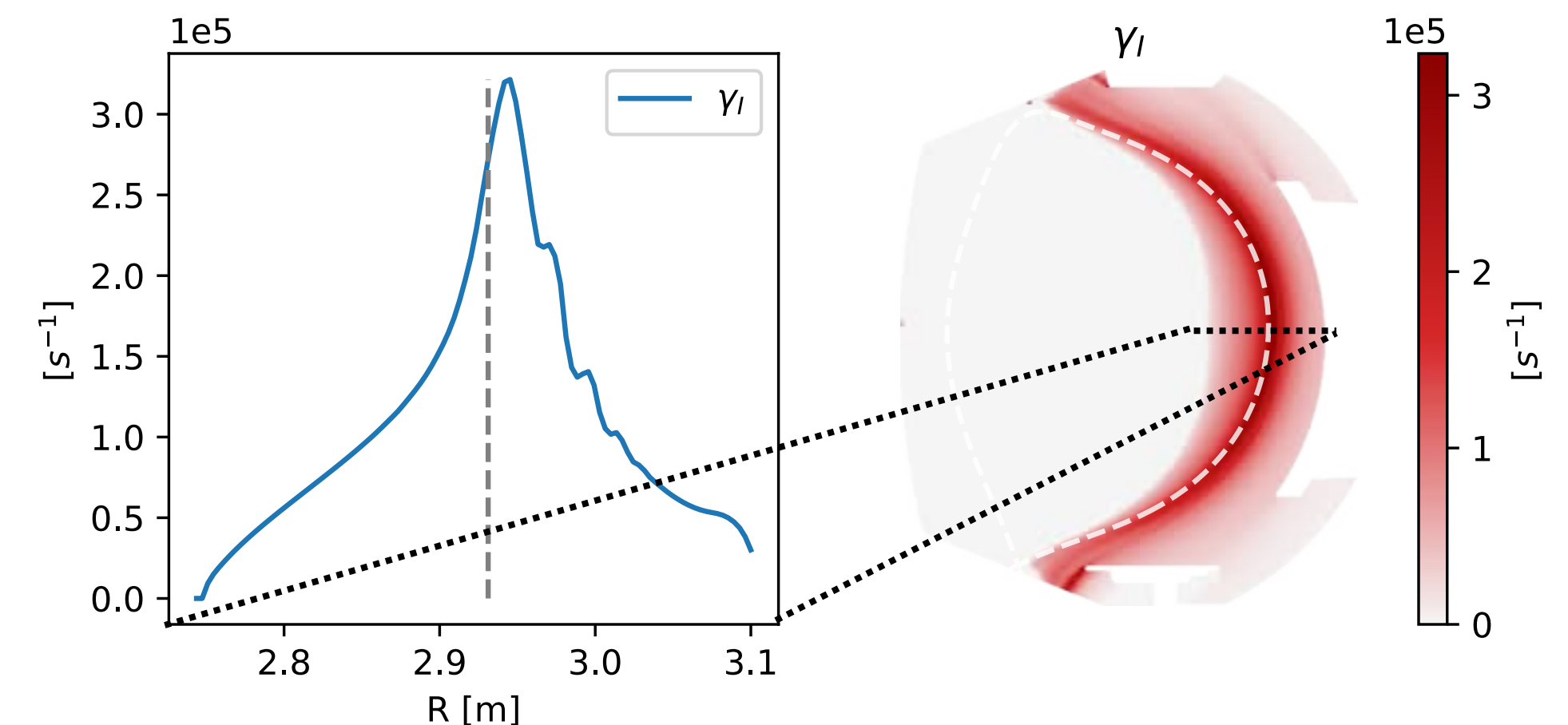
- Similar to the Rayleigh-Taylor instability
- Drives the ballooning effect the Low Field Side (LFS)
- Triggered by the pressure and the magnetic field gradient

Cross-section of WEST to illustrate the interchange instability.



$$\gamma_I = \frac{c_s}{R} \sqrt{\max \left(\underbrace{\frac{R^2}{pB} \frac{\partial p}{\partial R} \frac{\partial B}{\partial R}}_{\text{Positive in the LFS}} - \underbrace{5 \left(1 + \frac{T_i}{T_e} \right)}_{\text{Threshold}}, 0 \right)}$$

Garbet, X., et al., 1992



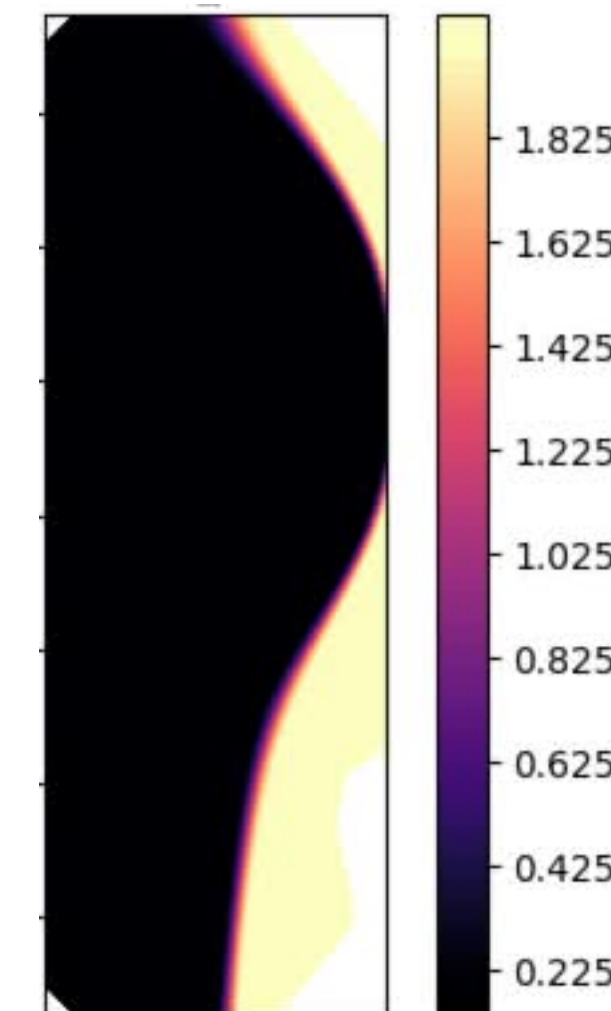
TCV-X23 simulations

SolEdge-HDG + TCV-X23

Setup

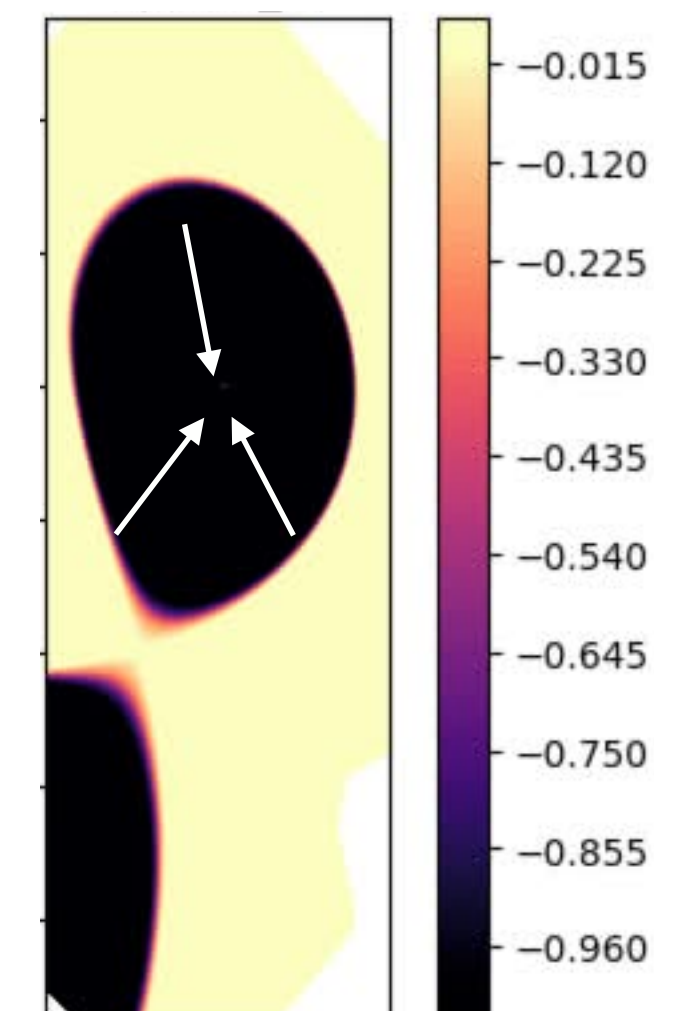
- TVC X23 scenario: L mode, long leg
- 2 regimes (the puff rate is manually tuned):
 - High density
 - Low density
- 2 turbulence models:
 - D=cst : $D = \mu = 0.2 \text{ m}^2/\text{s}$ and $\chi_i = \chi_e = 0.66 \text{ m}^2/\text{s}$ [Wensing21]
 - K- ϵ : $D = \mu = \chi_i = \chi_e = \max(k^2/\epsilon, 0.2\text{m}^2/\text{s})$
- No core transport (Bohm, Gyro-Bohm)

$D_{\text{num}} [\text{m}^2/\text{s}]$



Increase of the diffusivity in the far-SOL for numerical stability (doesn't impact the plasma)

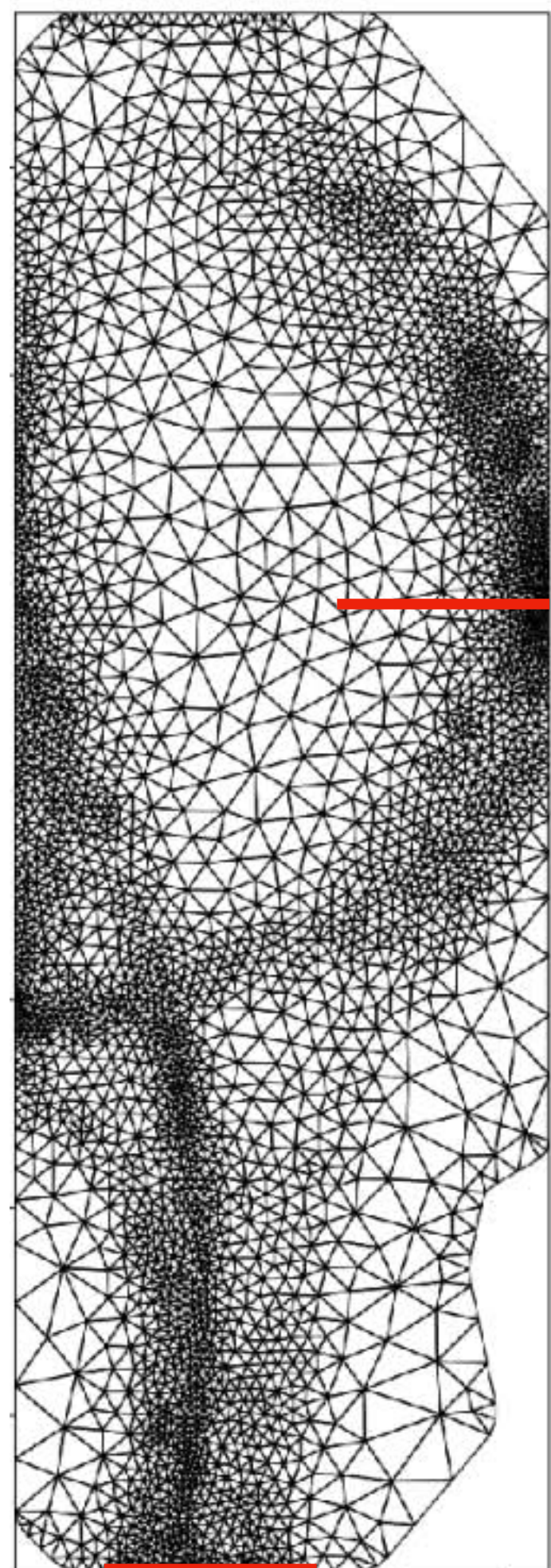
$v_{\text{pinch}} [\text{m/s}]$



Pinch velocity = 1m/s

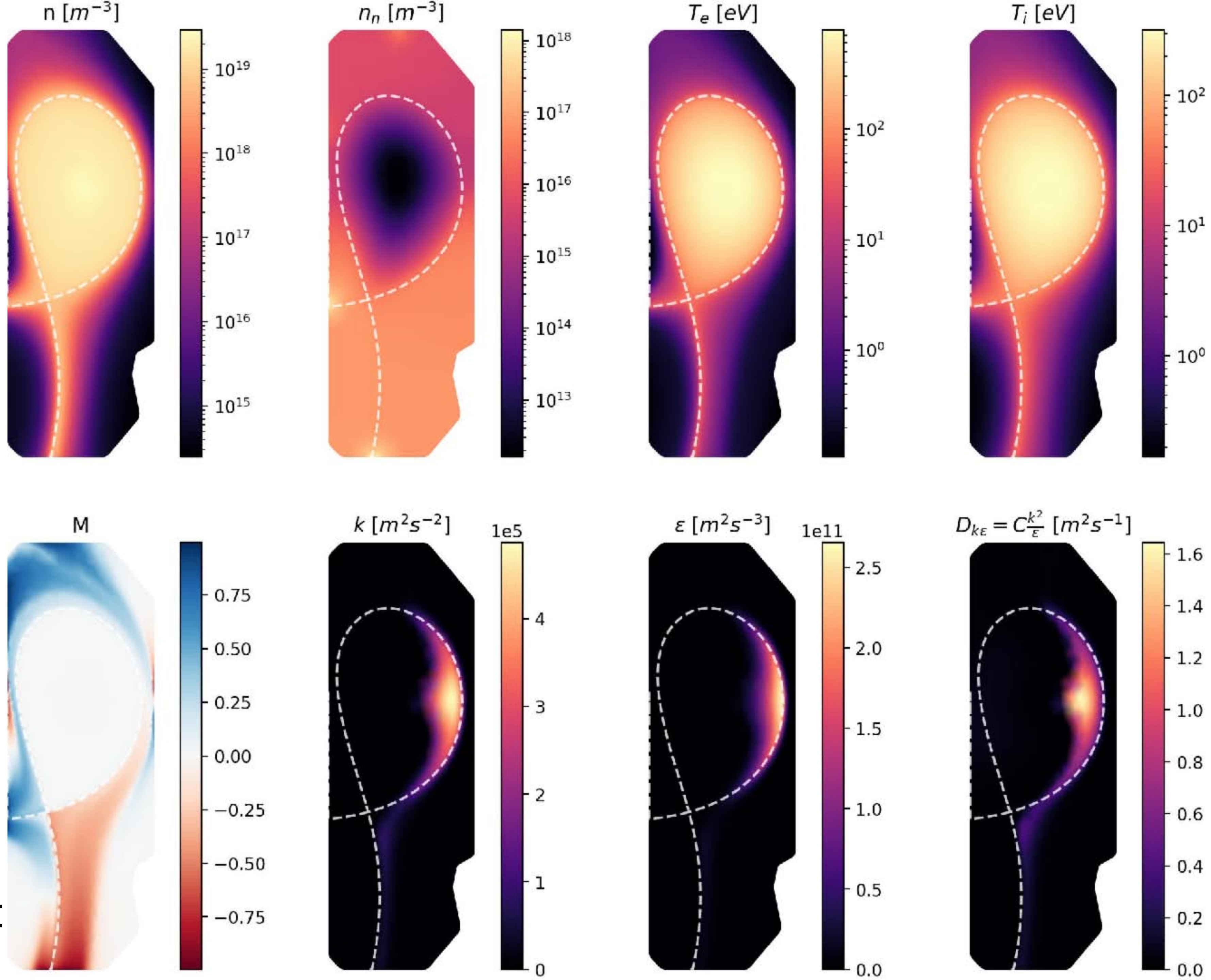
Effective on the density equation only

SolEdge-HDG + TCV-X23 Overview



Outer-
midplane

LFS target

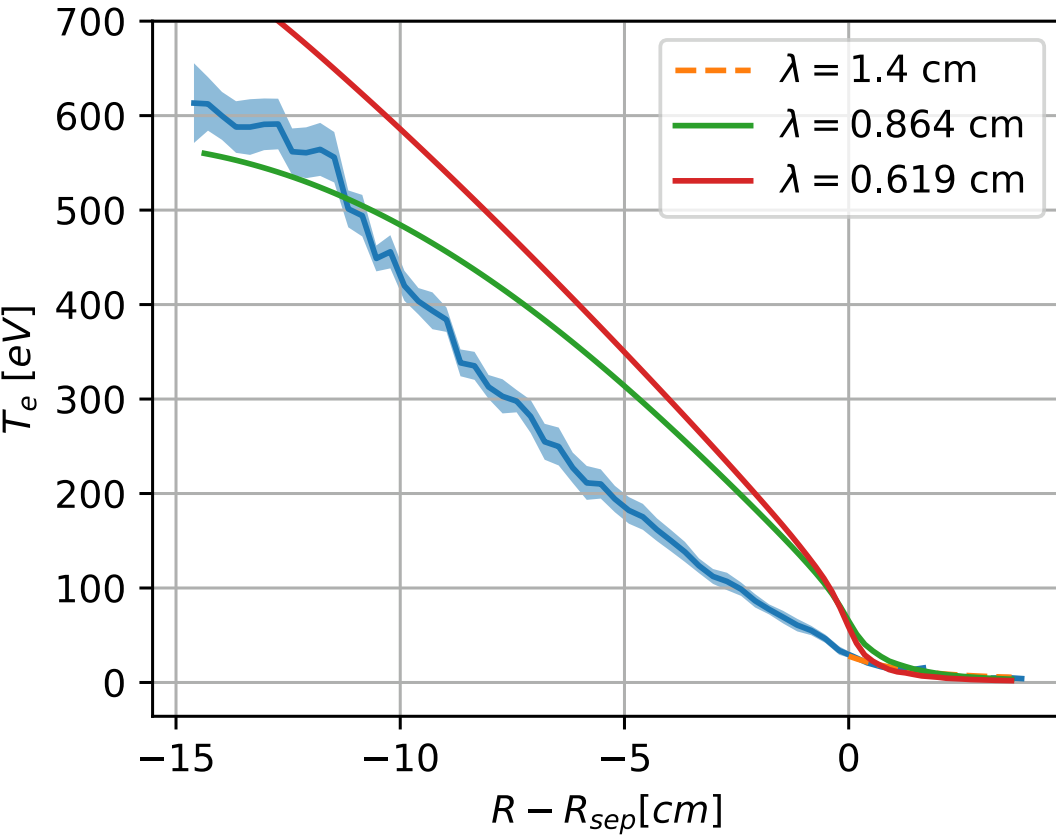
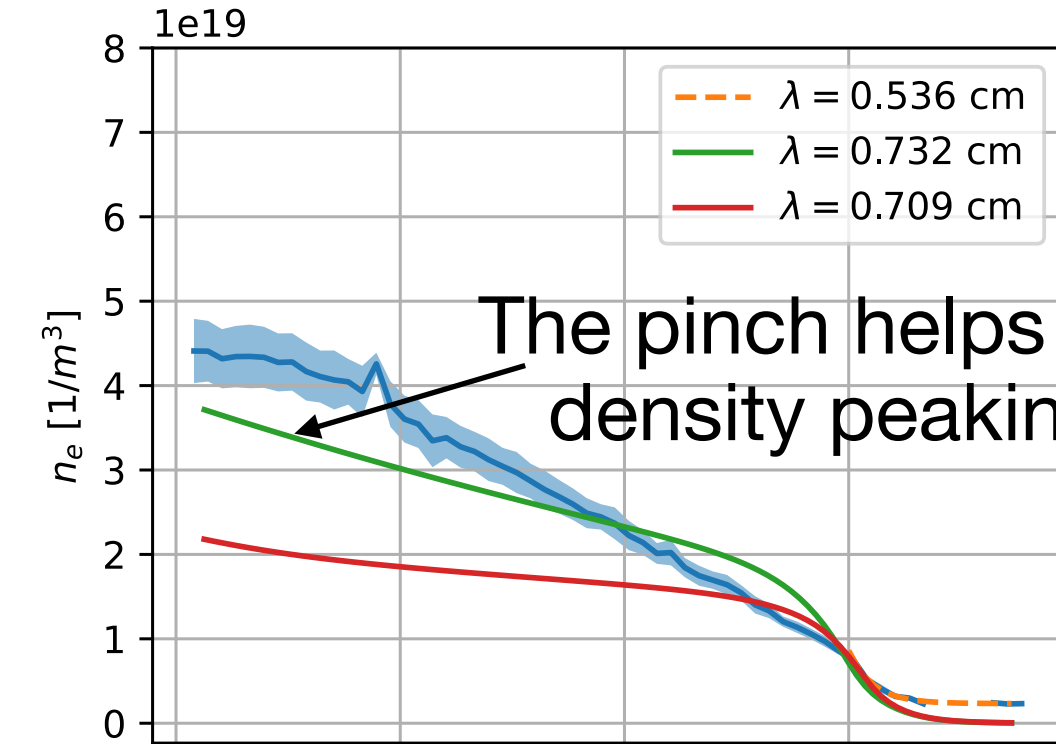


SolEdge-HDG + TCV-X23

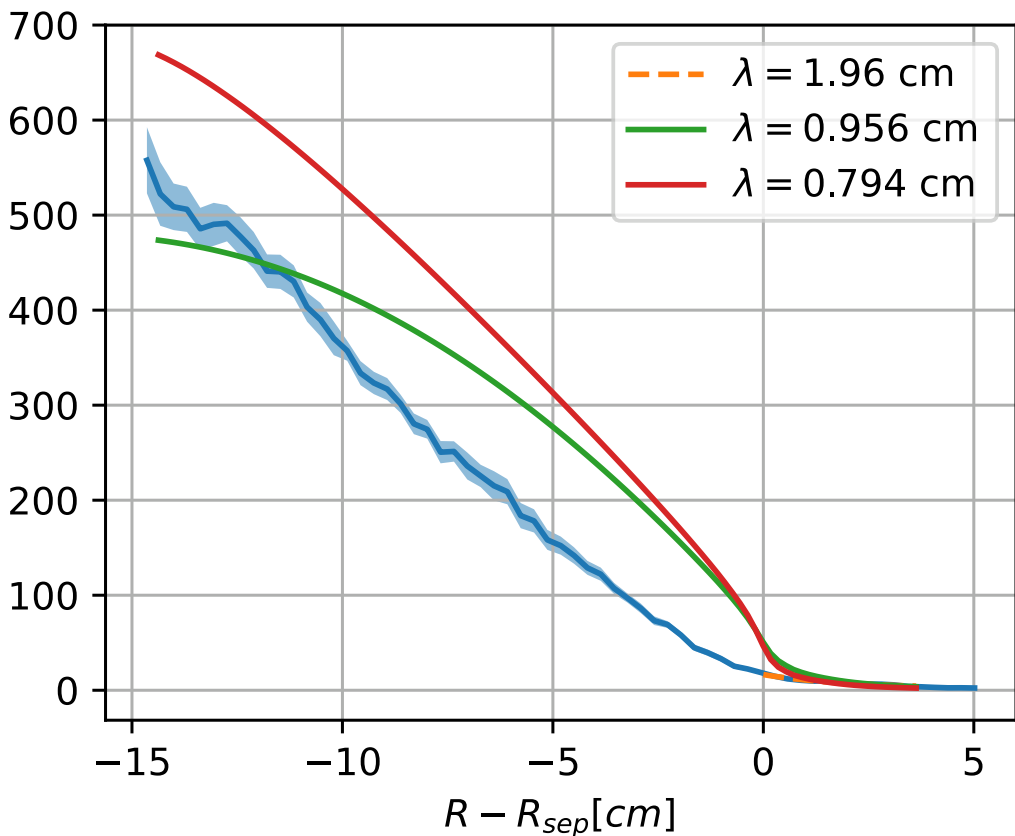
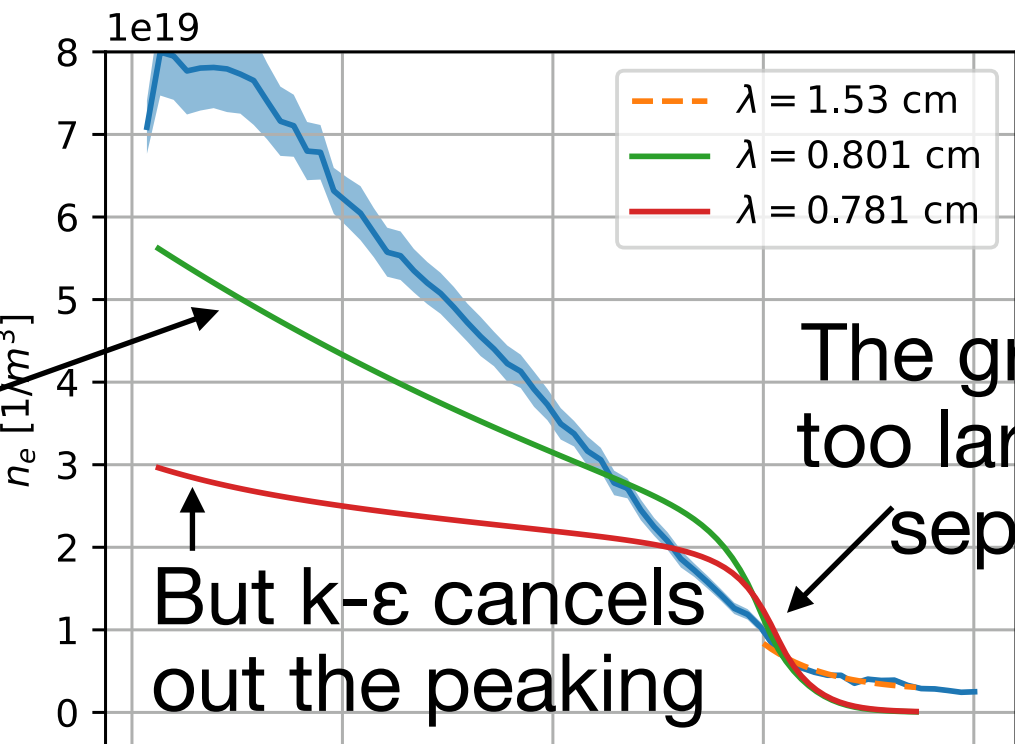
Plasma profiles

OMP

The slope of the density profile in the core represents the ratio of the pinch and the diffusion

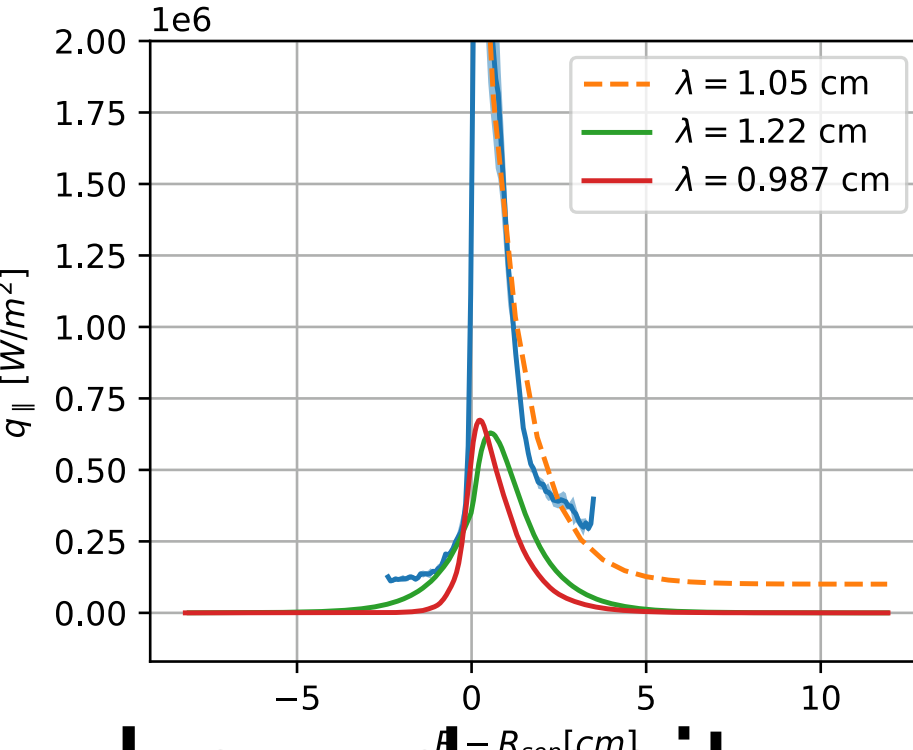
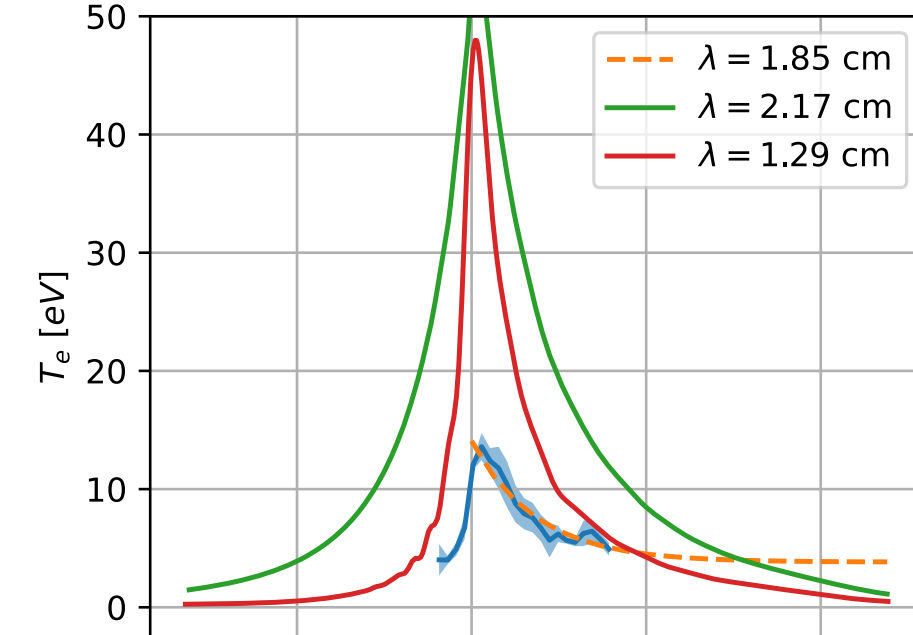
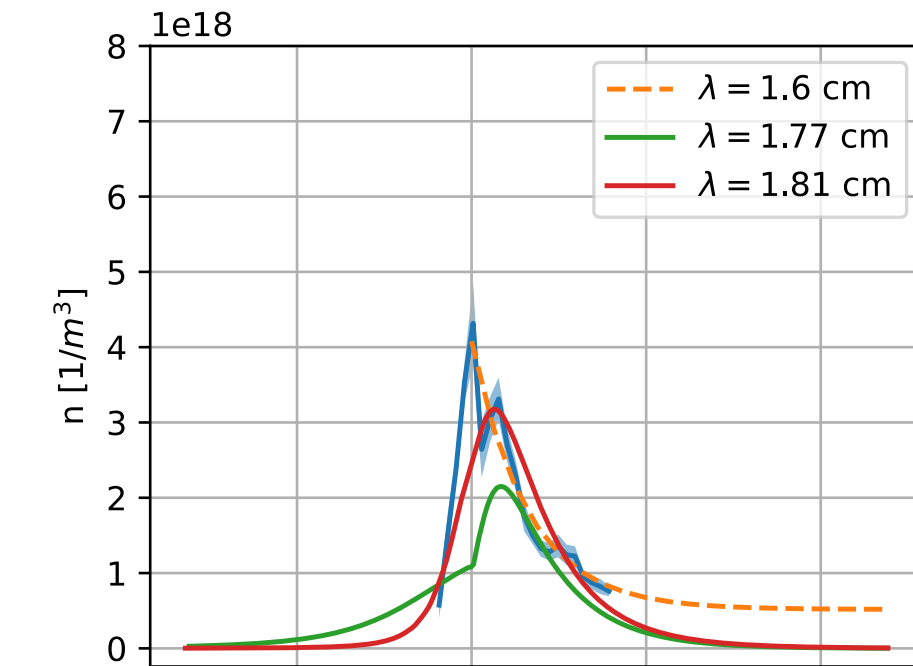


Low density

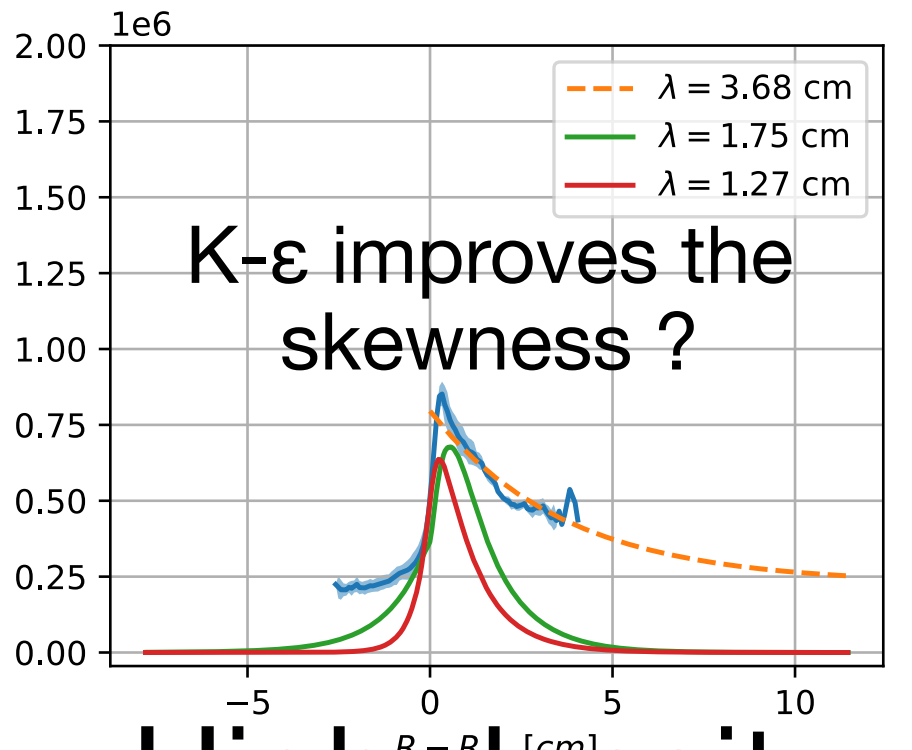
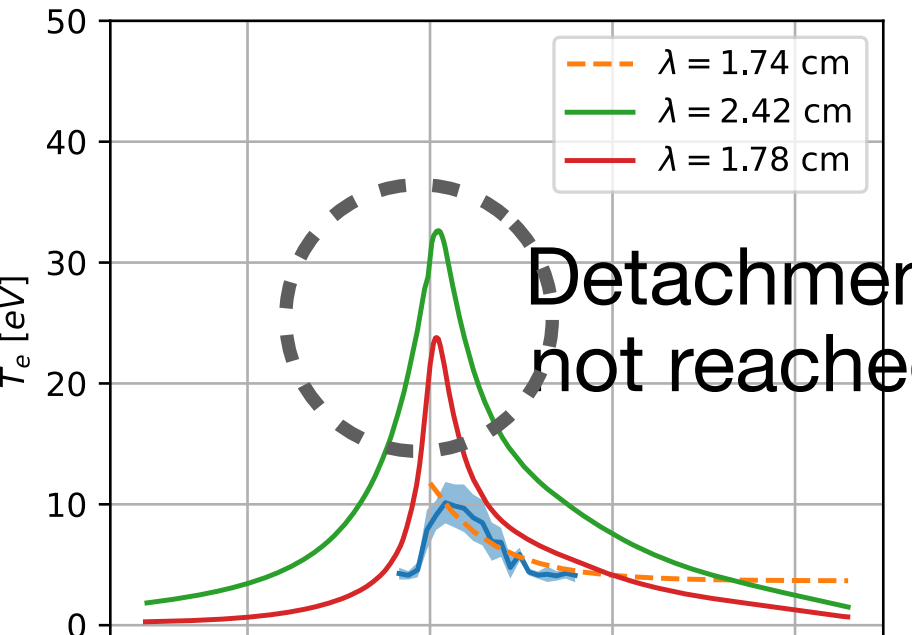
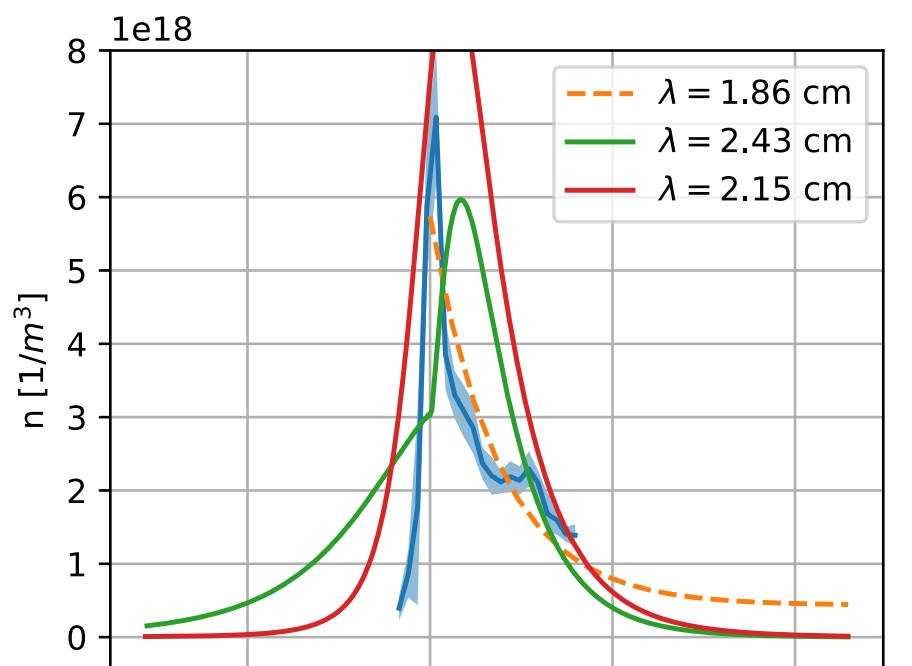


High density

LFS target



Low density

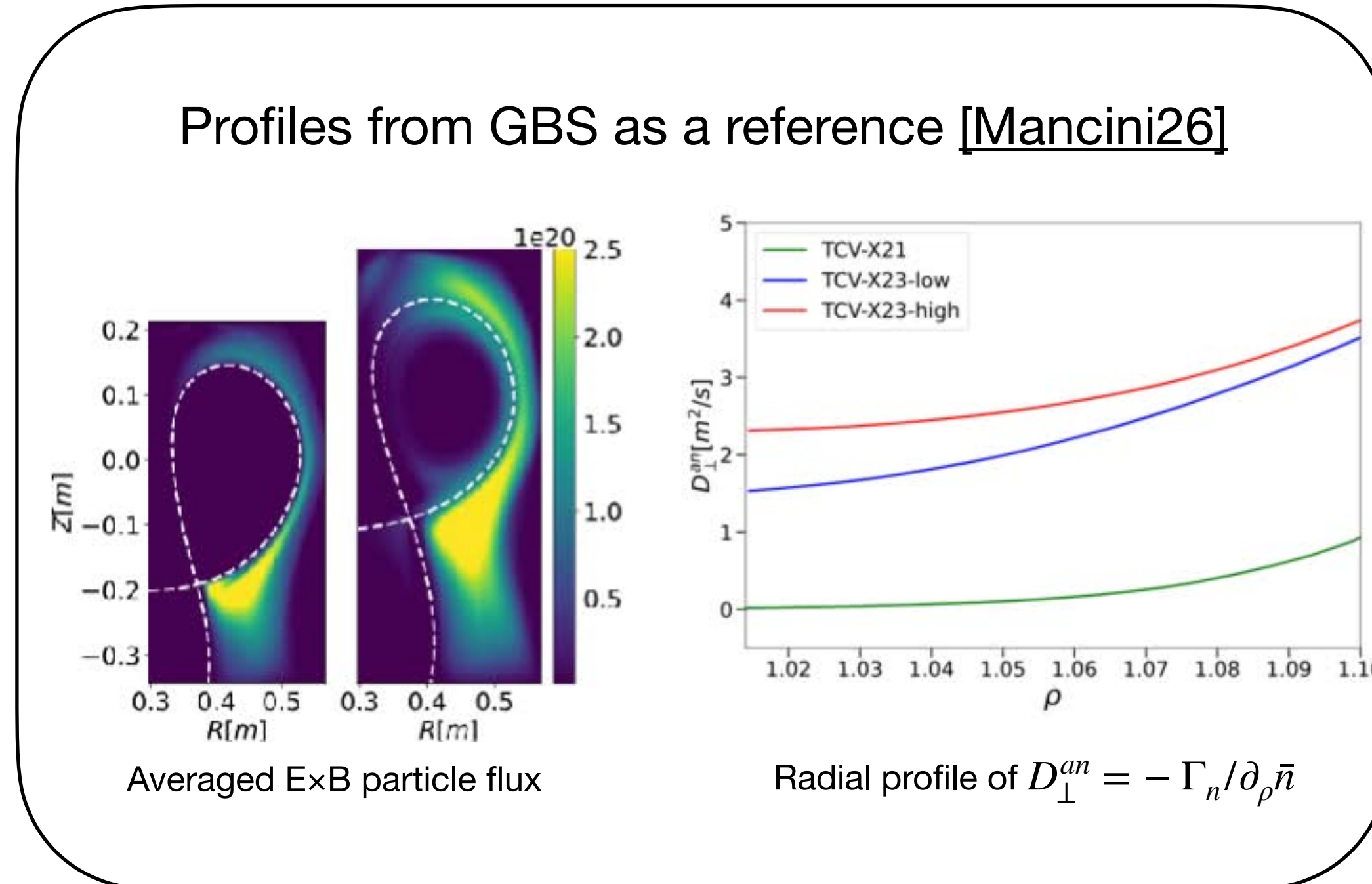
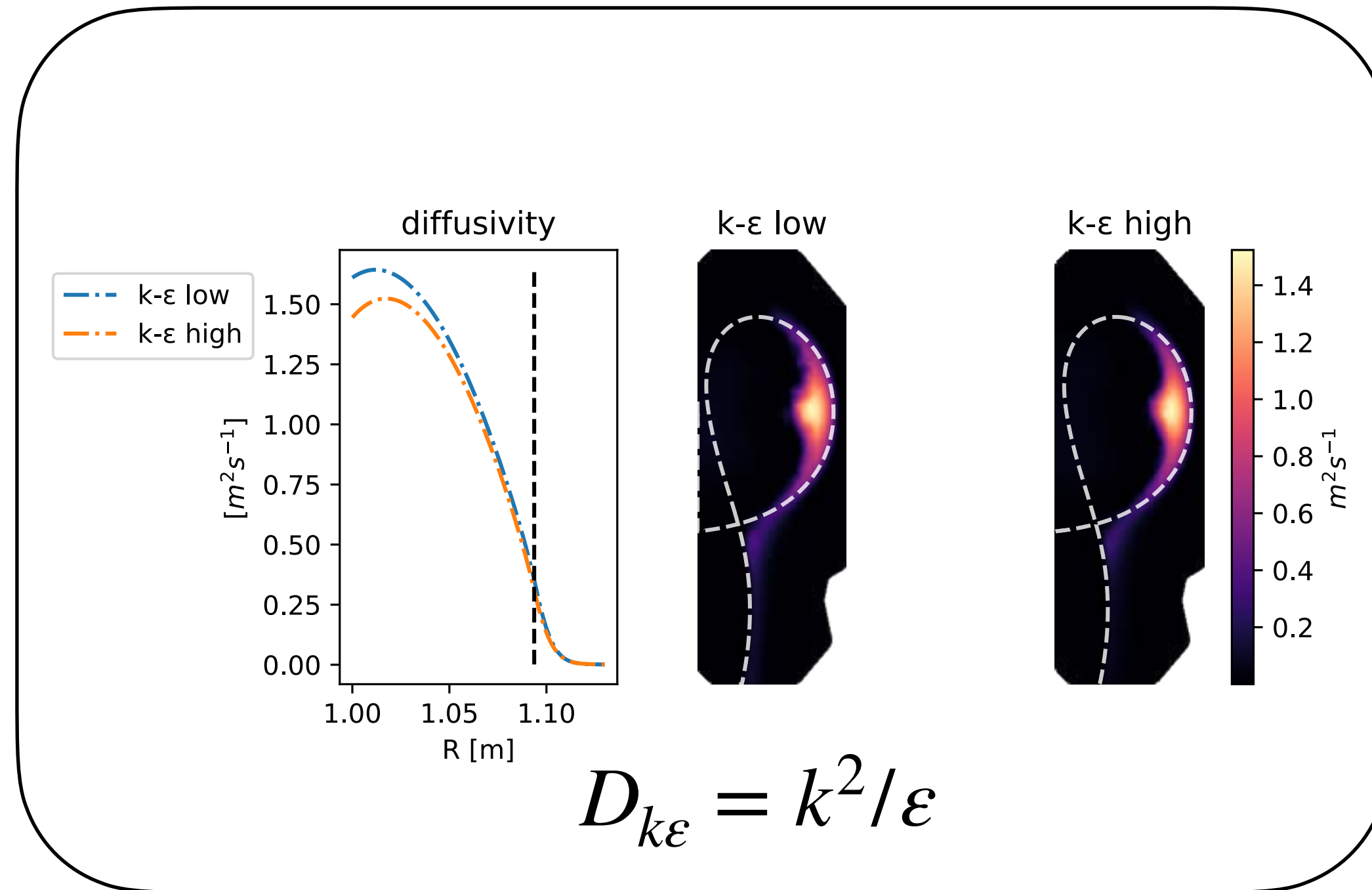


High density

SolEdge-HDG + TCV-X23

Diffusivity profiles

- No significant change between high and low density cases with k- ϵ
- $D_{k\epsilon}$ is larger inside of the separatrix whereas $D_{\perp}^{an}$ from GBS shows the opposite trend
- A larger D in the SOL and a lower D in the core could help HDG: it may increase the peaking in the core and reduce the gradient at the separatrix
- A proper comparison of HDG and GBS would be nice ...



Conclusion

- Although HDG shows some disagreements with the experimental data, the trend is correct
- The benefit of the k- ε model is not obvious compared to $D = \text{cst}$
- K- ε is a simplistic turbulence model which lacks some physics: ballistic transport, zonal flows, ...
- Perspectives:
 - Comparison with turbulence codes (SolEdge3X, GBS, ...)
 - Changing the closures in k- ε to decrease D in the core and increase D in the SOL.

Thank you

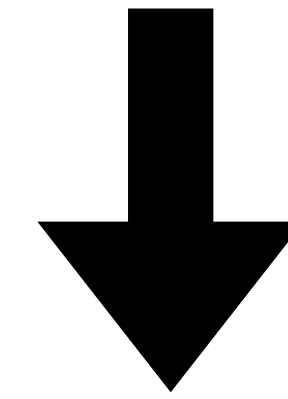
Backup slides

Fixed point solution

- Fixed point: analytical solution of k-ε without the transport terms
- The fixed point gives a larger D in the core
- The solved solution is close to the fixed point but “diffused”: smoothed and spread
- Therefore k-ε is quite local, is that ok?

$$0 = \gamma k - \varepsilon$$

$$0 = \gamma \varepsilon - V \varepsilon^2 k^{-3/2}$$



$$D_{FP} = \frac{k_{FP}^2}{\varepsilon_{FP}} = \frac{V^2}{\gamma} = \frac{\lambda^2}{\tau}$$

