



Application of RANS model to TCVX23 experiments and comparison to SOL scaling laws

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Outline

- RANS approach for plasma edge models
 - Basic $\kappa_{\perp}$ -model development
 - Model calibration through optimization
 - Application to TCV-X21/23
- Analysis of SOL decay lengths

Outcome RANS approach for mean-field codes

- Re-interpretation of evolution equations:

$$\begin{array}{l}
 \frac{\partial n}{\partial t} + \nabla \cdot \Gamma = S_n, \\
 \frac{\partial m n u_{||}}{\partial t} + \nabla \cdot \Gamma_m = S_m - \nabla_{||} p - \mathbf{b} \cdot \nabla \cdot \Pi_i, \\
 \frac{3}{2} \frac{\partial n T_e}{\partial t} + \nabla \cdot \mathbf{Q}_e = S_{H_e} + u_{||} \nabla_{||} p_e - p_e \nabla \cdot \mathbf{V}_E - j_{||} \nabla_{||} \phi,
 \end{array}
 \longrightarrow
 \begin{array}{l}
 \frac{\partial \bar{n}}{\partial t} + \nabla \cdot \bar{\Gamma} = \bar{S}^n, \\
 \frac{\partial m \bar{n} \bar{u}_{||}}{\partial t} + \nabla \cdot \bar{\Gamma}_m = \bar{S}_m - \nabla_{||} \bar{p}, \\
 \frac{3}{2} \frac{\partial \bar{n} \bar{T}_e}{\partial t} + \nabla \cdot \bar{\mathbf{Q}}_e = \bar{S}_{H_e} + \overline{u_{||} \nabla_{||} p_e} - \bar{p}_e \nabla \cdot \bar{\mathbf{V}}_E - \overline{p'_e \nabla \cdot \mathbf{V}'_E} - \bar{j}_{||} \nabla_{||} \bar{\phi} - \overline{j'_{||} \nabla_{||} \phi'},
 \end{array}$$

- Through 'smart' choice of Favre/Reynolds average, reduction of # closure terms:

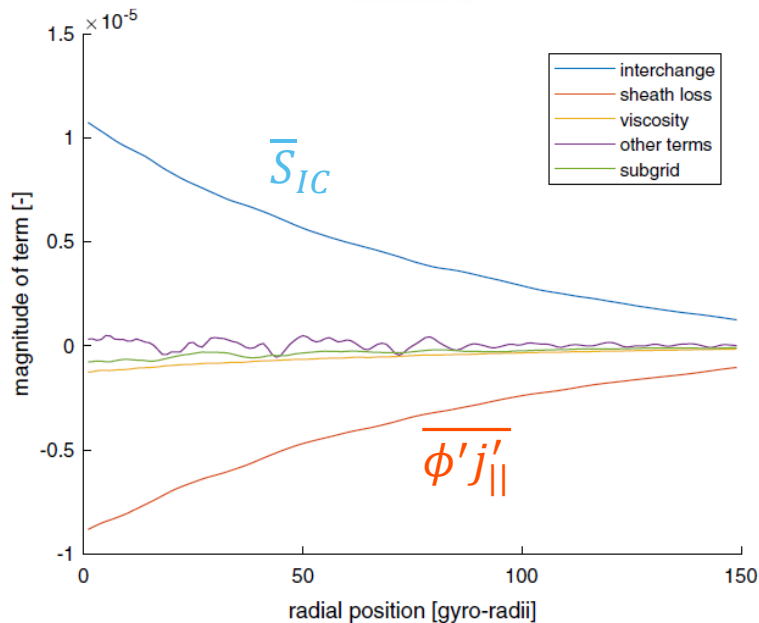
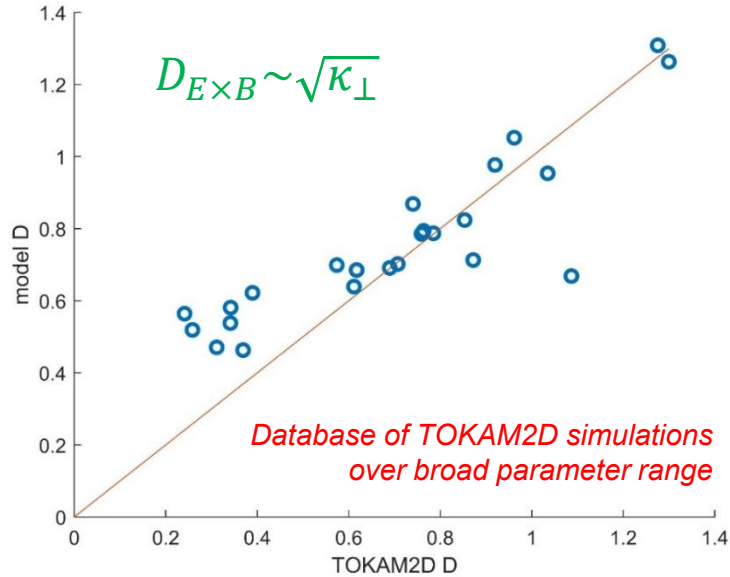
$$\bar{\Gamma} = \bar{n} \tilde{\mathbf{V}} = \bar{n} \tilde{u}_{||} \mathbf{b} + \bar{n} \bar{\mathbf{V}}_E + \overline{n' \mathbf{V}'_E} + \bar{n} \tilde{\mathbf{V}}_* + \bar{n} \tilde{\mathbf{V}}_p + \bar{n} \tilde{\mathbf{V}}_v, \quad \bar{\mathbf{V}}_E = \frac{\mathbf{B} \times \nabla \bar{\phi}}{B^2}, \quad \mathbf{V}'_E = \frac{\mathbf{B} \times \nabla \phi'}{B^2}, \quad \tilde{\mathbf{V}}_* = \frac{\nabla \bar{p}_i \times \mathbf{B}}{\bar{n} e B^2},$$

- (Implicit) Closure for most non-linearities:

$$\circ \quad \bar{\Gamma}|_{sheath} \sim \bar{n}_e \sqrt{\bar{T}_e + \bar{T}_i}, \quad S_{n,iz} = \bar{n}_e \bar{n}_n K_{iz}(\bar{T}_e, \bar{n}_e), \dots$$

- More consistent description of anomalous transport? Can we improve on convective-diffusive descriptions with user-specified anomalous coefficients?

RANS approach for electrostatic interchange turbulence



- Average fluxes, electrostatic turbulence (no B -fluctuations): fluctuating $E \times B$ -terms need closure
 - $\bar{\Gamma}_{i/e, E \times B} = \overline{n'_i \mathbf{u}'_{E \times B}} \sim -D_{E \times B} \nabla \bar{n}_i$
 - $\bar{Q}_{i/e, E \times B, c} = \overline{\frac{3}{2} n_i \mathbf{u}''_{E \times B} T_i''} \sim -\chi_{i/e, E \times B} \bar{n}_{i/e} \nabla_{\perp} \tilde{T}_{i/e}$
 - $\bar{Q}_{i/e, E \times B} = \overline{\frac{3}{2} \tilde{T}_{i/e} n'_i \mathbf{u}'_{E \times B}} + \bar{Q}_{i/e, E \times B, c}$ Note the factor 1.5 $\leftrightarrow$ 2.5
- Proposal: relate to turbulent kinetic energy $\kappa_{\perp}$ (and $\xi_{\perp}$) as measure(s) of local intensity of the turbulence/transport

$$\bar{n}_i \kappa_{\perp} = \frac{\overline{n_i m_i u_{E \times B}^{\prime \prime 2}}}{2} \quad \bar{n}_i \xi_{\perp} = \frac{\overline{n_i \omega^{\prime \prime 2}}}{2}$$

- Diffusive transport model based on 2D interchange simulations [Coosemans et al., CPP 2022, e202100193.]

$$\begin{aligned} \circ \quad D_{E \times B} &= C_D \rho_L \sqrt{\frac{\kappa_{\perp}}{m_i}} \quad \text{or} \quad D_{E \times B} \sim \frac{\kappa_{\perp}/m_i}{\sqrt{\xi_{\perp}}} \\ \circ \quad \chi_{i/e, E \times B} &= C_{i/e} D_{E \times B} \sim \frac{3}{2} D_{E \times B} \end{aligned}$$

- Need transport equation for $\kappa_{\perp}$ (and $\xi_{\perp}$)! Introduces non-local effects.

Sources and transport of $\kappa_{\perp}$ for interchange turbulence

[R. Coosemans et al., J. Plasma Phys. 90 (2024) 90590020.]

- Transport equation for $\kappa_{\perp}$ for 2D electrostatic interchange turbulence

$$\frac{\partial}{\partial t} \bar{n} \kappa_{\perp} + \nabla \cdot (\bar{\Gamma}_i \kappa_{\perp} + \overline{\phi' J'_{\parallel}}) = \bar{S}_{IC} + \bar{S}_{\parallel}$$

- Total heat flux due to $E \times B$ fluctuations drives production of $\kappa_{\perp}$

$$\bar{S}_{IC} = -\frac{2}{3} \bar{\mathbf{Q}}_{i+e, E \times B} \cdot \nabla \ln B^2$$

- (Typically) source in 'bad-curvature' regions, sink in 'good-curvature' regions

- Parallel transport of $\kappa_{\perp}$ governed by plasma conductivity

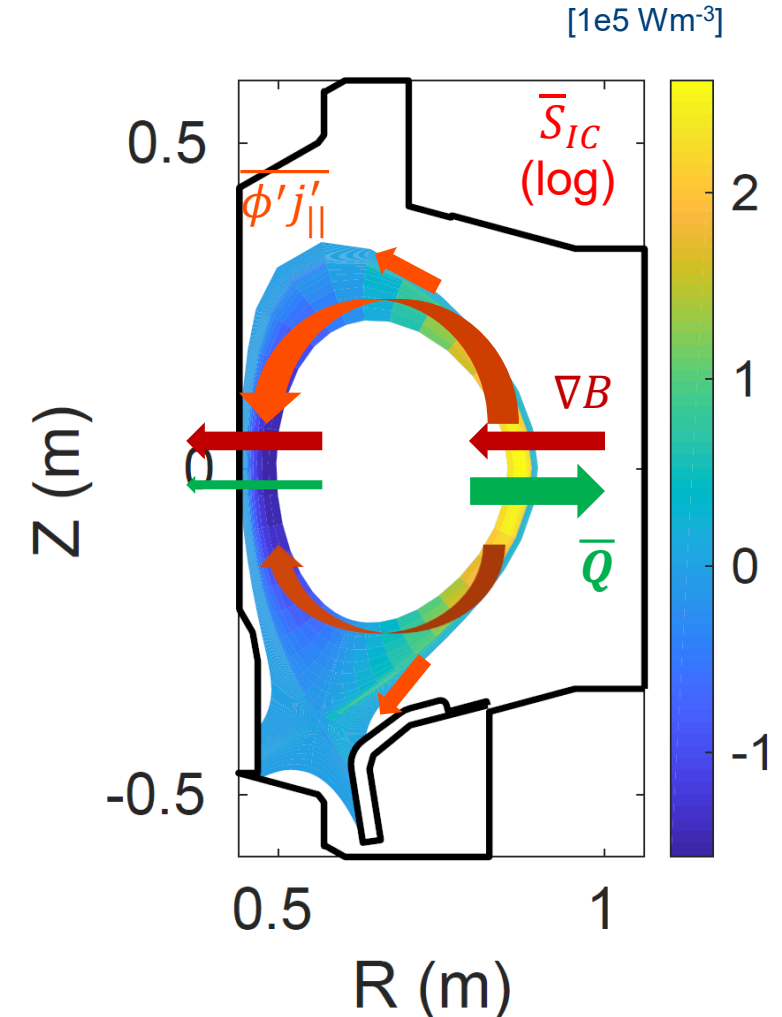
$$\overline{\phi' j'_{\parallel}} \sim -\sigma_{\parallel} \nabla_{\parallel} \frac{\overline{\phi'^2}}{2} \sim -C_{\sigma 1} \sigma_{\parallel} \rho_L^2 B^2 \nabla_{\parallel} \kappa_{\perp} \sim -C'_{\sigma 1} \sigma_{\parallel} \nabla_{\parallel} \left(\frac{\kappa_{\perp}^2}{\xi_{\perp}} \right)$$

- Strongly exceeds parallel convection with $\tilde{u}_{\parallel}$

- Proxy to account for mean ExB flow shear S_m suppressing anomalous transport

$$D_{\perp} = C_D \frac{\kappa_{\perp} / m_i}{\sqrt{\kappa_{\perp} / m_i / \rho_L + C_S |S_m|}}$$

...coupled to 'regular' mean field equations [Dekeyser et al., CPP 2022, e202100190.]



Intrinsic ballooning character and 'self-saturation'

Model calibration through optimization

- While (most) model parameters were calibrated for TOKAM2D → Application to SOLPS-ITER requires refinement
- Parameter calibration through optimization
[Carli et al., CPP 2021, e202100184.] ; [Paper to be submitted to NF: Carli et al., <https://arxiv.org/abs/2609.27435>]
 - Based on **PDE-constrained** optimization to minimize **cost function**

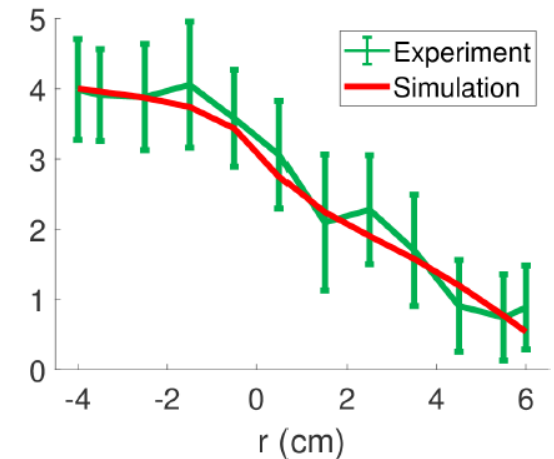
$$\min_{\theta, q} \mathcal{J}(\theta, q)$$

$$\text{s. t. } \mathcal{B}(\theta, q) = \mathbf{0}$$

$$\text{Least-squares: } \mathcal{J}(\theta, q) = \frac{1}{\Omega} \int_{\Omega} \omega_q \left(\frac{1}{\bar{D}^2} (q - D)^2 \right) d\Omega$$

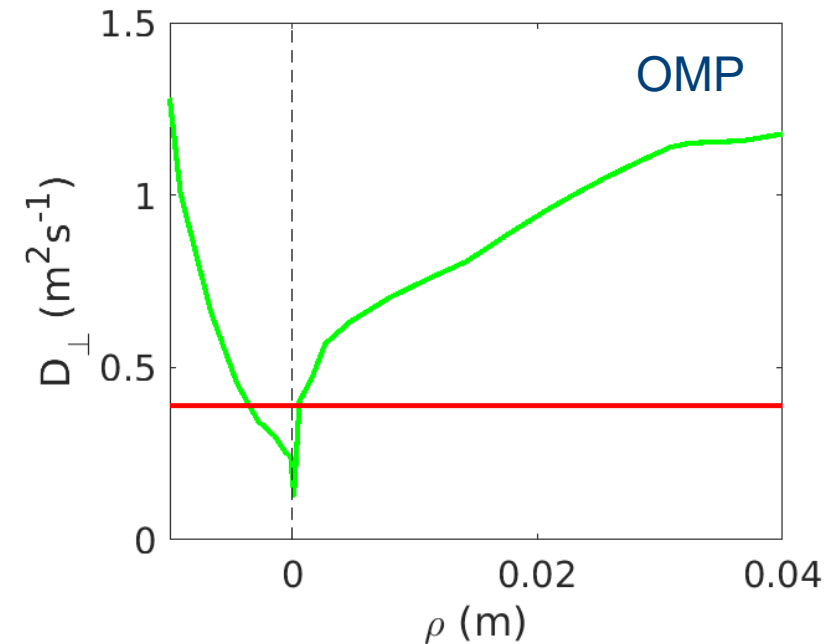
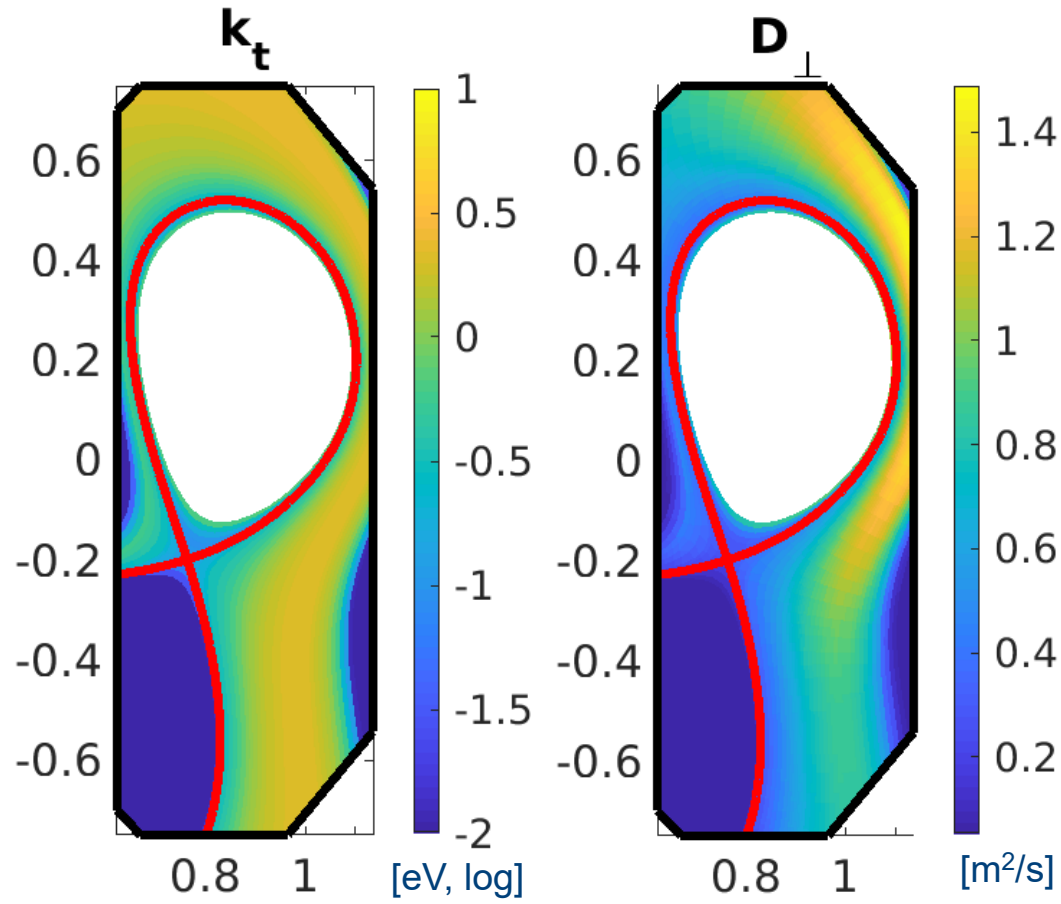
Maximum-a-posteriori (Bayesian), ...

- Unknown variables θ to be optimized: C_D , BC-parameters, ...
 - k -model parameters: $C_{i/e}$, C_v , C_κ , $C_{\sigma 1}$, $C_{\sigma 2(SOL\&core)}$, C_S , $\kappa_{\perp, core}$
 - Note: not C_D : ambiguity with level of $\kappa_{\perp}$ to be resolved through confrontation with data on $\kappa_{\perp}$ itself (!)
- Solved with gradient-based methods (AD version of code)

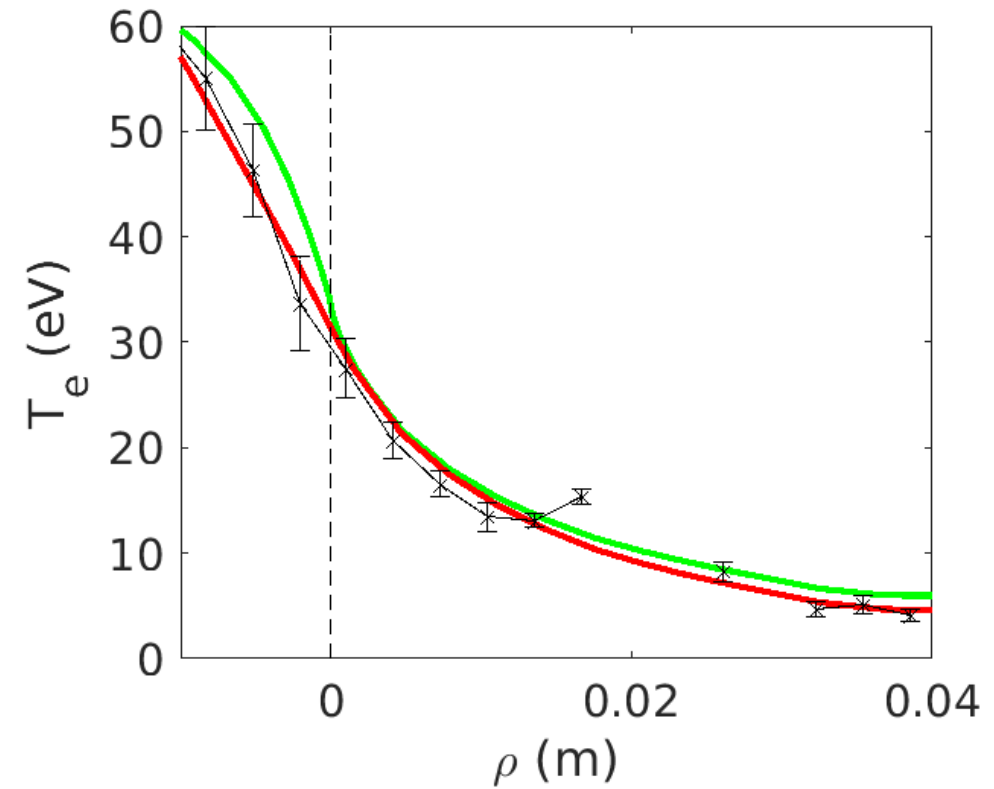
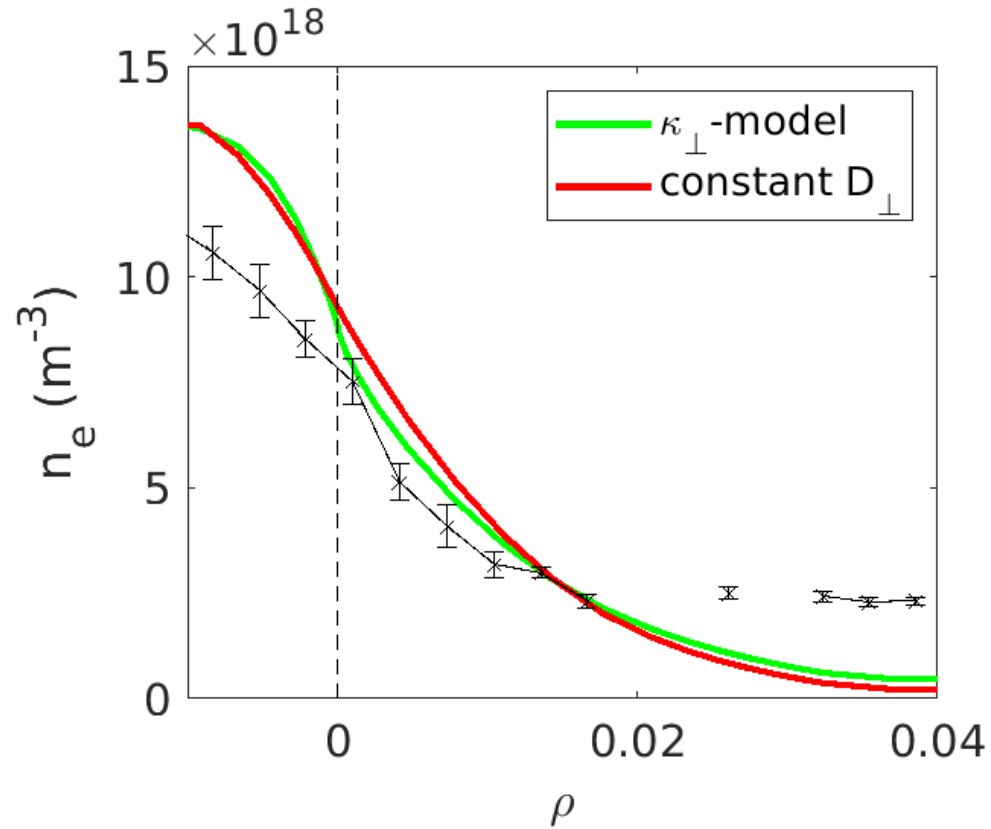


TCV-X23 showcase: turbulent kinetic energy and $D_{\perp}$

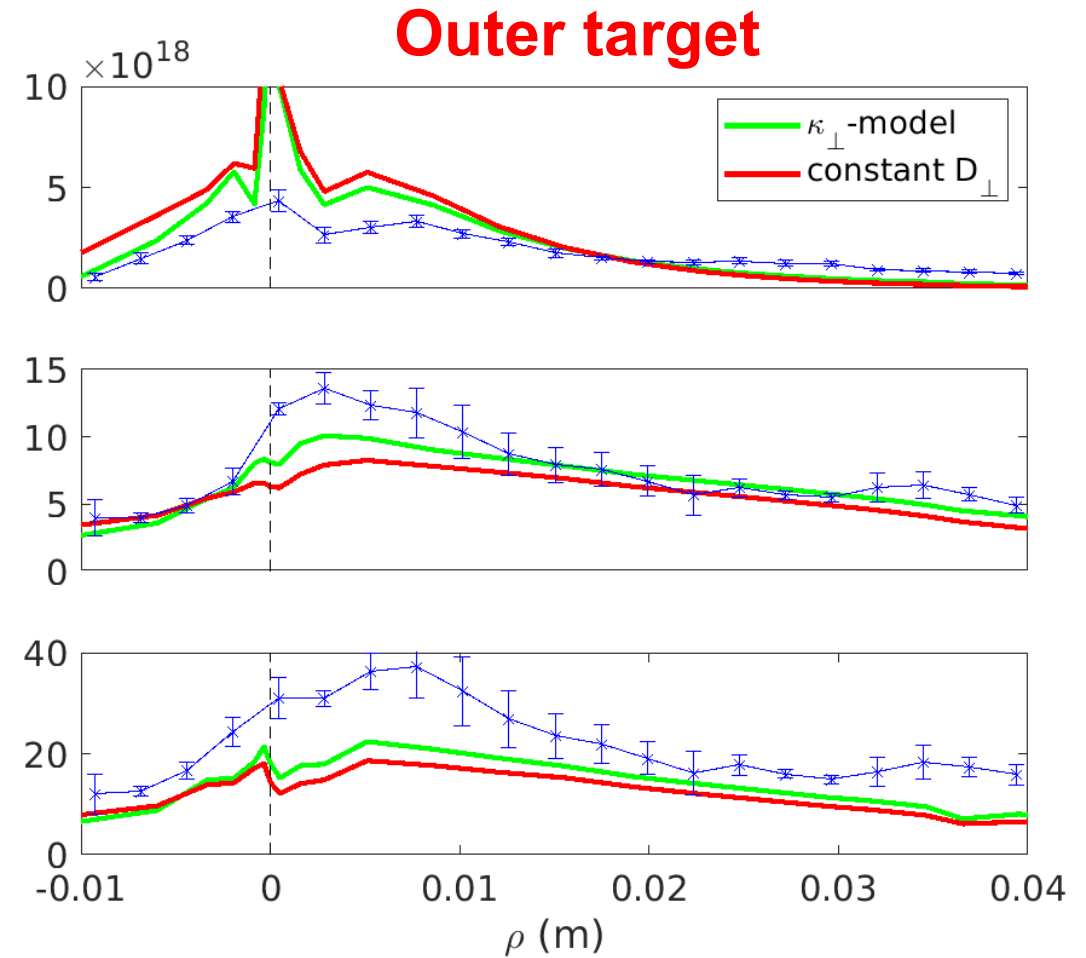
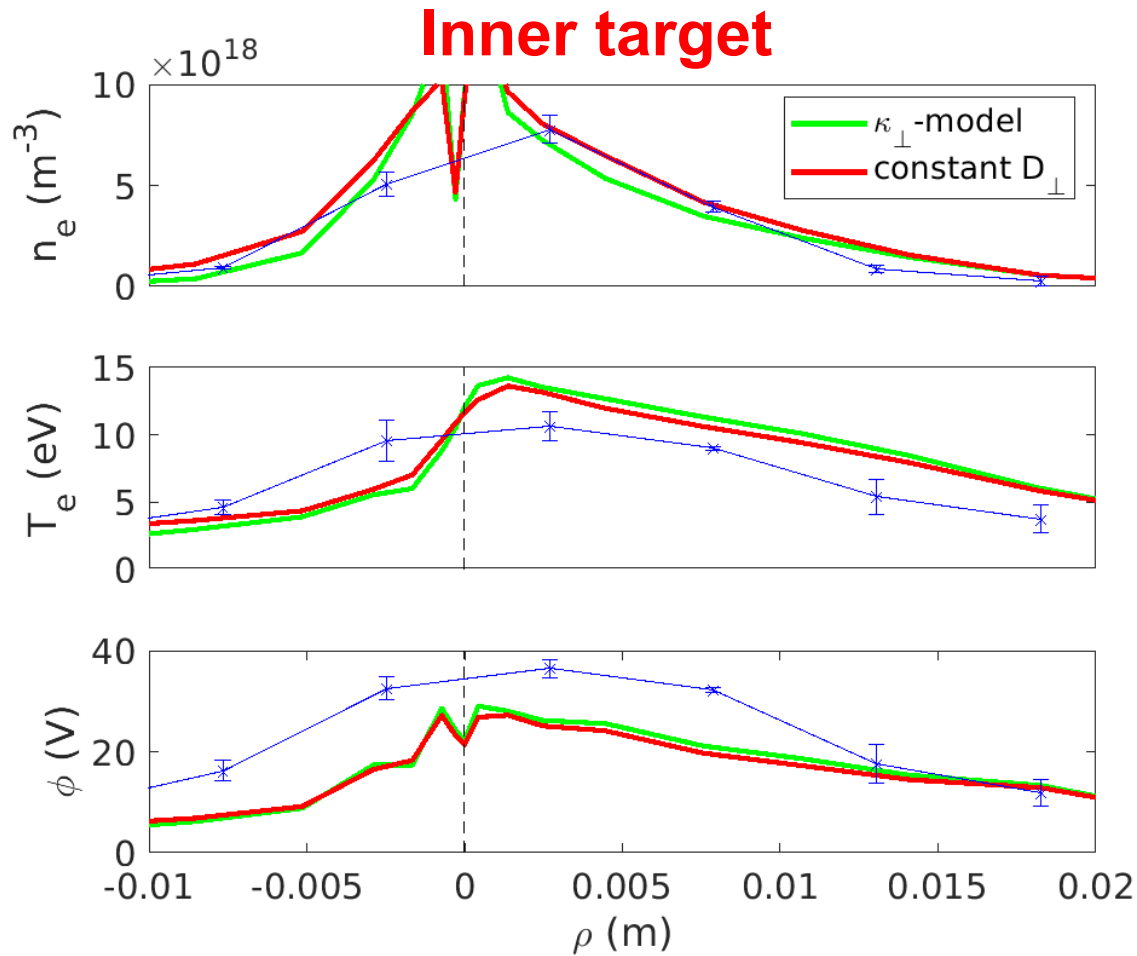
Using same coefficients as for TCV-X21, Note: low n -simulation here (~TCV-X21)



TCV-X23 showcase: midplane profiles

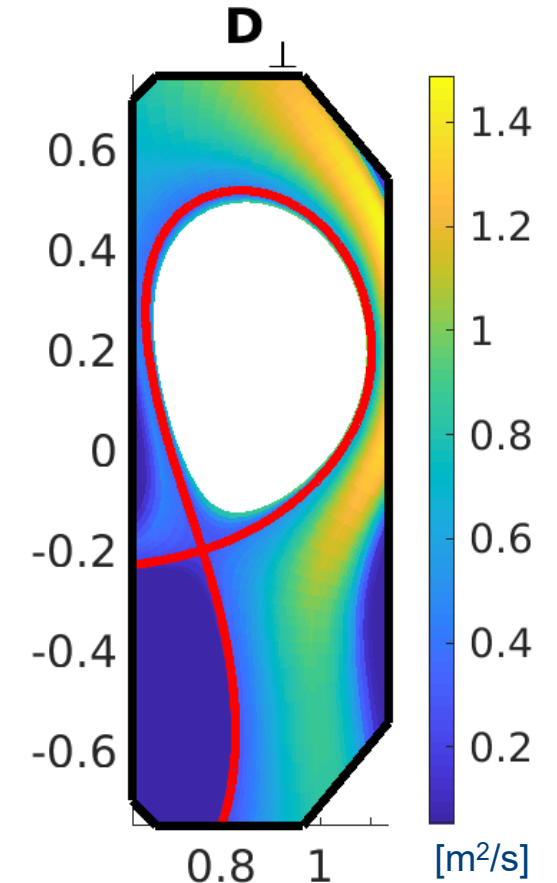
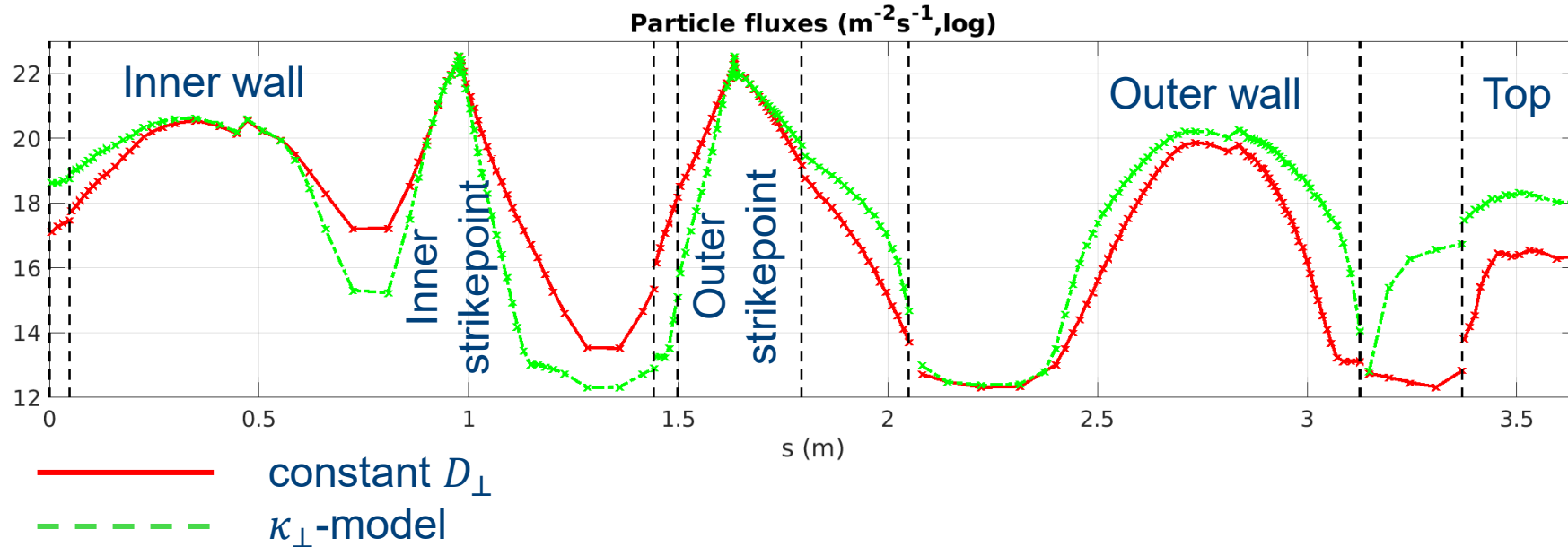


TCV-X23 showcase: target profiles



TCV-X23 showcase: wall fluxes

- Clear signature of magnetic field structure on wall loads
- Peak loads near strike points similar
- Up to factor two higher wall fluxes near outboard side for $\kappa_{\perp}$ -model, despite nearly identical density/temperature profiles



Summary and next steps

- Despite significantly different poloidal maps of the transport coefficients, both models behave similarly, but keep in mind:
 - Parameters optimized based on same data at OMP *and* targets (for TCV-X21)
 - Similar separatrix values of density, temperature, and transport coefficients with both models
- Extend parameter estimation framework to include data on $\kappa_{\perp}$, aiming to resolve ambiguity between $\kappa_{\perp} \leftrightarrow C_D$
- Derivation & analysis of improved (sheath) boundary conditions for $\kappa_{\perp}$ equation
- Improved expressions for $\kappa_{\perp}$ based on analysis SOLEDGE3X simulations
- Revisiting the need for $\xi_{\perp}$ equation (2-equation model)

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Assessment of decay lengths

- Using the optimised parameters of both the $\kappa_{\perp}$ -model and standard model, what do both models predict for the decay lengths?

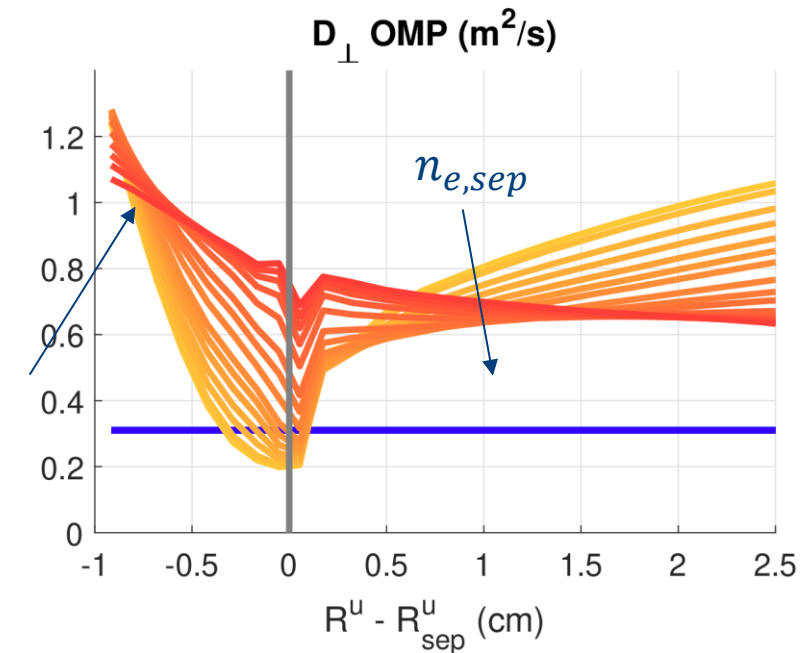
- $\lambda_q = 0.49 \cdot B_{tor}^{-0.69} q_{95}^{0.95} P_{SOL}^{0.05} R_{geo}^{0.29} \frac{n}{n_{GW}}^{-0.55}$ [T. Eich et al 2013 Nucl. Fusion **53** 093031]

- Density scan:**

- $P = 125 \text{ kW}$; constant SOL power
- n_e in $[1, 1.2, \dots, 2.8] \cdot 10^{19} \text{ m}^{-3}$; (core BC)

- Power scan:**

- P in $[50, 75, \dots, 200] \text{ kW}$
- $n_e = 0.932 \cdot 10^{19} \text{ m}^{-3}$ (optimised value)



Assessment of decay lengths

- Using the profiles from our simulations, **exponential fits** of the profiles are made at **different locations** in the vessel

- λ_n, λ_T : single/double exponential fit at OMP or divertor entrance

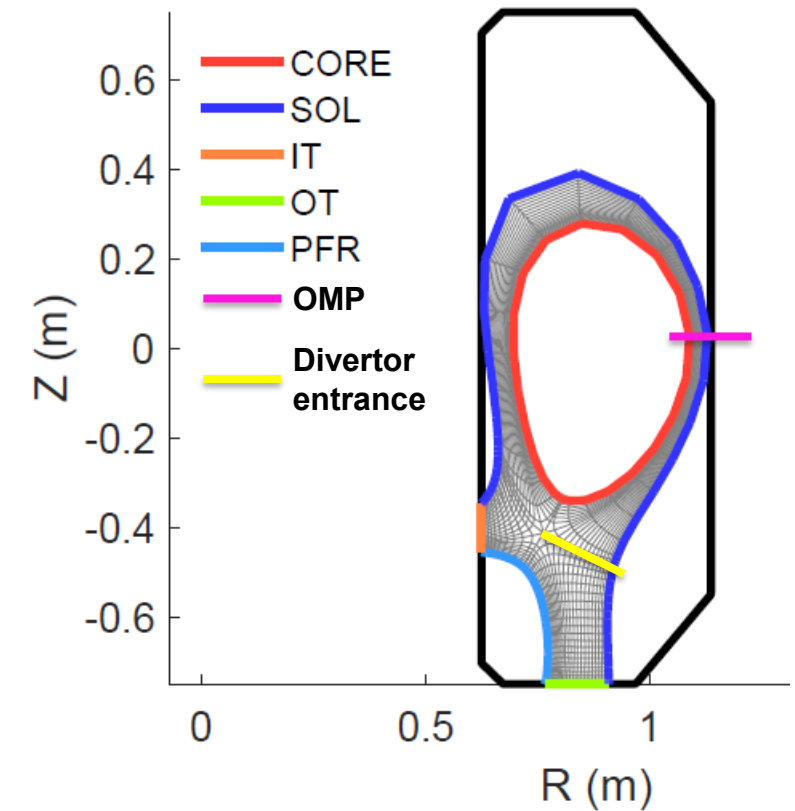
- $f(r) \cong f_0 \exp\left(\frac{-r}{\lambda_f}\right)$ or $f(r) \cong \frac{f_0}{2} \left(e^{\frac{-r}{\lambda_{f,near}}} + e^{\frac{-(r-r_0)}{\lambda_{f,far}}} \right)$

- λ_q : Convoluted gaussian fit at OT or

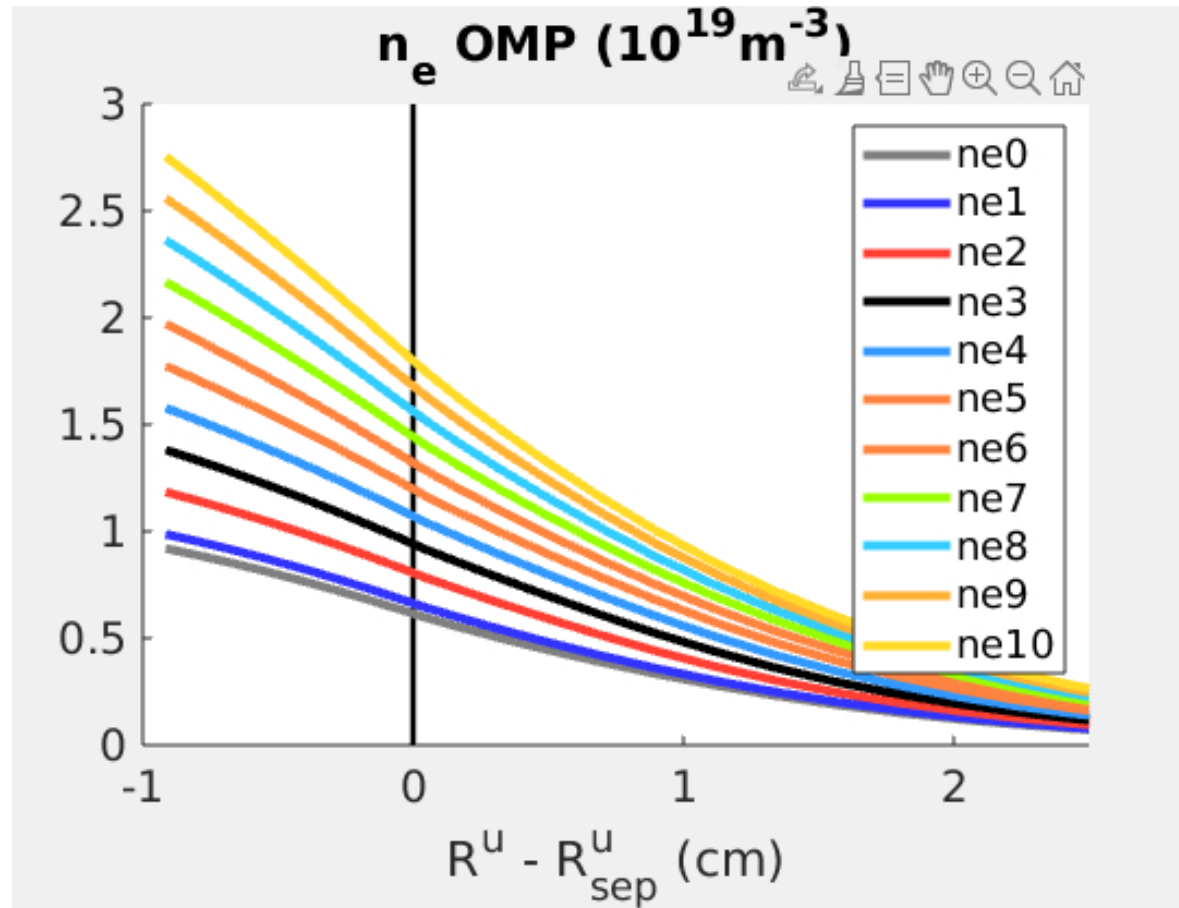
Single exponential at Divertor entrance

- $q(r) \cong \frac{1}{2} q_0 \exp\left(\left(\frac{S}{2\lambda_q}\right)^2 - \frac{r}{\lambda_q}\right) \text{erfc}\left(\left(\frac{S}{2\lambda_q}\right) - \frac{r}{\lambda_q}\right) + q_{BG}$

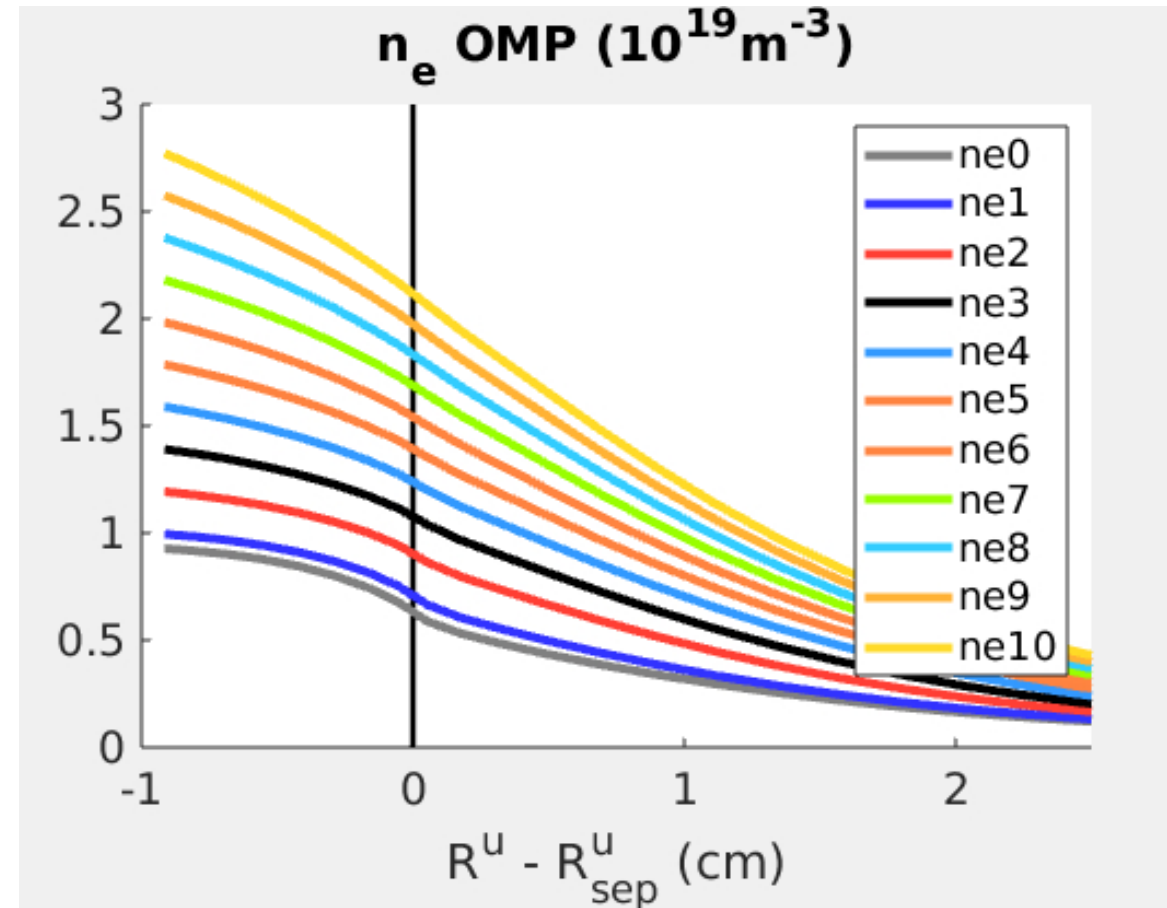
- $q(r) \cong q_0 \exp\left(\frac{-r}{\lambda_q}\right)$



Evolution OMP profiles – density scan

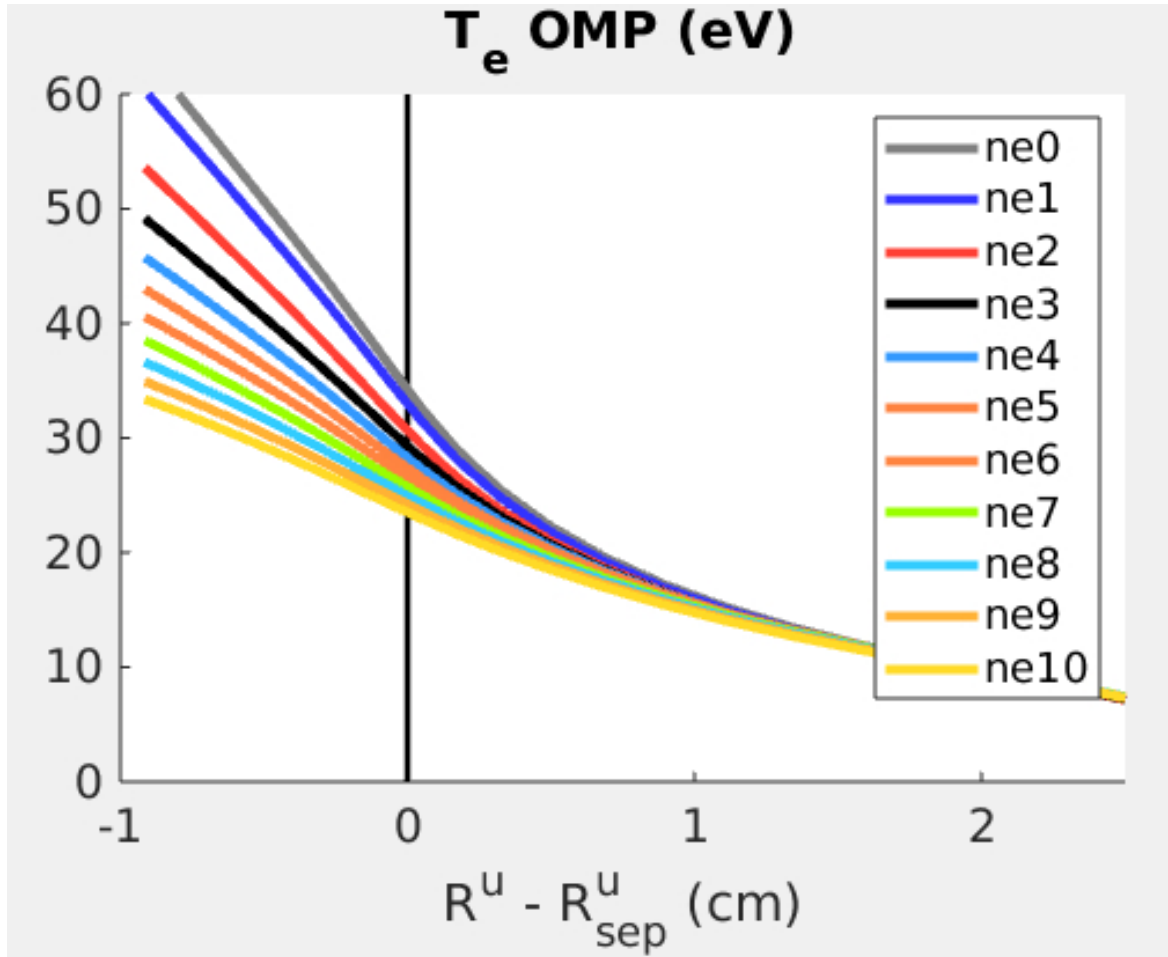


Standard model

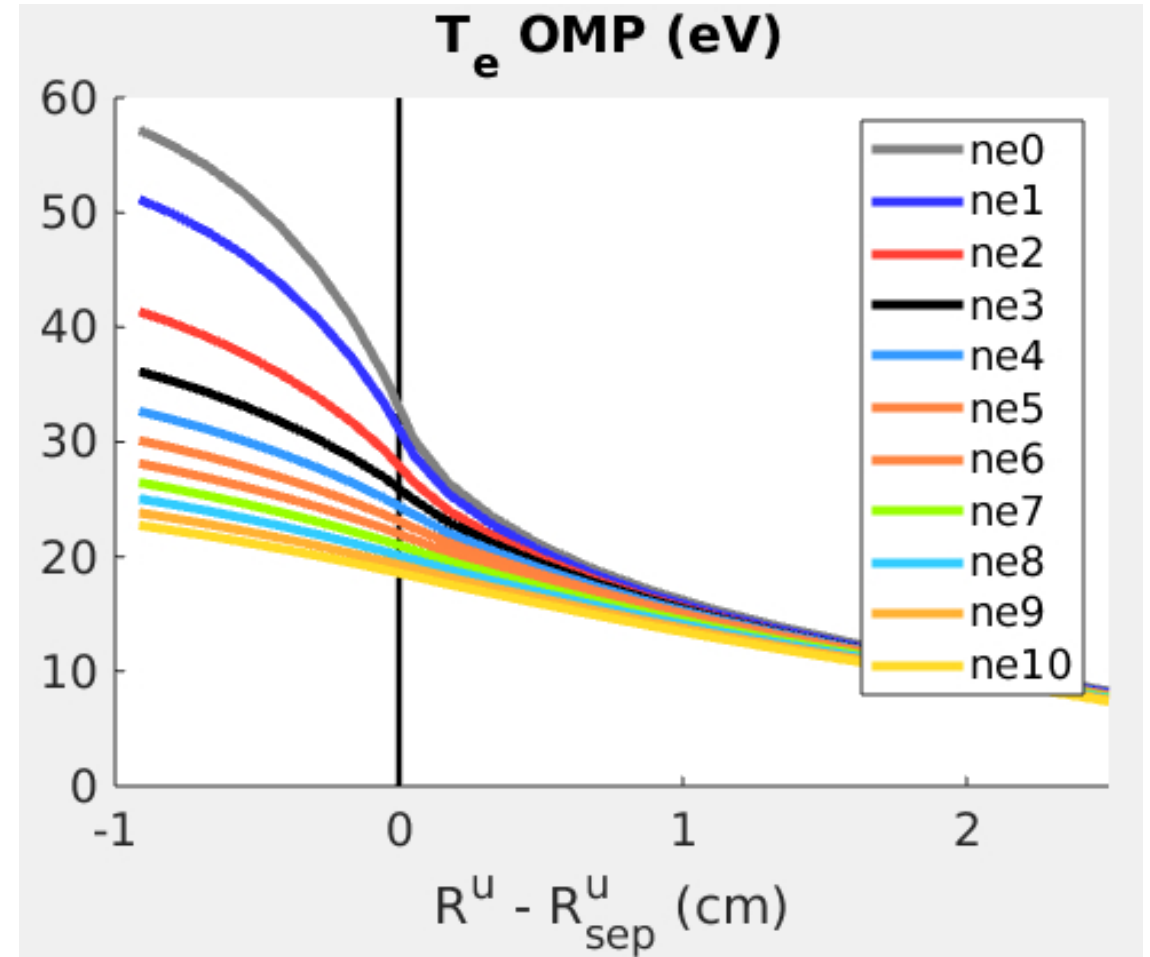


$\kappa_{\perp}$ model

Evolution OMP profiles – density scan

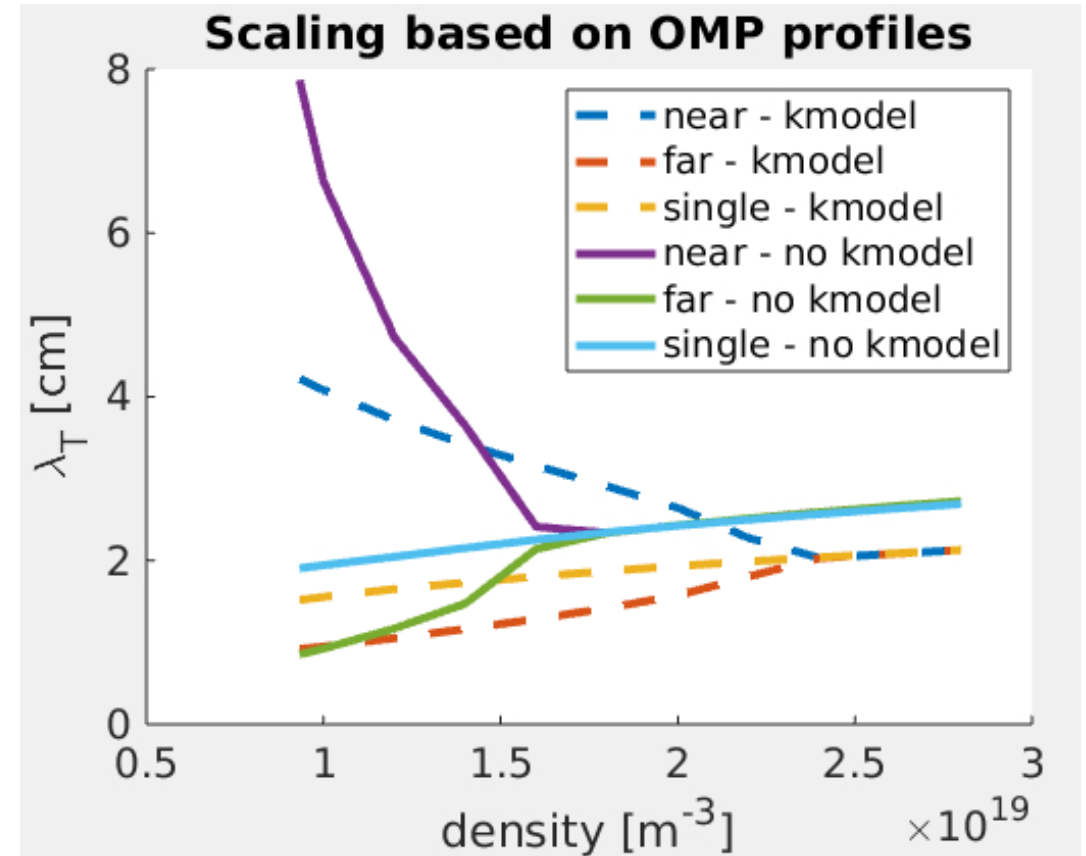
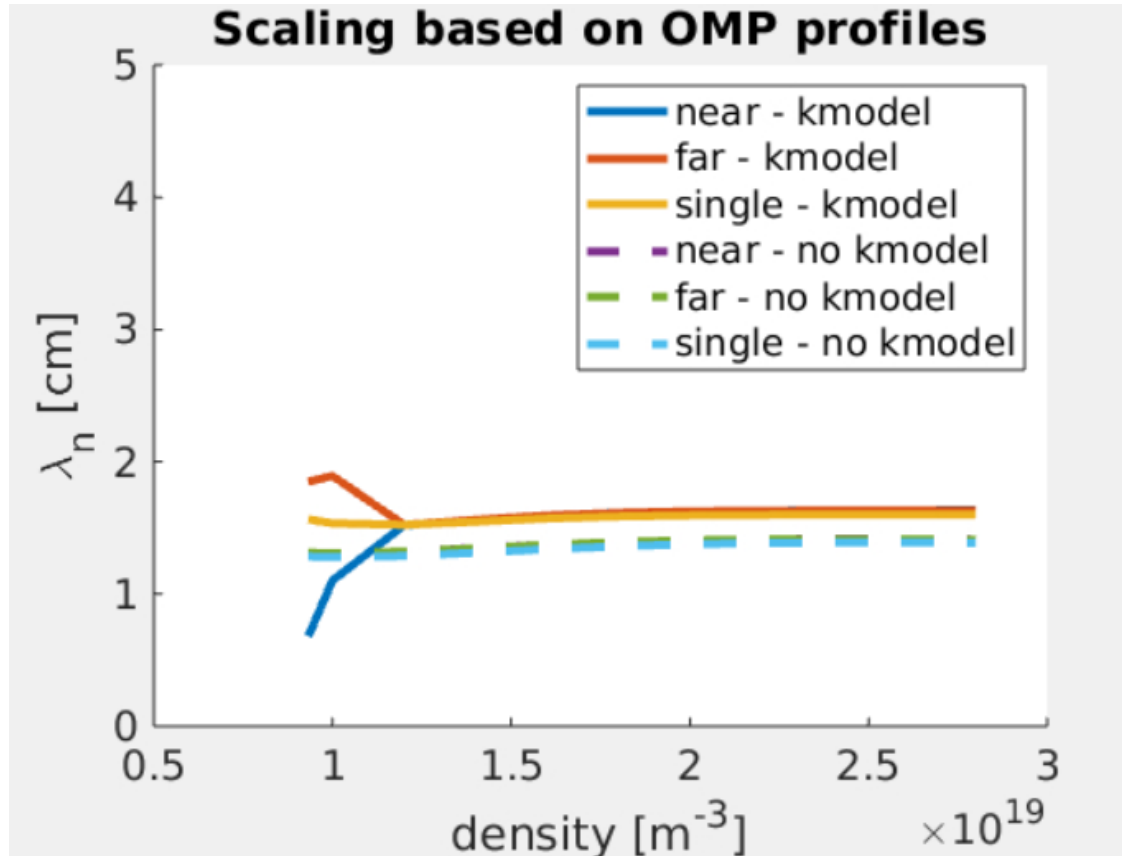


Standard model



$\kappa_{\perp}$ model

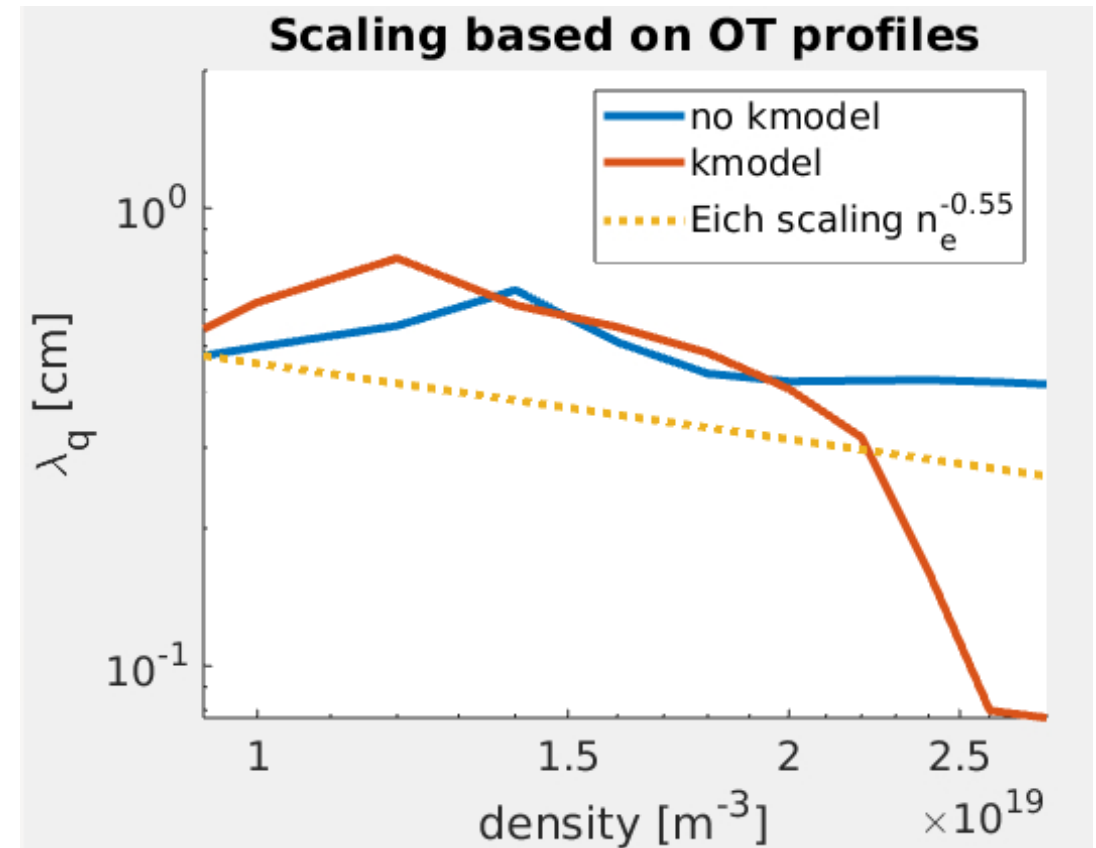
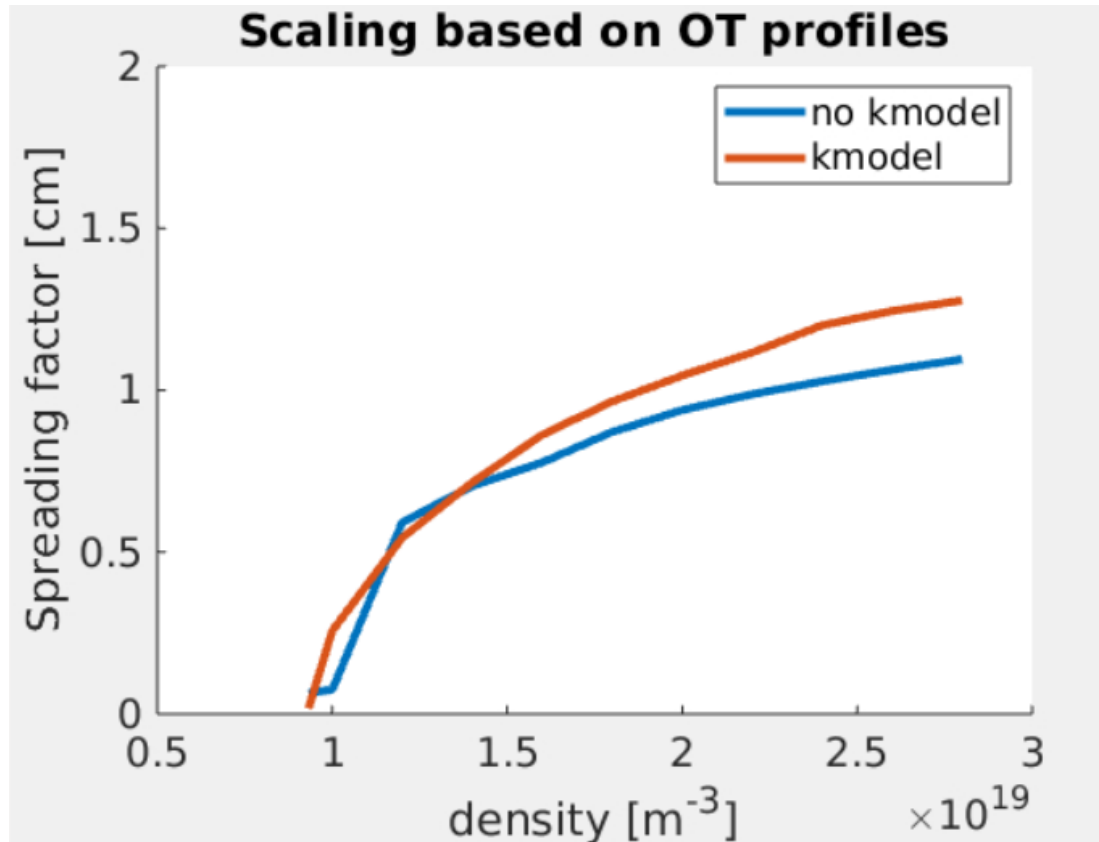
Scaling λ_n and λ_T based on OMP profiles



⇒ Suggests λ_q increasing with density based on OMP profiles?

⇒ Bifurcation at low densities with $\kappa_{\perp}$ model due to profile steepening (flow shear effect)

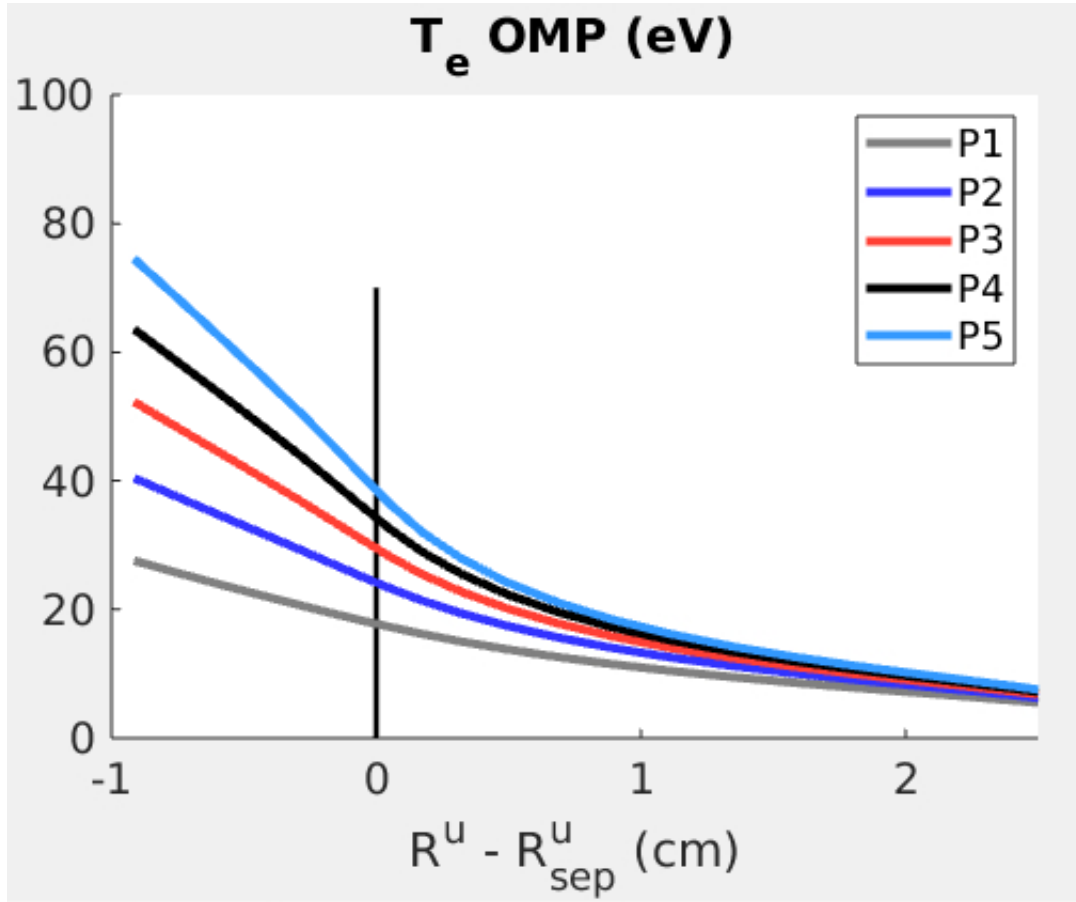
λ_q and spreading factor based on OT profiles



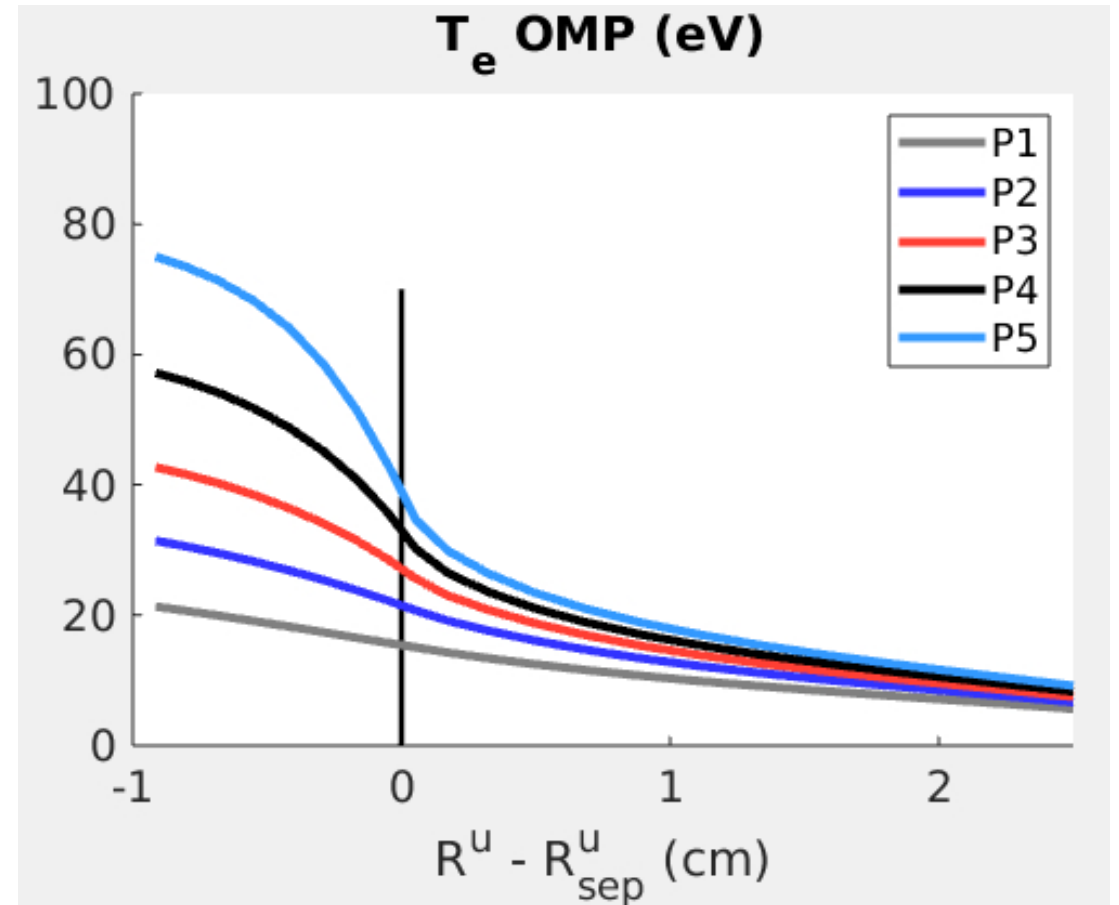
$\Rightarrow \lambda_q$ decreasing (or insensitive for standard model?) to density based on OT profiles?

$\Rightarrow$ Clear increase in spreading factor with density (higher recycling/detached)

Evolution OMP profiles – power scan

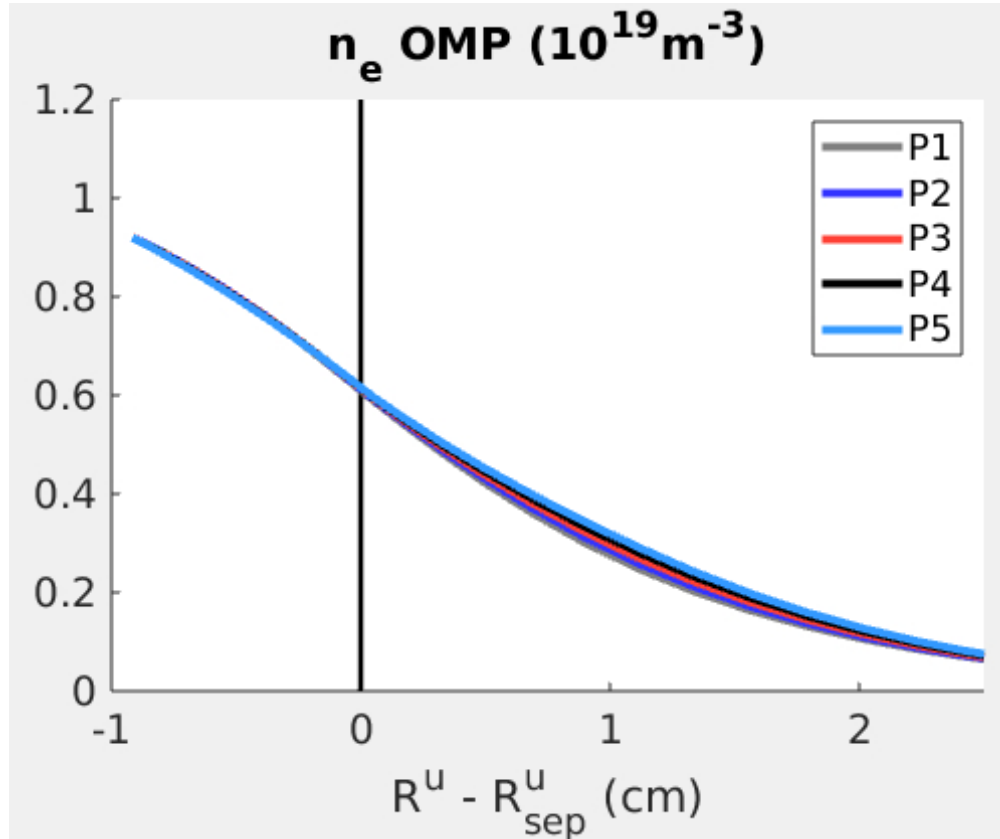


Standard model

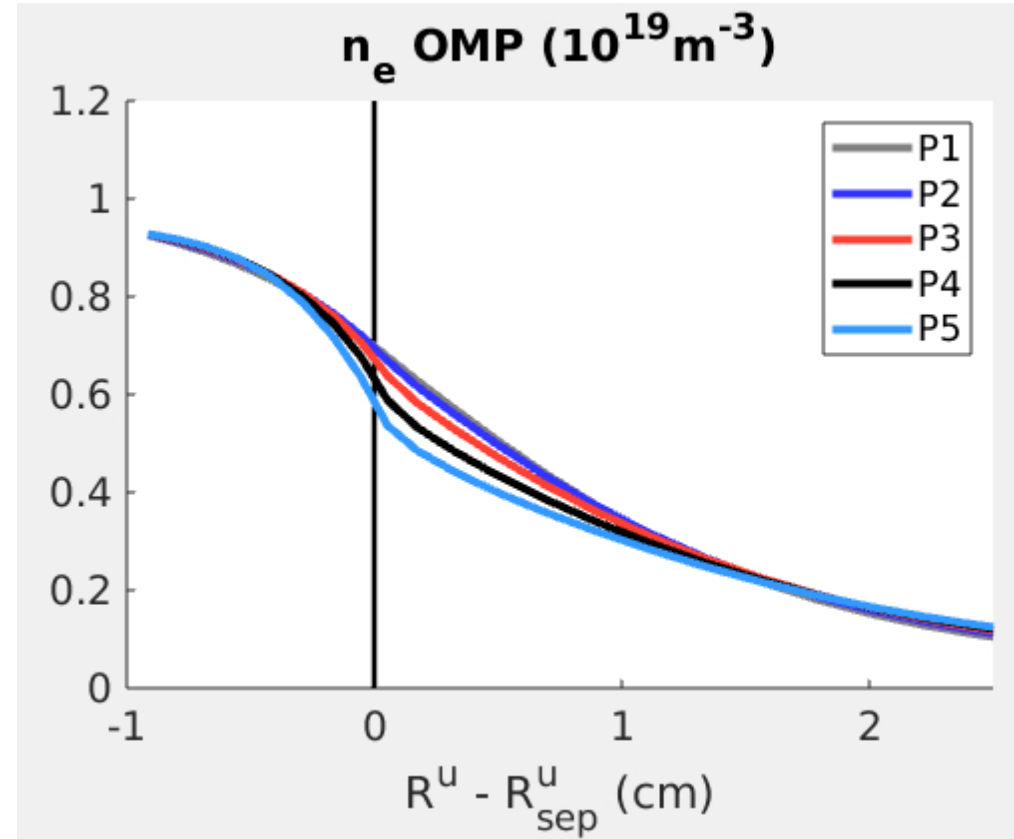


$\kappa_{\perp}$ model

Evolution OMP profiles – power scan

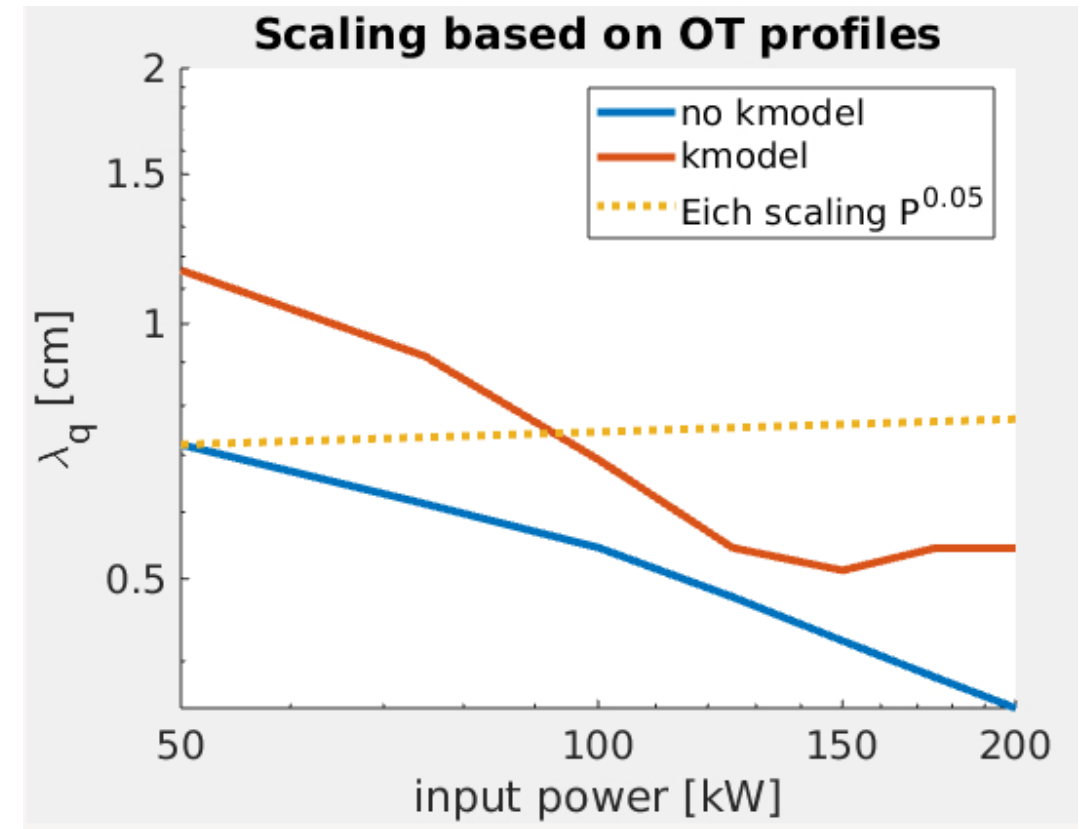
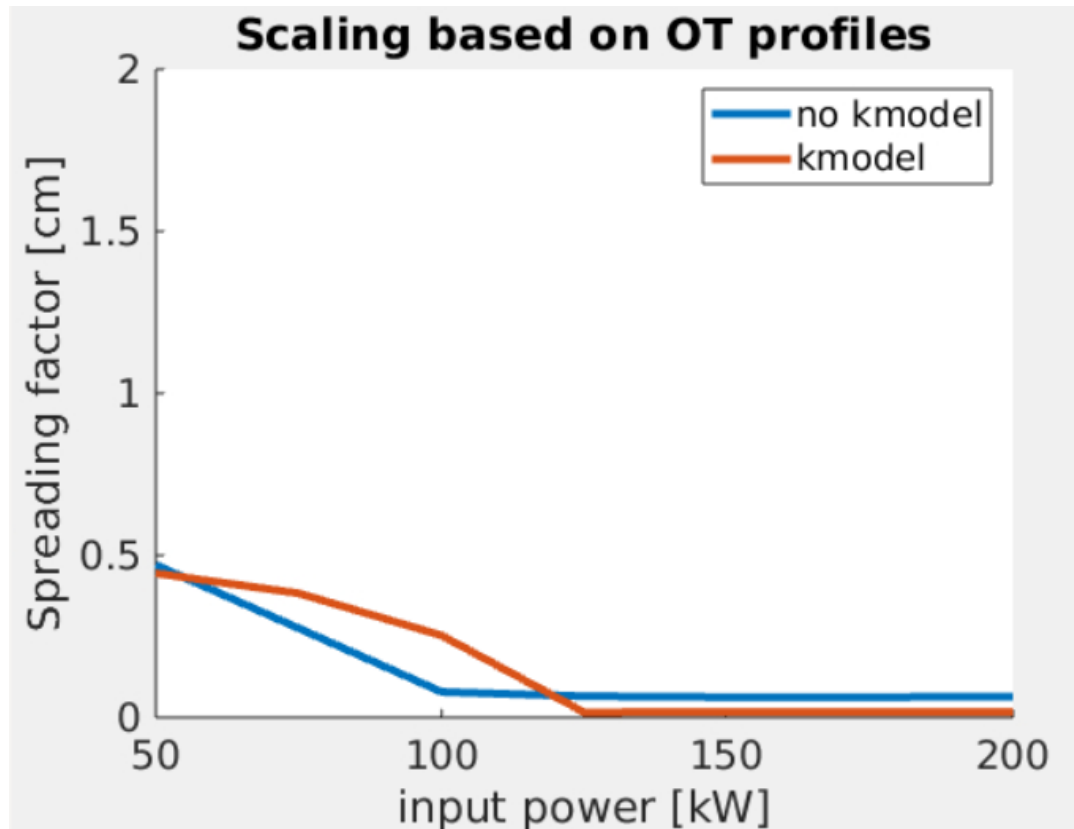


Standard model



$\kappa_{\perp}$ model

λ_q scaling: k-model vs constant transport



- ⇒ λ_q decreasing with power? But need to deconvolute with increasing separatrix density!
- ⇒ Clear increase in spreading factor towards lower power (higher recycling/detached)

Summary decay lengths

- Picture not yet very conclusive
 - λ_q scaling based on target profiles not necessarily consistent with OMP-based scaling
 - Decreasing λ_q with density broadly captured,
 - A stronger (and negative) sensitivity observed with P_{SOL} in the simulations (but still combined with decreasing separatrix density in the scan => to be deconvoluted)
- Simulation-based analysis does enable analysis of the decay lengths into higher recycling/detached regimes compared to experiment