



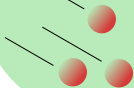
**CHALMERS**  
UNIVERSITY OF TECHNOLOGY



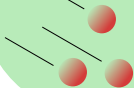
**A fluid-kinetic framework for disruption runaway  
electron simulations**

**M. Hoppe, O. Embreus and T. Fülöp**

# Runaway electron dynamics

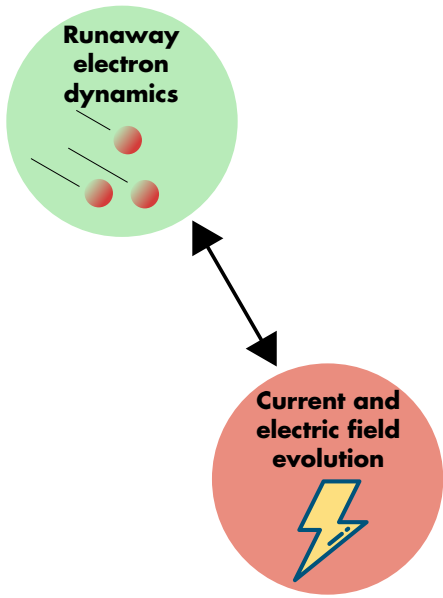


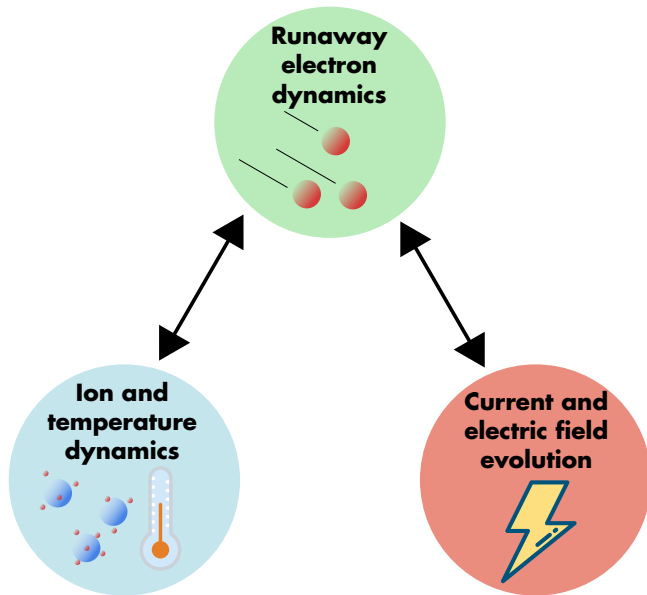
**Runaway  
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dynamics**

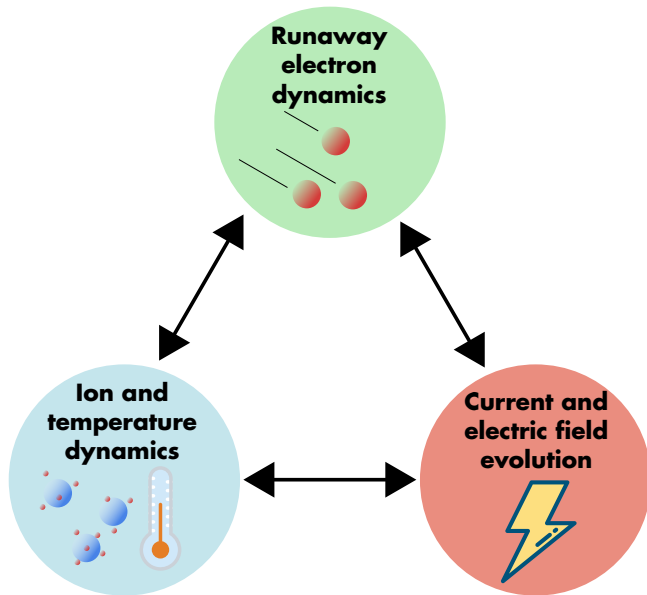


**Current and  
electric field  
evolution**







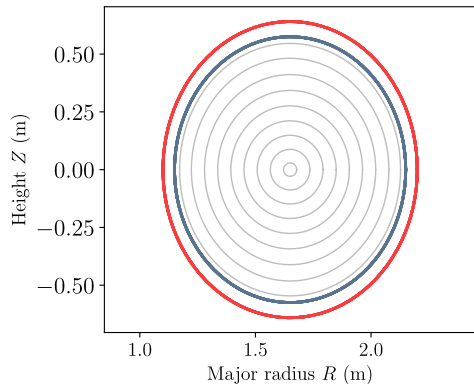


**DREAM**



1. Brief overview
2. The models
  - ▶ Electrons
  - ▶ Electric field ( $E$ ), poloidal flux ( $\psi$ ) and current density ( $j$ )
  - ▶ Ion and temperature dynamics
3. Model comparison





- 1D transport model in tokamak geometry for
  - ▶ Electric field  $E(r)$
  - ▶ Electron temperature  $T_e(r)$
  - ▶ Plasma current density  $j(r)$
- Fluid or kinetic (1D2P; bounce-averaged) electrons
  - ▶ Accurate treatment of transient runaway electron generation (e.g. hot-tail)
- Radial transport of electrons and heat

# **Electron models**

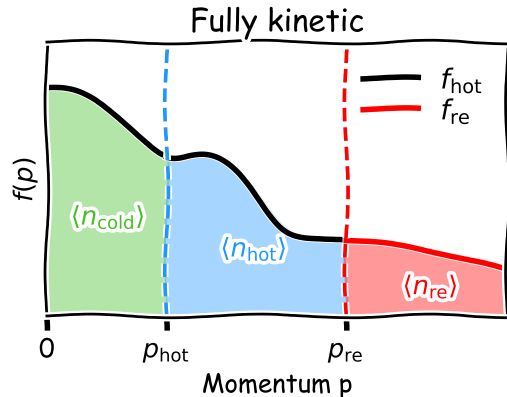
DREAM separates electrons into **three populations** based on their energy:

- **Cold:**  $p \sim p_{\text{thermal}}$
- **Hot:**  $p_{\text{thermal}} < p < p_c$
- **Runaway:**  $p > p_c$

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Electrons in each of these regions can be modelled either by solving the **kinetic equation** or be treated as a **fluid**



### **Fluid mode** (one cold+hot fluid)

Both cold and hot electrons are represented by density  $\langle n_{\text{cold}} \rangle$  and temperature

$T_{\text{cold}}$

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<sup>1</sup>Aleynikov & Breizman, [NF 57 \(2017\)](#)

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### **Superthermal mode** (“Aleynikov & Breizman mode”) <sup>1</sup>

- **Cold:**  $\langle n_{\text{cold}} \rangle$  and  $T_{\text{cold}}$  (starting at  $\langle n_{\text{cold}} \rangle = 0, T_{\text{cold}} = 0$ )
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**Isotropic mode** (reduced “Aleynikov & Breizman mode”)

Same as the *superthermal* mode, but  $f_{\text{hot}}$  solved for using an angle-averaged equation

(Why? Performance of fluid mode, but with accurate hot-tail!)

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<sup>1</sup>Aleynikov & Breizman, [NF 57 \(2017\)](#)



Runaway electrons are either treated as (independently of cold/hot electrons)

## Fluid

$$\frac{\partial \langle n_{re} \rangle}{\partial t} = \underbrace{F_{hot}}_{\text{flux from } f_{hot}} + \gamma_{\text{Dreicer}} + \underbrace{\Gamma_{ava} \langle n_{re} \rangle}_{\text{avalanche}} + \underbrace{\gamma_T + \gamma_{\text{compton}} + \dots}_{\text{other runaway sources}}$$

$$+ \underbrace{\frac{1}{V'} \frac{\partial}{\partial r} \left[ V' \left( -A_r \langle n_{re} \rangle + D_{rr} \frac{\partial \langle n_{re} \rangle}{\partial r} \right) \right]}_{\text{radial transport}}$$

## Kinetically

$$\frac{\partial f_{re}}{\partial t} + \underbrace{e \{E_{\parallel} \xi\}}_{E \text{ acceleration}} \frac{\partial f_{re}}{\partial p_{\parallel}} + \underbrace{\{\mathbf{F}_{\text{synch}}\} \cdot \frac{\partial f_{re}}{\partial \mathbf{p}}}_{\text{synchrotron radiation}} = \underbrace{C[f_{re}]}_{\text{collisions}} + \underbrace{S_{re}}_{\text{fluid RE sources}}$$

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# **Background plasma**

**Current density** (Ampère's law)

$$2\pi\mu_0 \langle \mathbf{B} \cdot \nabla \phi \rangle \frac{j_{\text{tot}}}{B} = \frac{1}{V'} \frac{\partial}{\partial r} \left[ V' \left\langle \frac{|\nabla r|^2}{R^2} \right\rangle \frac{\partial \psi}{\partial r} \right].$$

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**Electric field and poloidal flux**

$$\frac{\partial \psi}{\partial t} = -V_{\text{loop}},$$
$$2\pi \frac{\langle \mathbf{E} \cdot \mathbf{B} \rangle}{\langle \mathbf{B} \cdot \nabla \phi \rangle} = V_{\text{loop}}.$$

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**Wall coupling**

(with wall time  $\tau_{\text{wall}} = L_{\text{ext}}/R_{\text{wall}}$  as free parameter)

$$V_{\text{loop}}^{(\text{wall})} = R_{\text{wall}} I_{\text{wall}},$$
$$\psi_{\text{wall}} = -L_{\text{ext}} (I_{\text{p}} + I_{\text{wall}}).$$

## Ion (charge state) densities

$$\frac{\partial n_i^{(j)}}{\partial t} = \underbrace{\left( I_i^{(j-1)} \langle n_{\text{cold}} \rangle + \langle \sigma_{\text{ion},i}^{(j-1)} v \rangle \right) n_i^{(j-1)} - \left( I_i^{(j)} \langle n_{\text{cold}} \rangle + \langle \sigma_{\text{ion},i}^{(j)} v \rangle \right) n_i^{(j)}}_{\text{ionization}} + \underbrace{R_i^{(j+1)} \langle n_{\text{cold}} \rangle n_i^{(j+1)} - R_i^{(j)} \langle n_{\text{cold}} \rangle n_i^{(j)}}_{\text{recombination}}.$$

Ionization  $I_i^j$  and recombination  $R_i^j$  coefficients taken from ADAS. Kinetic ionization rates are calculated from

$$\langle \sigma_{\text{ion},i}^{(j)} v \rangle = \int dp \int_{-1}^1 d\xi_0 \frac{\mathcal{V}'}{V'} v \sigma_{\text{ion},i}^{(j)} f(r, p, \xi_0)$$

according to [Garland *et al*, [PoP 27 \(2020\)](#)]

**(Cold) electron temperature**

$$W_{\text{cold}} = \frac{3}{2} \langle n_{\text{cold}} \rangle T_{\text{cold}}$$

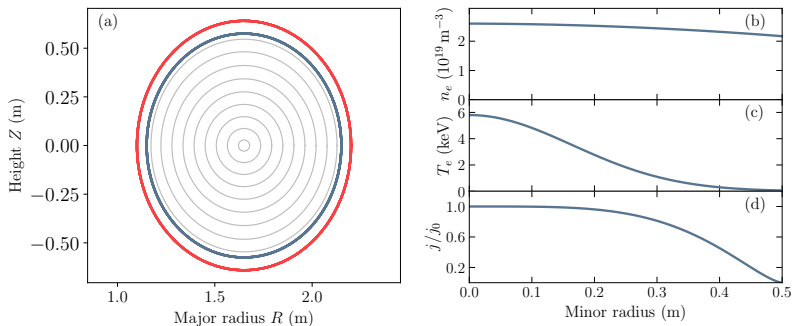
**Energy balance**

$$\begin{aligned} \frac{\partial W_{\text{cold}}}{\partial t} = & \underbrace{\frac{j\Omega}{B} \langle \mathbf{E} \cdot \mathbf{B} \rangle}_{\text{ohmic heating}} - \underbrace{\langle n_{\text{cold}} \rangle \sum_i \sum_{j=0}^{Z_i-1} n_i^{(j)} L_i^{(j)}}_{\text{line radiation}} + \underbrace{\langle Q_c \rangle}_{\text{coll. heat transfer}} \\ & + \underbrace{\frac{1}{V'} \frac{\partial}{\partial r} \left[ V' \left( A W_{\text{cold}} + D \frac{\partial W_{\text{cold}}}{\partial r} \right) \right]}_{\text{radial transport}} \end{aligned}$$

The collisional heat transfer  $\langle Q_c \rangle$  includes both e-e and e-i contributions.

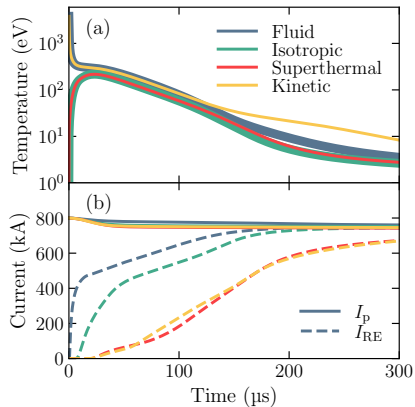


# **Comparison of electron models**

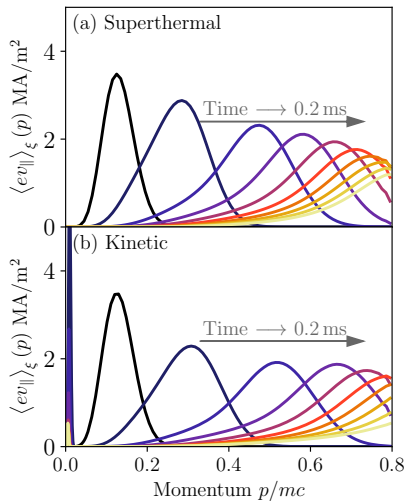
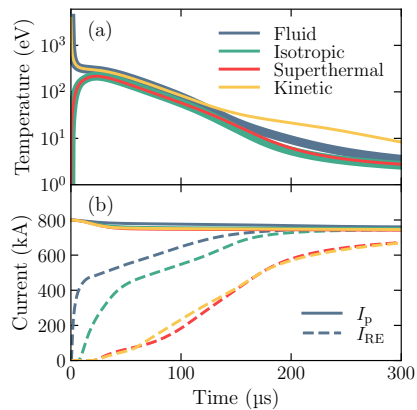


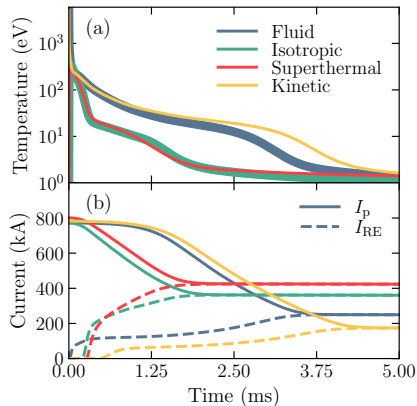
- Major radius  $R_m = 1.65$  m
- Plasma radius  $a = 0.5$  m
- Wall radius  $b = 0.55$  m
- Max. elongation  $\kappa_{\max} = 1.15$

- $B_0 = 2.5$  T
- $n_{e,0} = 2.6 \times 10^{19} \text{ m}^{-3}$
- $T_{e,0} = 5.8$  keV
- $I_p = 800$  kA

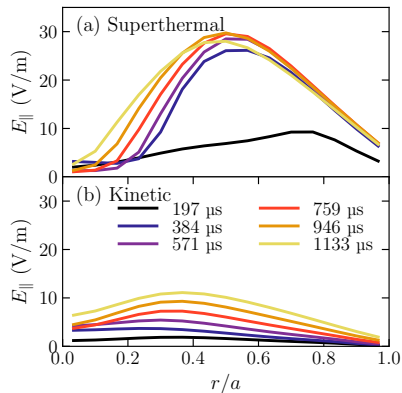
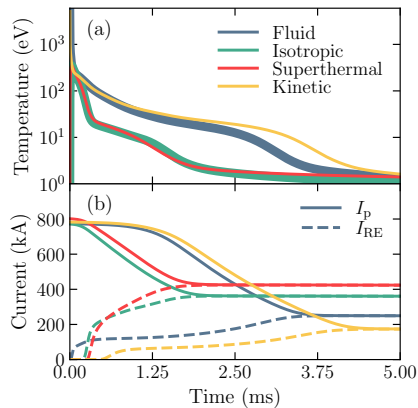


- Mixed D and Ar injection
- $n_D = n_{Ar} = 2.6 \times 10^{19} \text{ m}^{-3}$
- Almost full conversion  $I_p \rightarrow I_{re}$
- Good agreement between electron models





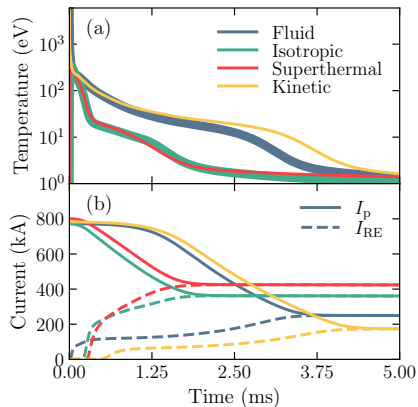
- More D, less Ar
- $n_D = 5.2 \times 10^{20} \text{ m}^{-3}$ ,  $n_{Ar} = 5.2 \times 10^{18} \text{ m}^{-3}$
- $I_{re}$  between 200-400 kA
- Major differences between electron models:  
due to the very different starting  
temperatures + less hot-tail



DREAM is a fluid-kinetic (1D2P) framework for self-consistent runaway electron simulation.

### What's new with DREAM?

- Background plasma and runaway evolution with **kinetic hot-tail**
- Radial transport of electrons
- Kinetic ionization
- Tokamak geometry (bounce and flux-surface averaged)



On arXiv: [2103.16457](https://arxiv.org/abs/2103.16457)

<https://github.com/chalmersplasmatheory/DREAM>

**Benchmark**



Comparison to Fig. 4 of GO simulations in ITER-like scenario [Vallhagen *et al*, [JPP 86 \(2020\)](#)].

- Mixed D and Ne injections
- Good agreement in  $I_p$  and  $j_{re,max}(r)$
- Some models have been upgraded in DREAM  $\implies$  agreement not perfect

